CSE 3101: Design and Analysis of Algorithms
Assignment 1 Do not submit - solutions will be posted soon

Notes:

- Feel free to refer to, and use any facts from the textbook.
- Please try to solve the problems before looking at the solutions.
- For each question, you must prove that your algorithm is correct, unless otherwise stated. Also, when you are asked to design an algorithm, you get full credit only if you produce the best possible algorithm (in terms of asymptotic running time).

Problems:

1. Prove that the following algorithm for exponentiation is correct.

```
POWER(y, z)
1  // return y^z where y ∈ R, z ∈ N
2  x ← 1
3  while z > 0
4  do if odd(z)
5        then x ← x * y
6     z ← ⌊z/2⌋
7     y ← y^2
8  return x
```

2. Prove that the following algorithm for evaluating the polynomial $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is correct. Assume that the coefficient a_i is stored in the array element $A[i]$ for $0 \leq i \leq n$.

```
POLY(A, n)
1  v ← 0
2  for i ← n to 0
3  do v ← A[i] + v * x
4  return v
```

3. Prove that $(n+1)^3 \in \Theta(n^3)$.

4. Find the relationship between $f(n) = n2^n$ and $g(n) = 3^n$ in terms of $O(), \Theta(), o(), \Omega(), \omega()$ and prove your answer.

5. What is the value returned by the following function? Express your answer as a function of n and give, using O-notation, the worst case running time.

```
PESKY(n)
1  r ← 0
2  for i ← 1 to n
3  do for j ← 1 to i
4     do for k ← j to i + j
5           do r ← r + 3
6  return r
```