Math/CS 1019
Homework 5
Due June 9, 2010

1. What is the cardinality of the set of all circles? Prove your answer. (Hint: A circle is defined by its radius.)

2. Prove by induction that

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Prove that if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets such that $A_i \subseteq B_i$ for $j = 1, 2, \ldots, n$ then

$$\bigcup_{j=1}^{n} A_j \subseteq \bigcup_{j=1}^{n} B_j$$

4. Subsets

(a) Prove by induction that a set with n elements has $\frac{n(n-1)}{2}$ subsets containing exactly two elements whenever n is an integer greater than or equal to 2.

(b) Prove by induction that a set with n elements has $\frac{n(n-1)(n-2)}{6}$ subsets containing exactly three elements whenever n is an integer greater than or equal to 3.

5. Assume that a chocolate bar consists of $n \times m$ squares arranged in a rectangular pattern. The bar can be broken along the vertical or horizontal lines separating the squares. Assuming that a break can only be made along one line at a time, determine the smallest number of breaks you must make to break the bar into $n \times m$ separate squares. Use strong induction to prove your answer.

6. from section 4.3

(a) Give a recursive definition of the function $ones(s)$, which counts the number of ones in a bit string s. (A bit string is a string of zeros and ones.)

(b) Use structural induction to prove that $ones(st) = ones(s) + ones(t)$.

7. A deck of cards has 52 cards in 4 suits (hearts, spades, diamonds, clubs). Each suit has 13 cards labeled with the numbers 2-10, Jack, Queen, King, and Ace. A *hand* of cards is a set of five cards taken from the deck.

(a) A hand in which four cards have the same label (but different suits) is called **four-of-a-kind**. How many different four-of-a-kind hands are there?

(b) A hand in which there are two pairs that have the same label, different from each other, and a fifth card with a third label is called **two pair**. Compute the number of hands that are two pair.

(c) A **full house** is a hand with three cards with one label and two cards with another label. Compute the number of full house hands.

8. How many functions are there from the set $\{1, 2, \ldots, n\}$ where n is a positive integer to the set $\{0, 1\}$? How many of these are onto?