

# Why study propositional logic?

- A formal mathematical "language" for precise reasoning.
- Start with propositions.
- Add other constructs like negation, conjunction, disjunction, implication etc.
- All of these are based on ideas we use daily to reason about things.

**Propositions** 

- Declarative sentence
- Must be either True or False.

#### **Propositions:**

- York University is in Toronto
- York University is in downtown Toronto
- All students at York are Computer Sc. majors.

#### Not propositions:

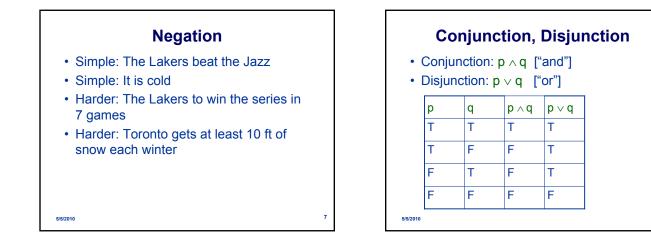
- Do you like this class?
- There are x students in this class.
- Read Ch 1 before the next class.

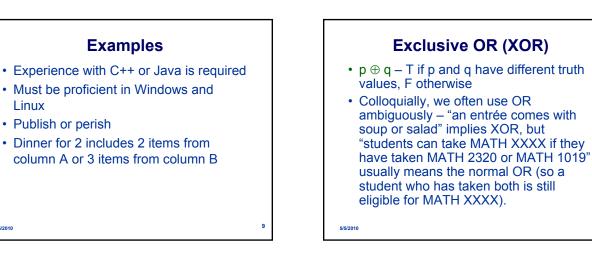
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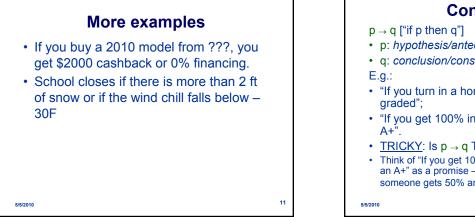
Propositions - 2 • Truth value: True or False • Variables: p,q,r,s,...• Negation: •  $\neg p$  ("not p") • Truth tables  $p \quad \neg p \ T \quad F \ F \quad T$ 

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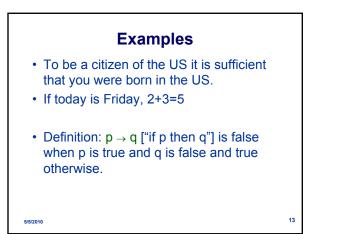


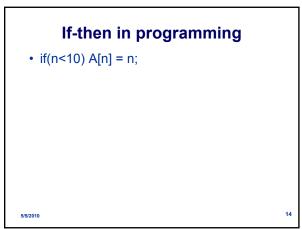


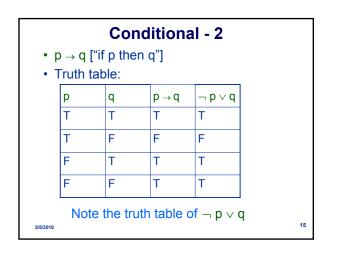
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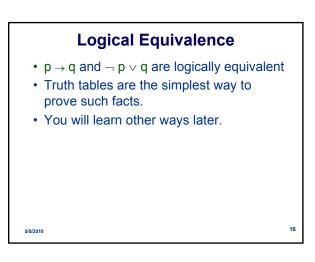
## Conditional

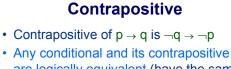
- p: hypothesis/antecedent/premise,
- q: conclusion/consequence
- "If you turn in a homework late, it will not be
- "If you get 100% in this course, you will get an
- TRICKY: Is p → q TRUE if p is FALSE? YES!!
- Think of "If you get 100% in this course, you will get an A+" as a promise - is the promise violated if someone gets 50% and does not receive an A+?











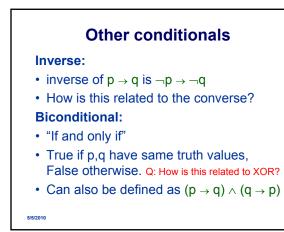
- are logically equivalent (have the same truth table) Check by writing down the truth table.
- E.g. The contrapositive of "If you get 100% in this course, you will get an A+" is "If you do not get an A+ in this course, you did not get 100%".

5/5/2010

Converse
Converse of p → q is q → p
Not logically equivalent to conditional
Ex 1: "If you get 100% in this course, you will get an A+" and "If you get an A+ in this course, you scored 100%" are not equivalent.
Ex 2: If you won the lottery, you are rich.



5/5/2010



## **Examples**

- For you to win the lottery, it is necessary and sufficient that you have a winning ticket.
- The train runs late on exactly the days when I take it
- You get promoted only if you have connections
- You get promoted if and only if you have connections.

