CSE 3101, Summer 2010

Sample Test 2

May 27, 2010

1. (2 points) Which is bigger, n or $(\lg n)^{\lg n}$? Justify your answer.

2. (3 points) For what constants a is the following true?

$$2^n + 3^{\frac{n}{2}} = O(a^n)$$

3. (8 points) Unroll the following recurrence and guess the answer. Then prove the answer using Mathematical Induction. $T(1) = 1$, and for $n > 1$,

$$T(n) = 2T(n-1) + 1.$$

4. (12 points) The following divide-and-conquer algorithm finds the maximum value in the array $S[1 \ldots n]$. The main body of the algorithm consists of a call to maximum (1,n).

Maximum(x, y)
// return maximum in $S[x \ldots y]$
if $y - x \leq 1$
  then return $max(S[x], S[y])$
  else $max1 \leftarrow maximum(x, \lfloor (x+y)/2 \rfloor)$
    $max2 \leftarrow maximum(\lfloor (x+y)/2 \rfloor + 1, y)$
    return $max(max1, max2)$

   (a) (3 points) Prove that the algorithm is correct. You should not assume n is a power of 2.

   (b) (5 points) Write down the recurrence for the worst case number of comparisons used by maximum (1,n). Solve the recurrence using the recursion tree method.

   (c) (4 points) Solve the recurrence by expanding (unrolling) the recurrence and using induction to prove your guess.