

Objective

Our goal is to find the boundary of salient objects in images of natural scenes using the Minimal Path algorithm[1].

Minimal Path Algorithm

The minimal path algorithm is based on

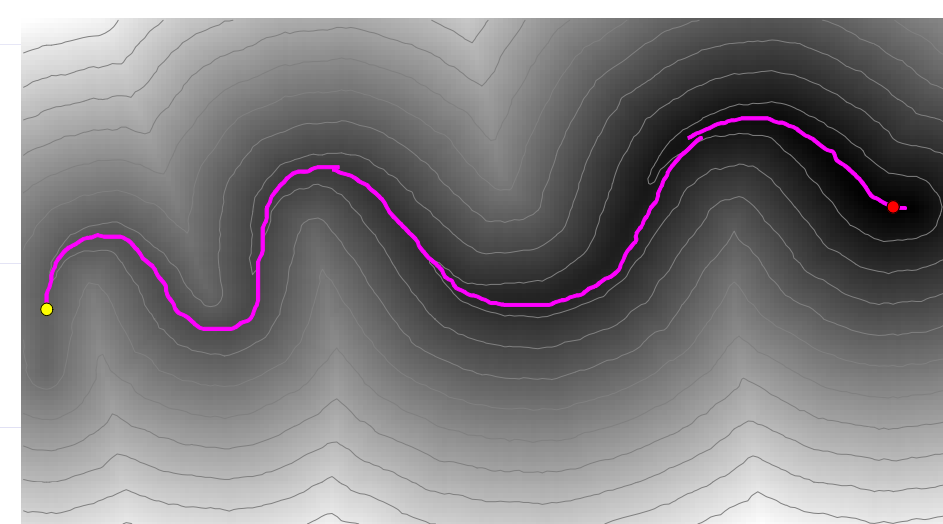
- A special case of Level Set methods[2] where speed (cost) is positive, and
- A fast computation method called the Fast Marching algorithm[3]

Given two points and a cost function, called the **potential** function **P**, the algorithm returns the path with minimum cost (shortest path). Compared with other shortest path methods, this approach does not suffer from digitization bias, and is guaranteed to converge to the true solution as the image grid is refined.

Surface of minimal action **U** is defined as the minimal energy integrated along a path from a starting point p_0 and any other point p .

$$U(p) = \inf_{A_{p_0,p}} E(C) = \inf_{A_{p_0,p}} \left\{ \int_{\Omega} \tilde{P}(C(s)) ds \right\}$$

where $A_{p_0,p}$ is the set of all paths between p_0 and p and $\tilde{P}$ is the smooth potential function.



Challenges

In order to use the Minimal Path (MP) algorithm for contour extraction, the following inputs must be provided to the MP algorithm:

- A potential function, which is defined everywhere on the image and has lower values on the object contours and high values elsewhere.
- A minimum of three key points on the object contour to obtain three partial contours that define the whole contour. These points are ideally equally distant on the object contour.

The following ideas can be helpful in overcoming the above challenges:

- Using Martin's Probability of boundary (Pb) map [4]
- Using convexity prior on paths
- Using contour hypotheses obtained from other segmentation methods, for example [5,6]

Potential Function

The potential function **P** is minimized along curves obtained by MP [1]. If **P** is a binary image of edge points representing a simple incomplete shape having small values along the edges and high values at the background, it has been shown that MP is sufficient to provide the complete contour of the shape [1]. Yet such potential function is not producible for images of natural scenes. We have tried the following to obtain a suitable potential function.

Pb map

In an effort to detect and localize boundaries in natural scenes, Martin [4] used the local color, brightness, and texture features to obtain the posterior probability of boundary (Pb) at each image location. These values provide a Pb map which can be used to design the potential function.



