

## Notes on Dijkstra's Algorithm

These are some notes on Dijkstra's mutual exclusion algorithm (CACM 8(9), Sept 1965, p. 569).

Below, I give Dijkstra's mutual exclusion algorithm using more modern notation. (The *Interested* array is the negation of Dijkstra's *b* array and the *Passed* array is the negation of Dijkstra's *c* array.) The shared registers *Passed*[*i*] and *Interested*[*i*] can be written by process *i* and read by every process. The shared register *K* can be read and written by every process.

### Shared variables:

*Interested*[1..*N*] % Boolean entries indicate which processes are interested in using the resource

*Passed*[1..*N*] % Boolean entries indicate which processes have passed Phase 1

*K* % integer is the name of a process allowed to try for the resource next

### Local variable:

*done* % Boolean variable that indicates when the process can enter the critical section

### Code for process *i*:

*Interested*[*i*] ← true

*done* ← false

loop

  loop % Entry Phase 1

    exit when *K* = *i*

    if not *Interested*[*K*] then % try to go next

*K* ← *i*

  end loop

*Passed*[*i*] ← true % begin Entry Phase 2

*done* ← true

  for *j* ← 1..*N* except *i* % check if anyone else has passed Phase 1

    if *Passed*[*j*] then % go back to Phase 1

*Passed*[*i*] ← false

*done* ← false

  end for

  exit when *done*

end loop

CRITICAL SECTION

*Passed*[*i*] ← false

*Interested*[*i*] ← false

REMAINDER SECTION

**Exclusion property:** No two processes are ever in the critical section simultaneously. The argument for this property is based entirely on the *Passed* array. (It has nothing to do

with the *Interested* array or  $K$ .) To derive a contradiction, suppose two agents  $i$  and  $j$  are in the critical section simultaneously. Then, consider the last time each of them set its entry of *Passed* to true prior to entering the critical section. Without loss of generality, suppose  $i$  did this after  $j$  did. Then, when  $i$  reads *Passed*[ $j$ ] during Phase 2, it will see that *Passed*[ $j$ ] is true, and it will go back to Phase 1. This contradicts the fact that  $i$  gets into the critical section.

**Progress property:** If anybody wants to enter the critical section, eventually somebody does. Suppose some processes are trying to get to the critical section (*i.e.*, in the big loop) and no process is in the critical section. We must show that some process eventually enters the critical section. Eventually  $K$  will get set to the name of one of the processes that is trying to enter the critical section. The value of  $K$  may change a few times (for example if several processes are about to write their own ids into  $K$ ) but, eventually,  $K$  will stop changing until some progress is made because *Interested*[ $K$ ] will be true. Once  $K$  has settled down to a particular value, that process must just wait until all other processes get kicked out of phase 2. Those processes won't be able to re-enter phase 2. So the process whose id is in  $K$  will eventually get into the critical section.

Notice that this algorithm allows starvation: one process may be left out of the critical section forever while another enters it infinitely many times.