1. Find these sums:

   (a) $\sum_{j=0}^{8}(1+(-1)^j)$

   (b) $\sum_{k=99}^{200} k^3$

2. Telescoping Sums

   (a) Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$ where $a_0, a_1, \ldots, a_n$ is a sequence of real numbers.

   (b) Use a direct proof to show that
   $$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}.$$

   (c) Use parts (a) and (b) to compute
   $$\sum_{k=1}^{n} \frac{1}{k(k+1)}.$$

3. What is the value returned by the following algorithm? Express your answer as a function of n only. What is the worst-case running time in big-O notation?

   Input: n: positive integer
   $r \leftarrow 0$
   for $i \leftarrow 1$ to n
       for $j \leftarrow 1$ to i
           for $k \leftarrow j$ to $i+j$
               $r \leftarrow r+1$
   return(r)

4. Write pseudocode for an algorithm that takes as input a list of n integers and finds the number of negative integers in the list. What is the worst-case running time in big-O notation?

5. Write pseudocode for an algorithm that determines whether a function from a finite set to another finite set is one-to-one. What is the worst-case running time in big-O notation?

6. Describe an algorithm (either in words or with pseudocode) based on the binary search algorithm for determining the correct position in which to insert a new element in an already sorted list. What is the worst case running time in big-O notation?

7. Show that x^3 is $O(x^4)$ but that x^4 is not $O(x^3)$.

8. Show that that $n^2 + 2$ is $\Omega(n^2)$.

9. Prove, using mathematical induction, that $3^n < n!$ if n is an integer greater than 6.

10. Prove, using mathematical induction, that 3 divides $n^3 + 2n$ whenever n is a positive integer.