CRAMÉR-RAO BOUND FOR TIME REVERSAL ACTIVE ARRAY DIRECTION OF ARRIVAL ESTIMATORS IN MULTIPATH ENVIRONMENTS

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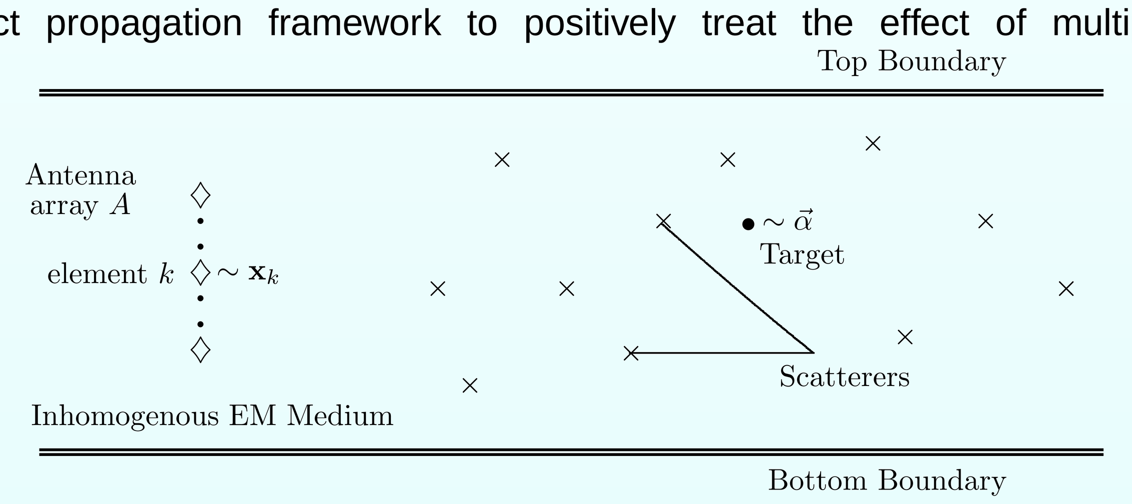
DOA Estimation in Multipath Environments

- The problem of estimating the direction of arrival (DOA) of a plane wave is important in fields as diverse as radar/sonar systems, seismic systems, electronic surveillance, medical diagnosis and treatment, and radio astronomy.
- In the conventional DOA estimators, multipath is either ignored or considered detrimental to the performance of the estimators.
- Only a handful DOA estimation approaches [1, 2] extend the single/direct propagation framework to positively treat the effect of multipath.



- Our approach is based on time reversal (TR) setup shown in Fig. 1 and does not assume any particular multipath model. The motivation for this algorithm comes from the waveform adaptation of TR to the multipath environment.
- In TR, the signal reflected from the target and observed at the antenna array during the probing stage is energy normalized, time-reversed, and retransmitted back into the medium. The backscatter of the time reversed signal, obtained from this TR stage, is used to estimate the location of the target.

Our Contributions

1. We derive the CRB of conventional DOA estimator for a passive target in a multipath environment.
2. We develop an analytical CRB expression for the TR DOA estimator for a passive target in the same multipath environment.
3. We express the structure present within the CRB expressions in terms of the multipath attenuation factors and associated delays
4. We compare the TR/DOA CRB with the conventional DOA CRB obtained by simulating a ground penetrating radar (GPR) system using the electromagnetic finite domain time domain (FDTD) model. Our results show that the TR/DOA CRB is lower by up to 15 dB compared to its conventional counterpart.

System Formulation

Forward Probing: The probing signal $f(t)$, transmitted by element k of Array A and backscattered by the target, is recorded by $(1 \le n \le P)$ elements of the same Array A after propagating through multiple paths in a inhomogeneous environment.

$$r_{(n,k)}(t) = \sum_{i=1}^{M_{(n,k)}} A_{(n,k,i)} f(t-\tau_{(n,k,i)}(\vec{\alpha})) + v_{(n,k)}(t), \quad (1)$$

for $(1 \le n \le P)$ and $(0 \le t \le T_0)$.

$M_{(n,k)}$	associated number of paths,
$A_{(n,k,i)}$	associated attenuation through path i,
$\tau_{(n,k,i)}$	associated delay through path i,
and; $v_{(n,k)}(t)$	associated observation noise.