Sample image from BSD [7]



Pb map

Blurring Pb

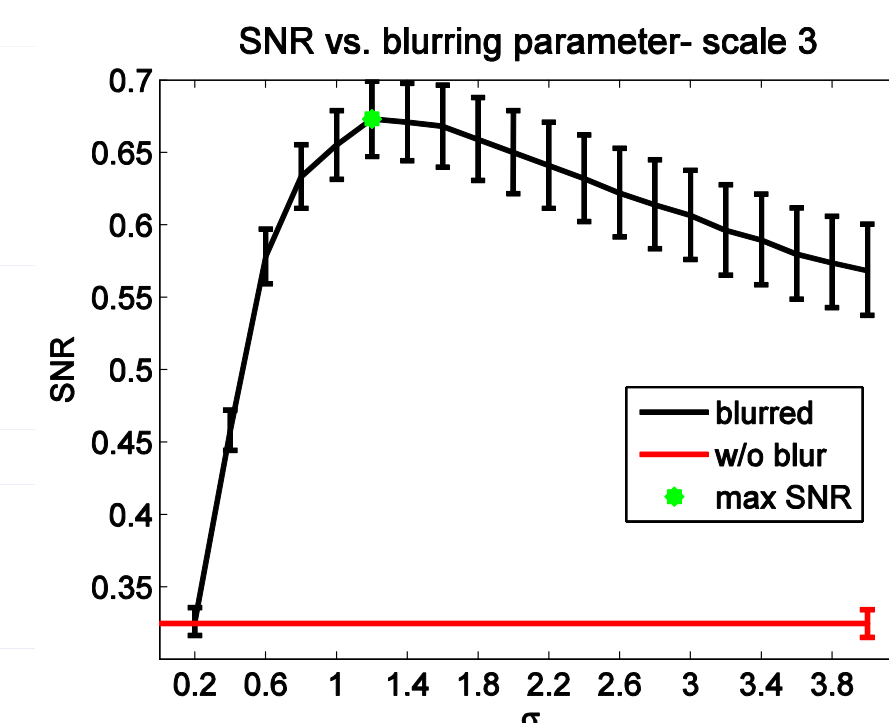
The Pb values are highly localized and therefore not suitable for use as a potential function. Discontinuities prevent MP from finding a good path. A smoother function is more desirable for which the values are minimum on the boundary location, but increase *gradually* as we move away from the boundary. A Gaussian kernel can be used to smooth Pb.

Optimizing Blur Parameters

In order to optimize the blur parameter σ for the Gaussian blurring kernel, the following signal to noise ratio was maximized:

$$SNR(\sigma) = \frac{mean(Pb_{on}) - mean(Pb_{off})}{\sqrt{var(Pb_{on}) - var(Pb_{off})}}$$

where Pb_{on} denotes the Pb value for pixels on the ground truth contours and Pb_{off} denotes Pb values for pixels off the ground truth contours after blurring Pb. We used the object contours (selected by 7 subjects in our lab) for 200 training images from BSD [7] as the ground truth dataset. The parameter σ was varied in [0.2 4] interval. This was done for scales 2 to 5 of a multi-scale pyramid which is equivalent to half the size of original images in BSD to 1/16 of the original size. The mean value across scales was selected as the optimum blurring parameter ($\sigma = 1.65$). The following Figure shows SNR vs. σ for scale 3 as an example.



