Math/CS 1019
Homework 1
Due May 12, 2010

1. Consider the following propositions:

- *f:* Bilbo will give the ring to Frodo.
- *g:* Bilbo will give the ring to Gandalf.
- *d:* Frodo will destroy the ring.

Use f, g, d and connectives to write down a formula representing each of the following sentences:

(a) Bilbo will give the ring to Frodo or Gandalf.

(b) If Bilbo gives the ring to Frodo, he will destroy it.

Translate the following formulas into English sentences:

(a) $g \oplus (f \wedge d)$

(b) $d \leftrightarrow (f \wedge \neg g)$

2. State the converse, contrapositive, and inverse of each of these statements.

(a) If I hand in my homework late, I will get a zero.

(b) If wishes were wings, pigs would fly.

3. Suppose you have three boxes labeled "pennies" and "dimes" and "pennies and dimes." The boxes are not empty and contain nothing other than pennies and dimes. It is possible to remove an item from a box without seeing any of the other items in the box. Consider the proposition c: The label is correct.

(a) Suppose you remove an item from the box labeled "pennies and dimes," and that item is a penny. Is this consistent with $\neg c$?

(b) Suppose the labels are mixed up so that each label is on the wrong box, and you can only remove one item from one box. Which box should you remove the item from in order to be sure that the information you obtain will enable you to fix the labels? Explain your answer.

4. Use a truth table to verify the associative law for OR: $(p \vee q) \vee r \equiv p \vee (q \vee r)$

5. Use De Morgan's laws to find the negation of the following statements:

(a) Canada exports hockey players and cold fronts.

(b) Canadians speak French or English.

6. Decide whether the following statements are tautologies, contradictions, or neither. Prove your answer in each case.

(a) $p \wedge \neg p$

(b) $(p \wedge q) \rightarrow (p \vee q)$

(c) $p \rightarrow (p \vee q)$

7. This problem deals with some important concepts not covered in lecture.

(a) Suppose that a truth table in n propositional variables is given. Explain how you can form a proposition from the truth table using only $\wedge, \vee$, and $\neg$ so that it is in a form like this:

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q),$$

a series of conjunctions of the variables or their negations connected by disjunctions. This is called *disjunctive normal form*.

(b) A collection of logical operators is *functionally complete* if every compound proposition is logically equivalent to a compound proposition using only those logical operators. Explain why $\wedge, \vee$, and $\neg$ are functionally complete.

(c) Use DeMorgan's Laws and part (b) to explain why $\neg$ and $\wedge$ are functionally complete. Note: the same argument works for $\neg$ and $\vee$.

8. For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

(a) Every shape has four angles.

(b) There is a shape with four angles.

(c) Exactly one shape has four angles.

9. Represent the statement "x loves y" with $L(x, y)$. Let the domain of x be all people in Canada and the domain of y be all kinds of food. Translate the following statements into English.

(a) $\forall x\ L(x,\text{donuts})$

(b) $\exists x\ L(x,\text{donuts})$

(c) $\forall x\ \exists y\ L(x, y)$

(d) $\exists x\ \forall y\ L(x, y)$