In the frequency domain, the received signal is given by

$$R_{(n,k)}(\omega) = \sum_{i=1}^{M_{(n,k)}} A_{(n,k,i)} e^{-j\omega\tau_{(n,k,i)}} F(\omega) + V_{(n,k)}(\omega). \quad (2)$$

Define the multipath response matrix as

$$\forall k, n = 1, \cdots, P \quad \mathbf{H}(\omega) \triangleq \{H(\omega; A_n \leftarrow A_k)\}, \quad (3)$$

where its (n,k) constituent element is given by

$$H_{nk}(\omega; A_n \leftarrow A_k) = \sum_{i=1}^{M_{(n,k)}} A_{(n,k,i)} e^{-j\omega\tau_{(n,k,i)}}. \quad (4)$$

Then, Eq. (2) is expressed in the matrix-vector form as

$$\mathbf{r}_k(\omega) = \mathbf{H}(\omega)\mathbf{e}_k F(\omega) + \mathbf{v}(\omega), \quad (5)$$

TR Probing: Following the principle of TR, the recorded signal $\mathbf{r}_k(\omega)$ is energy normalized, time reversed , and retransmitted back into the medium. The backscattered TR signal at Array A is

$$\begin{aligned}\mathbf{p}_k(\omega) &= c\mathbf{H}(\omega)\mathbf{r}_k^*(\omega) + \boldsymbol{\zeta}(\omega) \\ &= c\mathbf{H}(\omega)[\mathbf{H}^*(\omega)\mathbf{e}_k F^*(\omega) + \mathbf{v}^*(\omega)] + \boldsymbol{\zeta}(\omega)\end{aligned} \quad (6)$$

where constant c represents the signal gain used during the energy normalization step prior to time reversal. Eq. (6) is expressed as

$$\mathbf{p}_k(\omega) = c\mathbf{T}(\omega)\mathbf{e}_k F^*(\omega) + \mathbf{w}(\omega), \quad (7)$$

where $\mathbf{T}(\omega) = \mathbf{H}(\omega)\mathbf{H}^*(\omega)$ is the so called TR matrix.

CRLB: Single passive target in multipath

CRLB Expressions

Theorem 1. *Expressed in terms of the location parameters, the CRB of the direction of arrival of the target based on the forward observation vector $\mathbf{r}_k(\omega)$ (Eq. (5)) is given by*

$$CRB_{CV}(\vec{\alpha})^{-1} = \frac{N}{2\pi\sigma_v^2} \int |F(\omega)|^2 \mathbf{D}^H \mathbf{D} d\omega, \quad (8)$$

where the $(P \times 2)$ derivative matrix $\mathbf{D} = \left[\frac{\partial \mathbf{h}_k}{\partial R_t} \frac{\partial \mathbf{h}_k}{\partial Y_t}\right]$, and the vector $\mathbf{h}_k = \mathbf{H}(\omega)\mathbf{e}_k$ is the channel response vector in the forward probing stage.

Theorem 2. *Expressed in terms of the location parameters, the CRB of the DOA of the target based on the TR observation vector $\mathbf{p}_k(\omega)$ (Eq. (7)) is given by*

$$CRB_{TR}(\vec{\alpha})^{-1} = \frac{Nc^2}{2\pi\sigma_w^2} \int |F(\omega)|^2 \mathbf{E}^H \mathbf{E} d\omega, \quad (9)$$

where the $(P \times 2)$ derivative matrix $\mathbf{E} = \left[\frac{\partial \mathbf{t}_k}{\partial R_t} \frac{\partial \mathbf{t}_k}{\partial Y_t}\right]$ and vector $\mathbf{t}_k = \mathbf{T}(\omega)\mathbf{e}_k$ is the k'th column of the TR matrix $\mathbf{T}(\omega) = \mathbf{H}(\omega)\mathbf{H}^*(\omega)$.

Analytical Interpretation of the CRBs

Considering Eq. (4), the component $H_{ik}(\omega) = \mathbf{a}_{ik}^T \epsilon_{ik}$,where the attenuation coefficients and delays are grouped in vectors

$$\mathbf{a}_{ik} \triangleq [A_{(k,i,1)}, \cdots, A_{(k,i,M_{ik})}]^T \text{and} \quad \epsilon_{ik} \triangleq [e^{-j\omega\tau_{(k,i,1)}}, \cdots, e^{-j\omega\tau_{(k,i,M_{ik})}}]^T,$$

such that $H_{ik}(\omega) = \rho_{ik} e^{j\delta_{ik}}$.

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Both terms depend on the travel times $\{\tau_{(k,i,j)}\}_{j=1}^{M_{ik}}$ and consequently are functions of $\vec{\alpha}$. $H_{ik}^\bullet(\omega)$ can be expressed as $H_{ik}^\bullet(\omega) = e^{j\delta_{ik}}(\rho_{ik}^\bullet + j\rho_{ik}\delta_{ik}^\bullet)$. Then, we have

$$\mathbf{D}^H\mathbf{D} = \sum_{i=1}^{P} |H_{ik}^\bullet(\omega)|^2 = \sum_{i=1}^{P} (\rho_{ik}^\bullet)^2 + (\rho_{ik})^2(\delta_{ik}^\bullet)^2 \quad (10)$$

For the TR phase, we get the following result.

$$\mathbf{E}^H\mathbf{E} = \sum_{n=1}^{P} |T_{nk}^\bullet(\omega)|^2 = \sum_{n=1}^{P}(\sum_{i=1}^{P}[\rho_{ni}\rho_{ik}]_{\vec{\alpha}})^2 + (\sum_{i=1}^{P}(\rho_{ni}\rho_{ik})(\delta_{ni}^\bullet - \delta_{ik}^\bullet))^2, \quad (11)$$