SNR vs. σ the blurring parameter

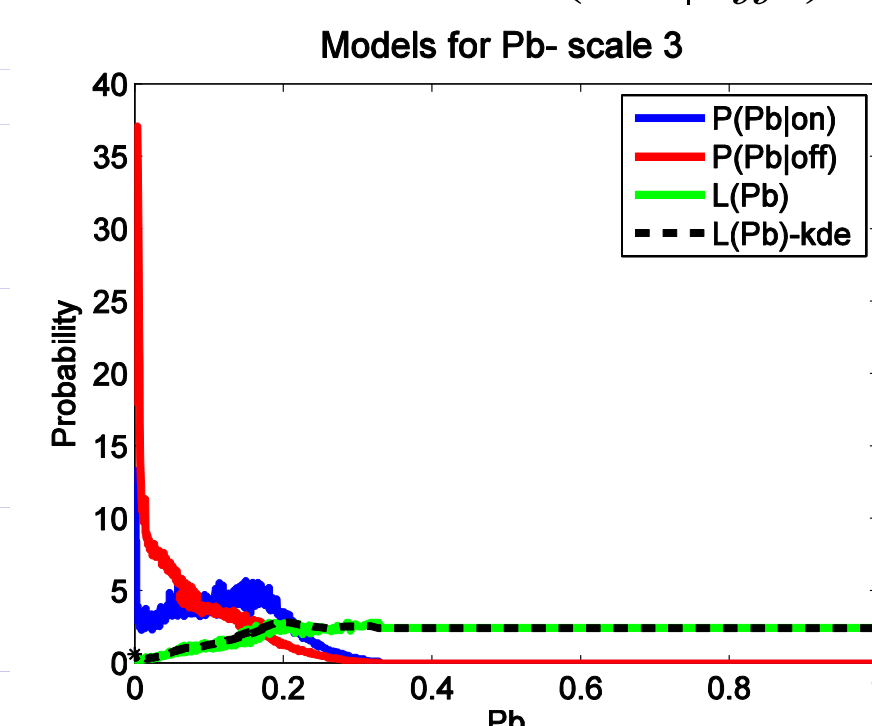


Blurred Pb map

Learning Likelihood of Pb

The likelihood ratio $L(Pb)$ indicates how likely a certain Pb value belongs to a pixel on the object boundary. This ratio can be used to define the potential function and is defined as probability of a (blurred) Pb value belonging to a pixel on an object contour to probability of it belonging to any pixel in the image. The probability distribution function (PDF) of pixels on the ground truth contours (ON) and random pixels (OFF) can be learned using our training dataset to provide us with an estimate of L . We used a nonparametric approach and a kernel density estimation (kde) to model the PDFs. These models are shown for scale 3 in the following Figure.

$$L(Pb) = \frac{P(Pb|on)}{P(Pb|off)}$$



PDFs for on and off distributions and the resulting likelihood model



Likelihood map

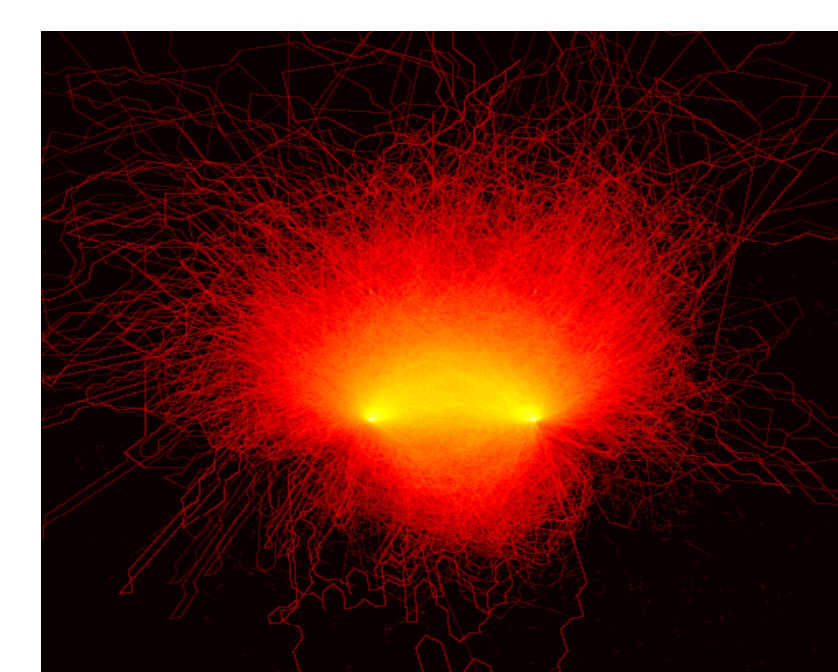
Convexity Prior

Preliminary experiments showed that constructing the potential function merely based on Pb will result in many self-intersecting contours. In order to solve this problem, we applied a convexity prior based on learning the location of curves between two points A and B on ground truth contours of 30 training images scaled based on distance $d(A,B)$ and oriented so as to make the third point C lie below line segment AB. The frequency F of curves traversing through a pixel can be used to determine the convexity prior.

The **potential function** at each pixel x is defined as:

$$P(x) = -\log(L(Pb(x))) - \log(1/F(x_{AB}))$$

Note that the potential function is defined piecewise for each pair of key points A and B.



