CSE 4215/5431: Mobile Communications
Assignment 1

Question 1

In this problem we will walk through the derivation of the medium utilization in ALOHA and slotted ALOHA. This is probably derived in many textbooks. You will learn much more if you work through this question without looking at other sources.
The analysis assumes that packets (frames) are of length L and the data transmission rate is R (note that this abstracts away the modulation scheme and other physical layer details). Since nodes in ALOHA do an exponential backoff upon collisions, it is reasonable to assume that the number of packets arriving in any interval follows the Poisson distribution. If you do not recall what this is, feel free to look up books, wiki or other sources. You need not cite these sources. Incidentally Poisson distributions are used to model the number of telephone calls placed in an unit of time in traditional telephony and decades of logs from phone companies have shown that this is a good model.
Notice that if a packet was transmitted at time t, no other packet should have been transmitted in the time interval $(t - X, t + X)$ where $X = R/L$ is the duration of a packet transmission.
(a) Using the Poisson transmission with rate λ, write down an expression for the probability that no other packets are transmitted in the interval $(t - X, t + X)$.
(b) Using calculus, derive the value of λ at which the expression derived in part (a) is maximized.
(c) What is the corresponding throughput? Hint: in a stable system, the rate at which packets enter the system is equal to the rate at which packets leave the system.
(d) Now consider slotted ALOHA. In this protocol, time is divided into slots, each of length X. A packet can arrive anytime but can only be transmitted in the next slot. Thus a packet being transmitted in slot k can collide only with another that arrived during slot $k - 1$. Assuming again that the arrivals obey a Poisson distribution with rate λ, write down an expression for the probability that there is no collision during a time slot.
(e) Optimize λ and find the maximum possible throughput.
Solution:
Part (a): Using the definition, the probability that k packets ($k = 0, 1, \ldots$) are transmitted in the time interval $(t - X, t + X)$ is given by

$$P(k \text{ transmissions in } 2X \text{ seconds}) = \frac{(2\lambda)^k}{k!} e^{-2\lambda}.$$

Therefore the probability that no packets are transmitted in a period of duration $2X$ is $e^{-2\lambda}$. Note that we have assumed that the average number of arrivals in a period of duration X is λ.
Part (b): We can maximize the last equation. What the question meant to say was maximize the throughput, which is given by $\lambda e^{-2\lambda}$. Using calculus the maximum value occurs at $\lambda = 0.5$.
Part (c): The optimal throughput value is $\frac{1}{2e} \approx 0.184$.
Part (d): Since nodes check at the beginning of time slots to decide whether the medium is available, collisions happen only when several packets arrive *during* a time slot. Using the Poisson distribution formula, the probability that there are no collisions is equal to the

probability that there are no arrivals during an interval of duration X, which in turn is given by $e^{-\lambda}$. The throughput is then given by $\lambda e^{-\lambda}$. As before we can compute the value of λ at which this is maximum, and using calculus, we see that the maximum throughput is $\frac{1}{\lambda} = 0.368$. Note: In general the arrival rate is not a design choice, but in this case we want to know how high the throughput can be in the *best* case and so it makes sense to optimize with respect to the arrival rate.

Question 2

In this problem we will learn a quantitative model often used in traffic engineering. The Erlang B equation expresses the relationship between the probability of blockage $B(N, \rho)$ to the number of channels N and the normalized call density $\rho = \lambda/\mu$ where λ is the call arrival rate and μ is the service rate or inverse of the average call duration. The Erlang B formula is

$$B(N, \rho) = \frac{\rho^N/N!}{\sum_{i=1}^{N} \rho^i/i!}$$

Use this formula to solve the following problem.
To provide telephone access to commercial ferries a company installs a multi-channel wireless telephone system in a ferry. The wireless radio system connects to a base station on the shore through the air. The base station is connected to the traditional phone system using wires.
(a) If the company installs a 4-channel system, what is the probability if having a person come to the phone and finding that all lines are busy? Assume that the average length of a telephone call is 3 minutes and each of the 150 passengers on the ferry make on average 1 call per hour.
(b) How many channels are needed to keep the blockage probability below 2 per cent?
Solution:
Part (a): Here $\lambda = 150/60 = 2.5$ calls per minute. The call finishing rate is $\mu = 1/3$. Therefore $\rho = \lambda/\mu = 7.5$. Putting these and $N = 4$ in the formula given, we get that the blocking probability is (approximately) 0.554.
Part (b): The easiest way to solve this is to write a program to evaluate the blocking probability for different N. Charts/tables may be used as well. Using any of these means, you should see that the minimum number of channels needed is 14.

Question 3

In the RMAC protocol covered in class, we assumed that the number of sources that had packets to send at any time step is completely unpredictable and therefore all numbers were equally likely (i.e. the a priori probabilities are all equal). Is this reasonable? Under what situations? Write down any improvements you can think of in this regard.
Solution: This is a open-ended question. The simplifying assumption that all a priori probabilities are equal is not really valid. For examples if a node has 10 packets to send in one time step, then it likely has about 10 packets the next time step. In particular the probability it has zero packets the next step is zero.
Thus the algorithm could potentially be improved by altering the a priori probabilities every time step. How this should be done is an open question.

Question 4

Assume a number of mobile users are on a cellular telephone system. The area has an uneven terrain including mountains and valleys. What implications would this have on signal propagation. Assume user A is using his phone to watch a video while user B is sending and receiving emails. From each of their perspectives, which is more preferable - CDMA, TDMA, FDMA? Justify your answer with arguments.
Solution: There is no one "right" answer here. In reality, each technology could be used as long as the system is engineered appropriately. Uneven terrain causes problems in several ways. Signal strengths vary a lot over the region. Further, multipath effects vary a lot over the region as well. If a large amount of bandwidth is being used different frequency bands are affected to different extents. CDMA and TDMA offer the advantages of allowing each user to utilize the whole bandwidth and therefore get performance at least as good as users using FDMA.
For a user with real-time communication sessions (e.g. user A) this means FDMA is less preferable. For users like user B it does not make a big difference, and in fact he might prefer FDMA if the costs are lower from the system being simpler.

Question 5

Suppose three users, each sending data at 1 MB/s are sharing a channel. Suppose that the system is implemented using CDMA with the chip period being one tenth of a bit period.
(a) Assuming BPSK coding, what is the bandwidth used by the company? List all simplifying assumptions you need to make to carry out this computation.
(b) What are the pros and cons from the company's point of view in supporting more subscribers? How would you expect the user experience to change as more subscribers are added?
Solution:
Part (a): Note that the bandwidth does not depend on the data rate in this case. Since the system generates 10 million chips a second, the minimum bandwidth required is 10 MHz. If we assume high quality hardware, and ignore factors like gaps between channels (to reduce interference) this is the bandwidth used by the company.
Part (b): Clearly the company can (in principle) support upto a maximum of 10 users. This is not achievable in practice due to factors like noise and interference. Nevertheless, the number of users can likely increase to beyond 3. The pros of increasing the subscribers to the company are of course higher revenues, better utilization of bandwidth and business advantage due to potential lowering of prices. The disadvantages are lower robustness (noise-tolerance) and higher probability of service degradation. This implies that users are more likely to see periods of inferior service.