Homework Exercise #1
Due: September 29, 2009

1. Recall the Two Generals problem from class. We shall consider a more general problem, where there are n generals instead of two. The generals of the red army have synchronized watches. Each general has an initial preference for what time to attack. Each general must eventually decide what time to lead his troops in an attack. Their decisions of all the generals must always be identical. Furthermore, if all generals have the same initial preference x and no communication failures occur, their common decision should be x.

In the Rarupongo Archipelago, the red army is spread out across n tiny islands, with one general on each island. The only means of communication is fireworks. A general on any island can shoot fireworks straight up into the air from his island. The fireworks can be seen anywhere within a radius of 30 kilometres. On each island, there are always islanders monitoring the sky in all directions for fireworks. Assume each island has an unlimited supply of fireworks. The algorithms followed by generals on different islands need not be identical.

Rarupongo is a very rainy place. Sometimes, when rainwater gets into a box of fireworks, the fireworks get damaged. Thus, when they are lit, they sometimes fizzle out without creating any visible signal. (This is considered a communication failure.)

Let G be a graph where each node represents an island. There is an edge connecting two nodes if and only if the two islands they represent are at most 30 kilometres apart. You may assume that all generals know the geography of their archipelago.

(a) State a necessary and sufficient condition on the graph G for the problem to be solvable.

(b) Show your condition is sufficient.

You can do this by giving an algorithm that works for every graph that satisfies the condition. You should give a detailed procedure for each general to follow that includes exactly how fireworks should be launched and how a general should decide on his attack time, based on the fireworks that were visible at his island. Briefly explain why your algorithm works.

(c) Show your condition is necessary.

This means you should prove that, for all graphs that do not satisfy the condition, there is no algorithm that solves the problem.