Frequency map

Key Points

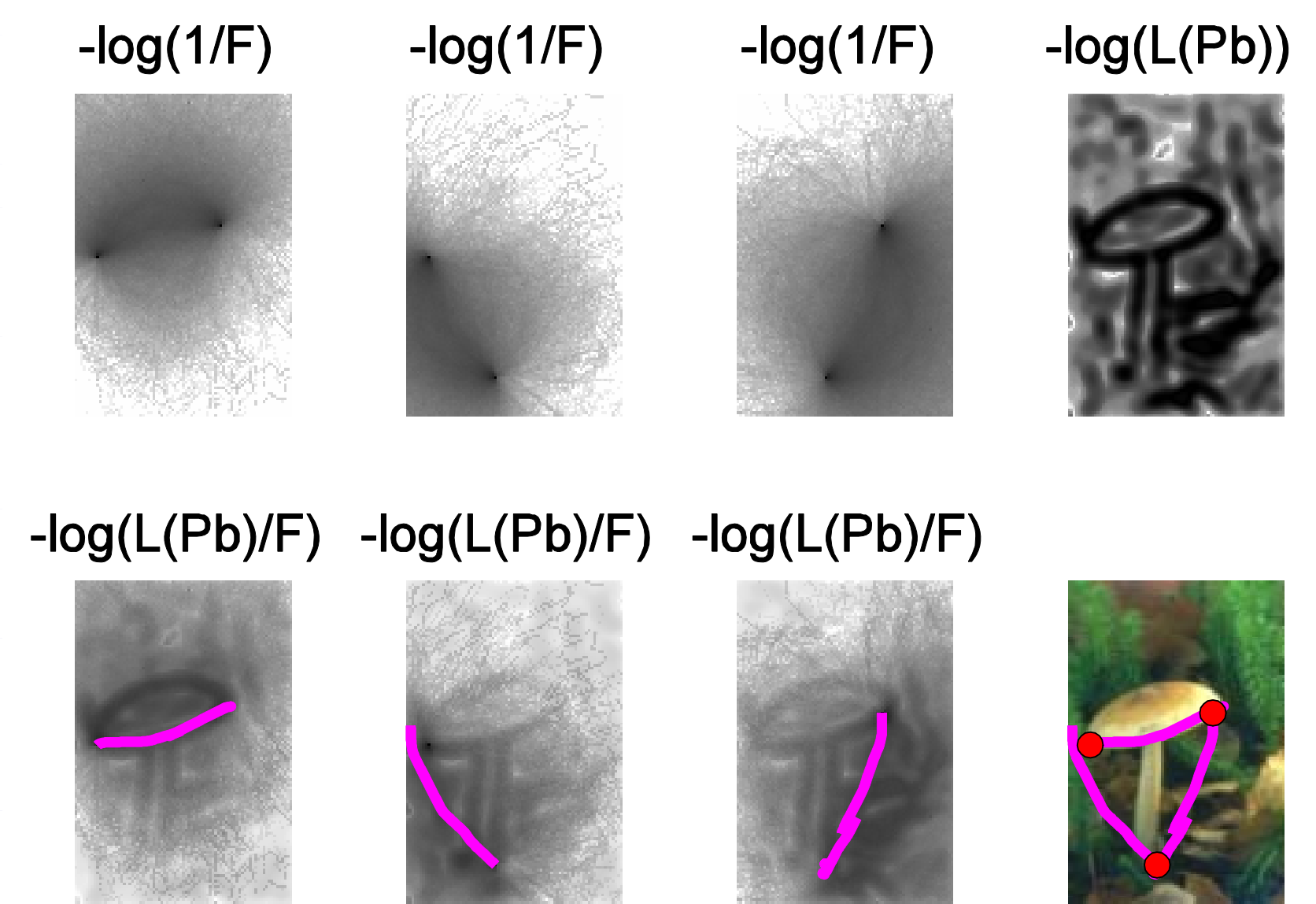
At this stage, we have selected key points as three equally distant points on ground truth contours, as well as on contour hypothesis provided by a contour grouping algorithm [5]. A set of possible triplets is obtained by considering rotations on the reference contours, leading to multiple contours obtained by above MP method. A further ranking step is required to sort the obtained contours and select the best. Future research is needed to automatically obtain key points.

Results

The piecewise potential functions are shown in the next Figure for a sample contour obtained for the sample image.

