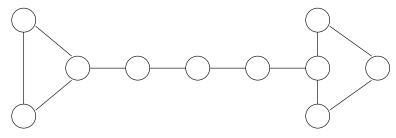
Homework Exercise #4 Due: October 27, 2009

1. In this question, we consider the problem of solving leader election in an anonymous, asynchronous, failure-free network.

The nodes in the network do not have a "sense of direction". This means that each node initially knows how many neighbours and it has one message channel connecting it to each of its neighbours. However, the node does not know which of its channels connects it to each neighbour. Thus, if a node has 3 neighbours, it can send messages along channel #1, #2 or #3, but it does not initially know which node each channel connects to.

(One way to think of this is that an adversary chooses the labels that each node uses to refer to its incident channels before the computation begins. Algorithms for this model should work regardless of how the adversary chooses those labels.)

(a) Show that leader election can be solved in networks that are shaped like an arrow:



Assume the number of nodes in the system is unknown. In other words, your algorithm can use the fact that the network looks like the picture above, but it should work regardless of the number of nodes along the "shaft" of the arrow. (Try to make your solution as simple as possible.)

(b) Consider a cubic network consisting of 8 nodes positioned at the corners of a cube. Each node is connected to three neighbours, namely, the ones to which it would be joined by an edge of the cube.

Is leader election possible? Prove that your answer is correct.