Homework Exercise #7
Due: November 19, 2009

1. Consider an asynchronous shared-memory system in which any number of processes can experience crash failures.

Define a VECTOR object type which stores a vector of m components. Each component stores a natural number. The VECTOR object provides two operations:

- BLOCKWRITE(i, j, v), where $1 \leq i \leq j \leq m$ and v is a natural numbers, changes all of the components $i, i+1, \ldots, j$ of the vector to v and returns ACK, and
- SCAN returns the contents of the entire vector without changing the object's state.

(a) Suppose you have a wait-free consensus algorithm that uses VECTOR objects. Suppose C is a multivalent configuration such that every successor of C is univalent. Prove that each process's next step after C must update a component that is not updated by any other process's next step.

(b) Determine the largest number k such that wait-free consensus can be solved in a system of k processes using VECTOR objects. (Your value of k may or may not depend on m.) Prove your answer is correct.

Hint: in designing a consensus algorithm, it is sometimes helpful to have a subset of the processes first agree among themselves and then communicate with the processes outside the subset to ensure all the processes agree on the output.