Using the regional error measure [8] between the region A defined by an algorithm boundary and region B defined by a ground truth boundary as

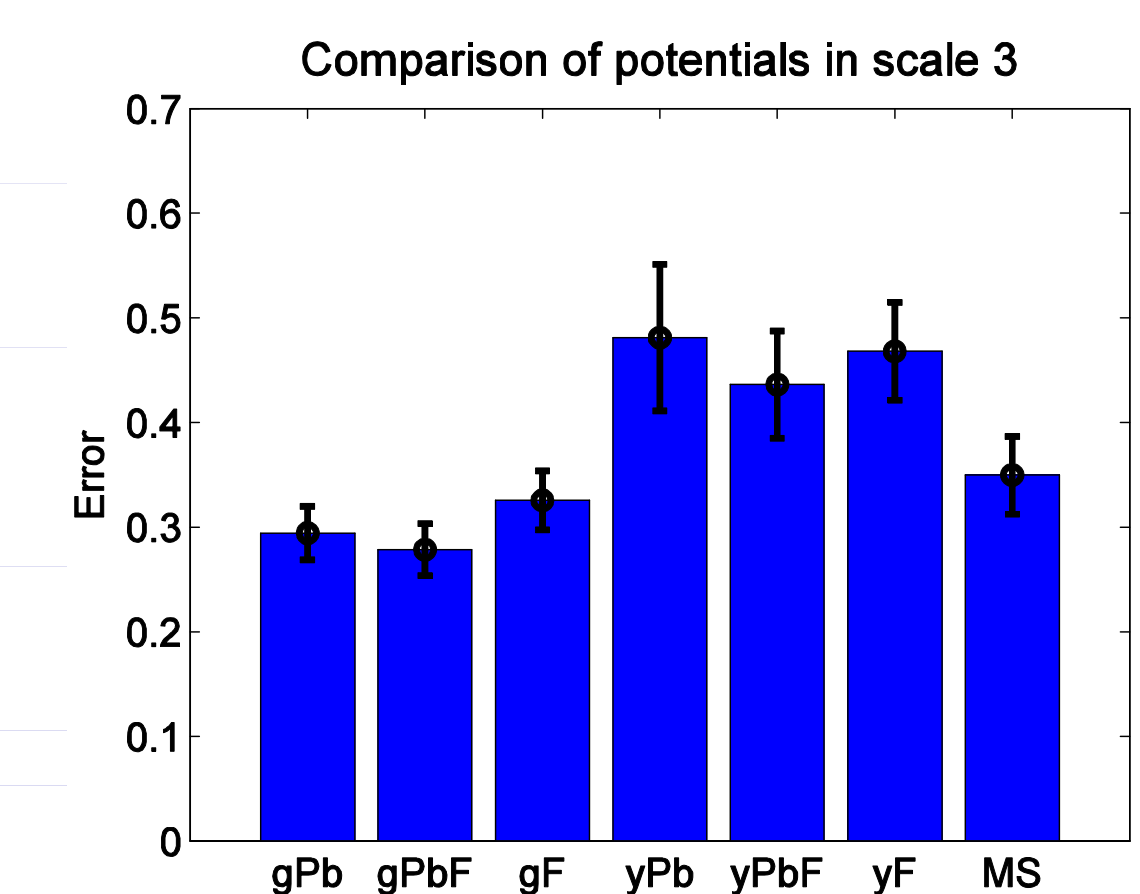
$$error = 1 - \frac{|A \cap B|}{|A \cup B|}$$