5/5/2010

5/5/2010

19

21

23

# Proof using contrapositive Prove: If x<sup>2</sup> is even, x is even Proof 1: x<sup>2</sup> = 2a for some integer a. Since 2 is prime, 2 must divide x. Proof 2: if x is not even, x is odd. Therefore x<sup>2</sup> is odd. This is the contrapositive of the original assertion.

## **Readings and notes**

- Read pages 1-12.
- Think about the notion of truth tables.
- Master the rationale behind the definition of conditionals.
- Practice translating English sentences to propositional logic statements.

#### Next

#### Ch. 1.2: Propositional Logic - continued

- Compound propositions, precedence rules
- Tautologies and logical equivalences

### Ch. 1.3: Predicate Logic

- Predicates and quantifiers
- Rules of Inference

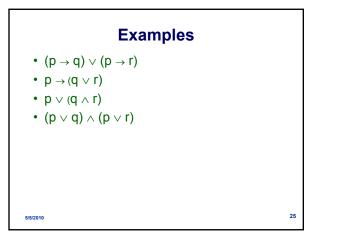
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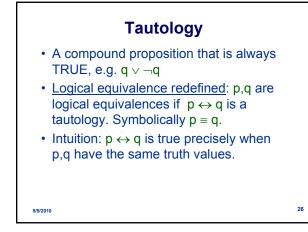
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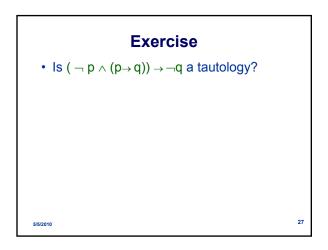
Compound Propositions

- Example: p \langle q \langle r : Could be interpreted as (p \langle q) \langle r or p \langle (q \langle r)
- precedence order:  $\neg \land \lor \rightarrow \leftrightarrow (IMP!)$ (Overruled by brackets)
- We use this order to compute truth values of compound propositions.

20











- Identity  $p \land T, q \lor F$
- Domination  $p \lor T$ ,  $q \land F$
- Idempotence  $p \lor p, p \land p$
- Double negation ¬(¬p)

5/5/2010

- Commutativity  $p \lor q \equiv q \lor p$ , etc
- Associativity  $p \lor (q \lor r) \equiv (p \lor q) \lor r$ , etc

 $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ Intuition (not a proof!) – For the LHS to be true: p must be true and q or r must be true. This is the same as

saying p and q must be true or p and r must be true.

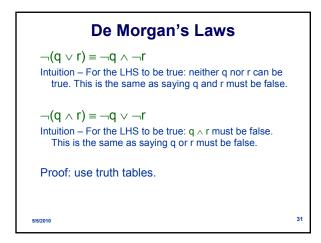
**Distributive Laws** 

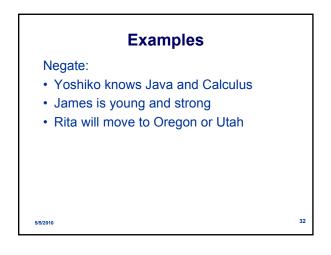
 $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ 

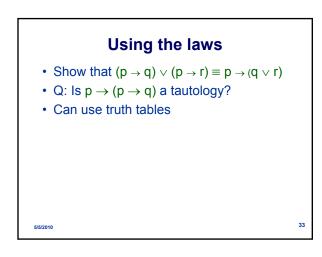
Intuition (less obvious) – For the LHS to be true: p must be true or both q and r must be true. This is the same as saying p or q must be true and p or r must be true.

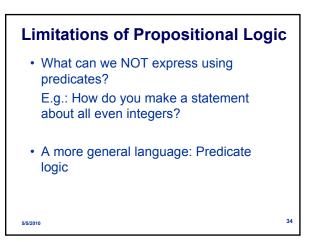
Proof: use truth tables.

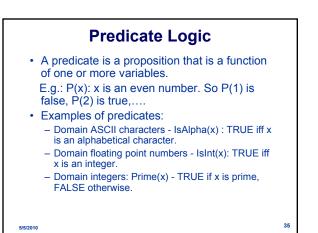
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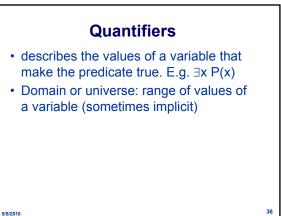












## **Two Popular Quantifiers**

- Universal: ∀x P(x) "P(x) for all x in the domain"
- Existential: ∃x P(x) "P(x) for some x in the domain" or "there exists x such that P(x) is TRUE".
- Either is meaningless if the domain is not known/specified.
- Examples (domain real numbers)
  - ∀x (x² >= 0)
  - ∃x (x >1)
  - $(\forall x>1) (x^2 > x)$  quantifier with restricted domain

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37

# **Using Quantifiers**

38

Domain integers:

• Using implications: The cube of all negative integers is negative.

#### $\forall x (x < 0) \rightarrow (x^3 < 0)$ • Expressing sums :

$$\forall n \left(\sum_{i=1}^{n} i = n(n+1)/2\right)$$

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