Assignment 2
Due: October 4, 9:30 am

1. (8 points) Let F(x,y) be the statement "x can fool y," where the domain consists of all people in the world. Use F(x,y), quantifiers and logical operators to express each of the following statements:

   (a) Everybody can fool Tom.
   (b) Not everybody can fool Tom.
   (c) Nobody can fool Tom.
   (d) Everybody can fool somebody.
   (e) There is no one who can fool everybody

   Translate the following formulas into English sentences:
   (f) $\neg\exists x(F(x,Tom)\wedge F(x,Jerry))$
   (g) $\forall y\exists x F(x,y)$
   (h) $\neg\exists x F(x,x)$

2. (6 points) Let Q(x,y) be the statement "x+y = x-y", where the domain for both variables consists of all integers. Determine the truth values of each of the following statement:
   (a) Q(256, 128)
   (b) $\forall x Q(x, 0)$
   (c) $\forall x\exists y Q(x,y)$
   (d) $\exists x\forall y Q(x,y)$
   (e) $\exists y\forall x Q(x,y)$
   (f) $\forall x\forall y\neg Q(x,y)$

3. (4 points) Rewrite these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
   a. $\neg\exists y(Q(y)\rightarrow\forall x R(x,y))$
   b. $\neg\forall x(\exists y\forall z P(x,y,z)\vee\exists z\forall y P(x,y,z))$

4. (4 points) Prove that if n is an integer and 13n+2 is even, then n is even using
   (a) a proof by contraposition
   (b) a proof by contradiction