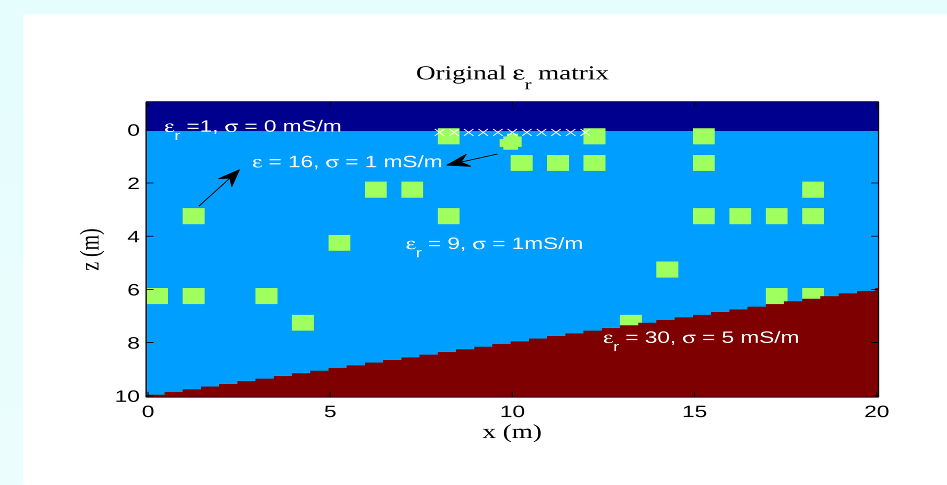
For computing the partial derivate of the conventional Jacobian matrices $\mathbf{D}$, we use the following finite difference discretization expressions

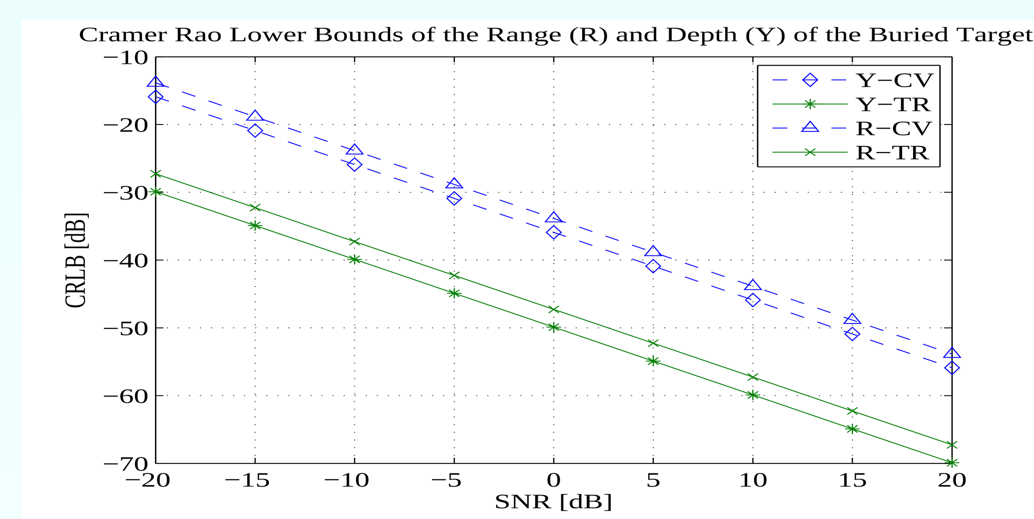
$$\frac{\partial H_{ik}(\omega)}{\partial R} = \frac{H_{ik}(\omega; R_1) - H_{ik}(\omega; R_2)}{\Delta R} \quad (12)$$

$$\text{and} \quad \frac{\partial H_{ik}(\omega)}{\partial Y} = \frac{H_{ik}(\omega; Y_1) - H_{ik}(\omega; Y_2)}{\Delta Y} \quad (13)$$

Similar expressions are used for the partial derivate of the Jacobian matrix $\mathbf{E}$.

FDTD Electromagnetic Simulations





Summary

- The CRBs for the range and DOA of a passive target embedded in a rich multipath environment are derived for the conventional and TR DOA estimators.
- The CRBs are expressed in terms of the contributions made by the multipath parameters to the FIM matrix.
- Our FDTD simulation results illustrates the potential of better performance with the TR/DOA estimator compared to the conventional estimator.

References

[1] M. J. D. Rendas and J. M. F. Moura, "Cramer-Rao bound for location systems in multipath environments,", *IEEE Trans. on Signal Processing*, vol. 39, no. 12, pp. 2593-2610, Dec. 1991.

[2] F. Foroozan and A. Asif, "Time Reversal Ground Penetrating Radar: Range Estimation with Cramer-Rao Lower Bounds", *IEEE Trans. on Geoscience and Remote Sensing*. Accepted.