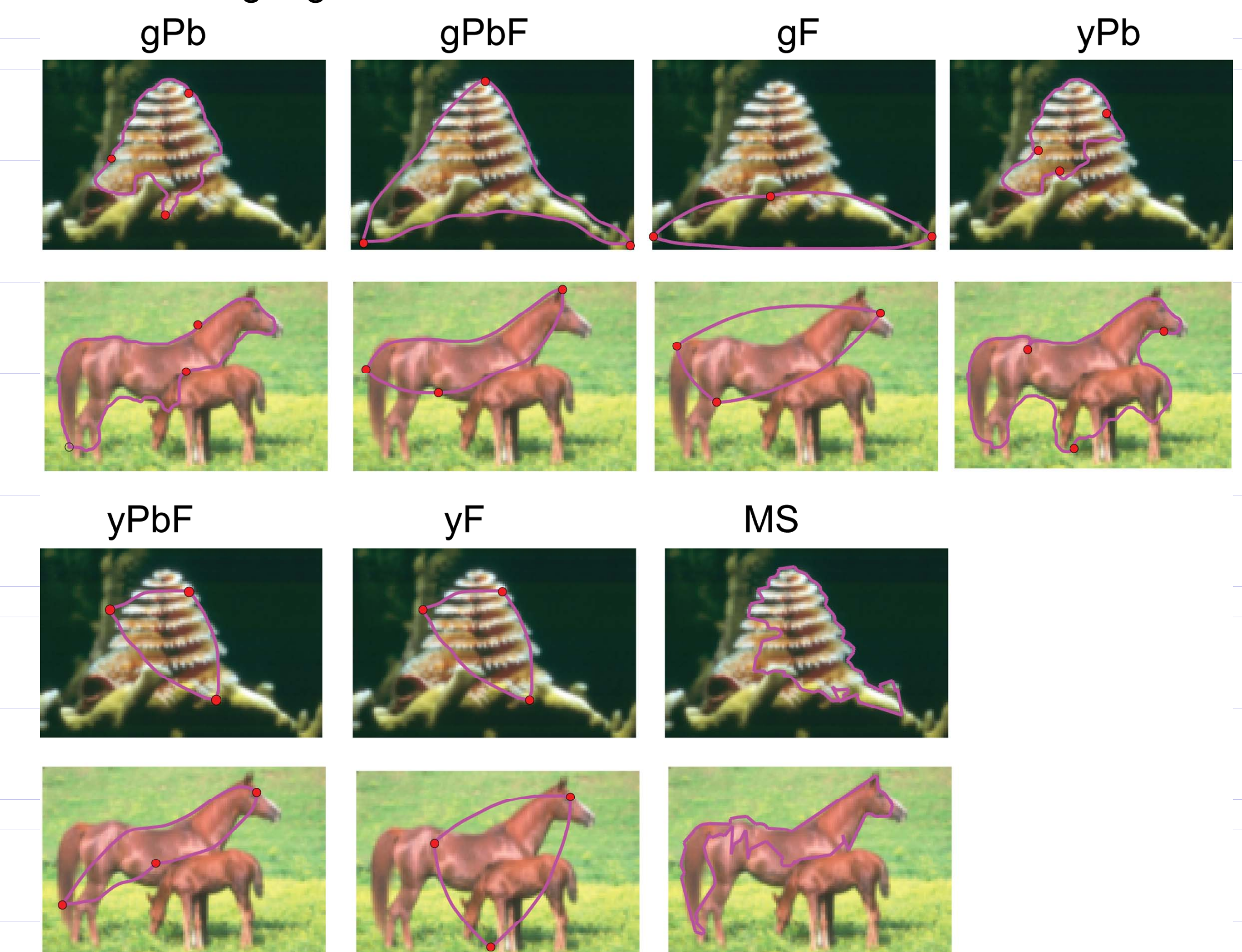
where $|A|$ denotes the number of pixels in a region A, the *best* performance obtained by this method on 15 training images is shown in the following plot. The notations used are as follows:

- g Key points rotated on ground truth contours
- y Key points rotated on a contour hypothesis
- Pb Used Pb likelihood term in calculation of potential function
- F Used convexity prior term in calculation of potential function

The results are compared with MS algorithm [6]. It should be noted that without the convexity prior in the potential function, there is no guarantee that MP would provide at least one simple contour.



The following Figures show some results:



Future work

The high error values above suggest future research is needed to

- Improve the potential function
- Provide key points automatically
- Study the possible ranking criteria for sorting obtained contours
- Study the effect of a multi-scale implementation similar to [6]

Conclusion

Challenges for applying the MP algorithm to object segmentation of images with natural settings are i) designing a suitable potential function and ii) selecting suitable key points on objects contour. Although providing the right key points can lower the segmentation error, the lowest achievable error is highly dependent on the potential function used. If a set of guesses for key points are available, contour hypotheses obtained need to be ranked and sorted.

References

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