

Math/CS 1019
Homework 2
Due May 19, 2010

1. Express the negations of each of these statements so that the $\neg$ symbol does not appear before any quantifier.

- (a) $\forall x \exists y P(x, y)$
- (b) $\exists x \exists y (Q(x, y) \iff Q(y, x))$
- (c) $\exists z \forall y \forall x T(x, y, z)$
- (d) $\exists y (\forall x \exists z T(x, y, z) \wedge \exists x \forall z U(x, y, z))$

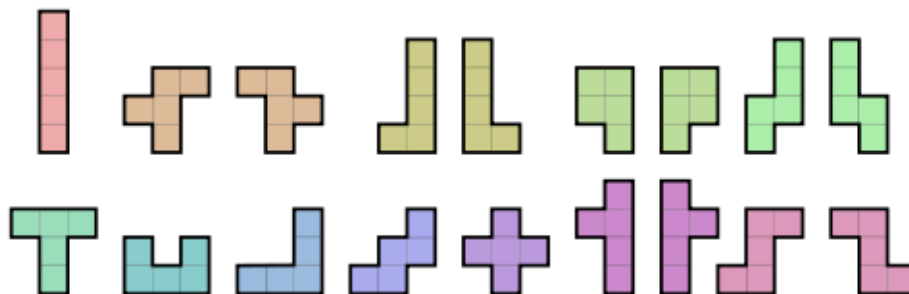
2. Each argument below is either correct or has a fallacy (but not both!). Write the argument in symbols and then determine whether the argument is valid. If it is valid, write whether it uses *modus ponens* or *modus tollens*. If it is not valid, write whether it is a fallacy of affirming the conclusion or a fallacy of denying the hypothesis.

- (a) If I play hockey, then I am sore the next day. I am sore. Therefore, I played hockey yesterday.
- (b) All insects have six legs. Dragonflies are insects. Therefore, dragonflies have six legs.
- (c) All insects have six legs. Spiders have eight legs. Therefore, spiders are not insects.
- (d) If I eat spicy foods, I get a stomachache. I didn't eat spicy foods. Therefore, I don't have a stomach ache.
- (e) All healthy food tastes bad. Carrots taste good. Therefore, carrots are not healthy.
- (f) All healthy food tastes bad. Carrots taste bad. Therefore, carrots are healthy.

3. Apply the rules of inference, as was done in Example 6 on page 67, to Lewis Carroll's argument:

All hummingbirds are richly coloured.
No large birds live on honey.
Birds that do not live on honey are dull in colour.
Therefore, hummingbirds are small.

- 4. Prove that if m and n are integers and mn is even, then m is even or n is even.
- 5. Find a counterexample to the statement that every positive integer can be written as the sum of squares of three integers.
- 6. Prove by contradiction that if x^3 is irrational, then x is irrational.
- 7. Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.
- 8. Pentominoes are like dominoes except that they are made up of five squares rather than two. The complete set of pentominoes is shown below.



Is it possible to tile a standard checkerboard with one of these pentominoes? Prove your answer.