Propositional Logic (cont.)
&
Propositional Equivalences (skim)

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Review

- Proposition and truth values
- Boolean logic: $\wedge, \vee, \neg, \oplus$
- Truth tables

Conditional

- Conditional: $p \rightarrow q$ ("if p then q")
- p: hypothesis, q: conclusion
- $p \rightarrow q$ is true when both p and q are true and when p is false

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional - 2

- $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent (Useful!)

$$p \rightarrow q \equiv \neg p \vee q$$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Conditional - 3

- Conditional: $p \rightarrow q$

  If you are a CS student, then you take CSE1019.

- Contrapositive of $p \rightarrow q$: $\neg q \rightarrow \neg p$

  If you do not take CSE1019, then you are not a CS student.

- Converse of $p \rightarrow q$: $q \rightarrow p$

  If you take CSE1019, then you are a CS student.

- Inverse of $p \rightarrow q$: $\neg p \rightarrow \neg q$

  If you are not a CS student, then you do not take CSE1019.

Conditional - 4
Logical Equivalence

		Conditional	Contrapositive	Converse	Inverse
p	q	$p \to q$	$\neg q \to \neg p$	$q \to p$	$\neg p \to \neg q$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Biconditional

- Biconditional: $p \leftrightarrow q$ ("p if and only if q")
  iff
- $p \leftrightarrow q$ is true if and only if p and q have same truth values
- Also defined as $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical operators (review)

- Negation

  $\neg p$ "not p"
- Conjunction

  $p \wedge q$ "p and q"
- Disjunction

  $p \vee q$ "p or q or both"
- Exclusive or

  $p \oplus q$ "p or q, but not both"
- Conditional statement

  $p \rightarrow q$ "if p then q"
- Biconditional statement

  $p \leftrightarrow q$ "p if and only if q"

Compound Propositions

- Example: $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
- Precedence order: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ (Overruled by parenthesis)

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Tautology

- A compound proposition that is always TRUE.
- Examples:
    - $p \vee T$
    - $p \vee \neg p$
    - $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

Propositional Equivalence

- Logical equivalence redefined: p,q are logical equivalent if $p \leftrightarrow q$ is a tautology, denoted by:

$$p \equiv q$$

Not a logical operator!!!!

  - $\Leftrightarrow$ is sometimes used instead of $\equiv$
  - Truth tables are the simplest way to prove such facts
  - We will learn other ways later

Manipulating Propositions

- Compound propositions can be simplified by using simple rules.
- Read page 24-25
- Some are obvious: Identity, Domination, Idempotent, double negation, commutativity, associativity, negation
- Less obvious: distributive, De Morgan's laws, Absorption

Distributive Laws

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Intuition (not a proof!) – For the LHS to be true: p must be true and q or r must be true. This is the same as saying p and q must be true or p and r must be true.

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Intuition (less obvious) – For the LHS to be true: p must be true or both q and r must be true. This is the same as saying p or q must be true and p or r must be true.

Proof: use truth tables.

De Morgan's Laws

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Intuition - For the LHS to be true: neither p nor q can be true. This is the same as saying p and q must be false.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Intuition - For the LHS to be true: $p \wedge q$ must be false.This is the same as saying p or q must be false.

Proof: use truth tables.

De Morgan's Laws - 2

$$\neg(p_1 \vee p_2 \ldots \vee p_n) \equiv \neg p_1 \wedge \neg p_2 \ldots \wedge \neg p_n$$

$$\neg(p_1 \wedge p_2 \ldots \wedge p_n) \equiv \neg p_1 \vee \neg p_2 \ldots \vee \neg p_n$$

Example

Is $p\vee(\neg(p\wedge q))$ a tautology?

Solution:

$p\vee(\neg(p\wedge q)) \equiv$

$p\vee(\neg p\vee\neg q) \equiv$

$(p\vee\neg p)\vee\neg q \equiv$

$T \vee\neg q \equiv$

T

Readings and notes

- Read Section 1.1 and 1.2
- Master the rationale behind the definition of conditionals
- Practice proving logical equivalence by manipulating compound propositions
- Recommended exercises: 1.1:5,9,19,23,27,44,49,55-59; 1.2:1,3,4,5,7