Accumulators
More on Arithmetic
and
Recursion

listlen (L , N)

◊ **L** is a list of length **N** if ...

```
listlen ( [] ,  0 ).
listlen ( [ H | T ] , N ) :-  listlen ( T , N1 ) , N is N1 + 1.
```

> **On searching for the goal, the list is reduced to empty**

> **On back substitution, once the goal is found, the counter is incremented from 0**

◊ Following is an example sequence of goals (**left hand column)** and back substitution (**right hand column**)

listlen([a, b, c] , N).	**N <== N1 + 1**
listlen([b, c] , N1).	**N1 <== N2 + 1**
listlen([c] , N2).	**N2 <== N3 + 1**
listlen([] , N3).	**N3 <== 0**

Abstract the counter

- The following abstracts the counter part from listlen.

```
addUp ( 0 ).
addUp ( C )  :-  addUp ( C1 ) , C is C1 + 1.
```

- Notice the recursive definition occurs on a counter one smaller than in the head.

Count Up

- An alternate method is to count on the way to the fixed point value in the query
- The auxiliary counter accumulates the result on the way to the goal.

```
adder ( C )  :-  adder ( 0 , C ).  ;Introduce auxiliary counter
adder ( C , C )  :-  nl , write ( 'a ' ).
```

  - The goal is reached when the auxiliary counter reaches the fixed point count value

```
adder ( Acc1 , C )  :-  write ( 'b ' ) , Acc2 is Acc1 + 1
                                      , adder ( Acc2 , C ).
```

  - The predicates in black always succeed, side effect is to write to the terminal – can see order of rule execution

listLen(L,N) – 2

- We can define list length using an accumulator

```
listln ( L , N )  :-  lenacc ( L , 0 , N ).
```

  - Introduce the auxiliary counter – length of list L when added to the accumulator is N

```
lenacc ( [] , A , A ).
lenacc ( [ H I T ] , A , N )  :-  A1 is A + 1
                                , lenacc ( T , A1 , N ).
```

- Following is a sequence of goals

```
listln  ( [ a , b , c ] ,  N ).
lenacc ( [ a ,  b , c ] , 0 , N ).    N <== N1
lenacc ( [ b , c ] , 1 , N1 ).        N1 <== N2
lenacc ( [ c ] , 2 , N2 ).            N2 <== N3
lenacc ( [] , 3 , N3 ).               N3 <== 3
```

Sum a List of Numbers – no accumulator

◊ **sumList(List, Total)** asserts **List** is a list of numbers and **Total = + / List** .

```
 sumList([], 0).

 sumList(  [ First | Rest ], Total) :-
     sumList(Rest, Rest_total)

    , Total is First + Rest_total.
```

Sum a List of Numbers – with accumulator

- ◊ **sumList(List, Total)** asserts **List** is a list of numbers and **Total = + / List** .
  - » **Use an accumulator**
  - » **Here sumList asserts Total = (+ / List) + Acc**

```
sumList(List, Total) :- sumList(List, 0, Total).

sumList([], Acc, Acc).

sumList(  [ First I Rest ], Acc, Total) :-
     NewAcc is Acc + First
   , sumList(Rest, NewAcc, Total).
```

A base case stops recursion

- A base case is one that stops recursion
  - **This is a more general notion than the smallest problem.**
- Generate a sequence of integers from 0 to N, inclusive.
  - **Need to stop recursion when we have reached N.**

```
numInRange(X,N) :- addUpToN(0,X,N).
addUpToN(X,X,_).
addUpToN(Acc,X,N) :- Acc < N
                   , Acc1 is Acc + 1
                   , addUpToN(Acc1,X,N).
```

Base case, no recursion

Need guard to prevent selecting this rule to prevent recursion

Accumulator – Using vs Not Using

- The definition styles reflect two alternate definitions for counting
  - **Recursion – counts (accumulates) on back substitution.**
    - **Goal becomes smaller problem**
    - **Do not use accumulator**
  - **Iteration – counts up, accumulates on the way to the goal**
    - **Accumulate from nothing up to the goal**
    - **Goal "counter value" does not change**
- Some problems require an accumulator
  - **Parts explosion problem**
  - **Summing a list of numbers**

Factorial using recursion

- Following is a recursive definition of factorial

  Factorial (N) = N * Factorial (N – 1)

```
factr ( N , F)  -- F is the factorial of N
 factr ( 0 , 1 ).
 factr ( N , F )  :-  J  is  N – 1  ,   factr ( J , F1 )
                      , F  is  N * F1.
```

- The problem **(J , F1)** is a smaller version of **(N , F)**
- Work toward the fixed point of a trivial problem
- Does not work for **factr (N ,120)** and **factr (N , F)**.
  - **Cannot do arithmetic J is N – 1 because N is undefined.**

Factorial using iteration – accumulators

- An iterative definition of factorial

```
facti ( N , F )  :-  facti ( 0 , 1 , N , F ).
facti ( N , F , N , F ).
facti ( I , Fi , N , F )  :-  invariant ( I , Fi , J , Fj )
                          ,  facti ( J , Fj , N , F ).

invariant ( I , Fi , J , Fj )  :-  J  is  I + 1 ,  Fj  is  J * Fi.
```

- The last two arguments are the goal and they remain the same throughout.
- The first two arguments are the accumulator and they start from a fixed point and accumulate the result
- Works for queries **factr (N ,120)** and **factr (N , F)** because values are always defined for the **is** operator.

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Fibonacci – Ordinary Recursion

- Following is a recursive definition of the fibonacci series. For reference here are the first few terms of the series

Index –	0	1	2	3	4	5	6	7	8	9	10	11	12
Value –	1	1	2	3	5	8	13	21	34	55	89	144	233

**Fibonacci (N) = Fibonacci (N – 1)
+ Fibonacci (N – 2).**

```
fib ( 0 , 1 ).
fib ( 1 , 1 ).
fib ( N , F )  :-  N1  is  N – 1 ,  N2  is  N – 2
                 ,  fib ( N1 , F1 ) , fib ( N2 , F2 )
                 ,  F  is  F1 + F2.
```

- Does not work for queries **fib (N , 8)** and **fib (N , F)**
  - **Values for is operator are undefined.**

Fibonacci – Tail Recursion

- A tail recursive definition of the fibonacci series
  > **Tail recursion is equivalent to iteration**

```
fibt ( 0 , 1 ).
fibt ( 1 , 1 ).
fibt ( N , F )  :-  fibt ( 2 , 1 , 1 , N , F ).

fibt ( N , Last2 , Last1 , N , F )  :-  F  is  Last2 + Last1.
fibt ( I , Last2 , Last1 , N , F )  :-  J  is  I + 1
                                      ,  Fi  is  Last2 + Last1
                                      ,  fibt ( J , Last1 , Fi , N , F ).
```

- Works for queries **factr (N , 120)** and **factr (N , F)**
  - » **values are always defined for is operator.**

Parts Explosion – The Problem 1

- ◊ Parts explosion is the problem of accumulating all the parts for a product from a definition of the components of each part
- ◊ Consider a bicycle we could have
  - > **the following basic components**

basicPart(spokes). basicPart(rim). basicPart(tire).
basicPart(inner_tube). basicPart(handle_bar).
basicPart(front_ fork). basicPart(rear_fork).

  - > **the following definitions for sub assemblies**

assembly(bike, [wheel, wheel, frame]).
assembly(wheel, [spokes, rim, wheel_cushion]).
assembly(wheel_cushion, [inner_tube, tire]).
assembly(frame, [handle_bar, front_fork, rear_fork]).

Parts Explosion – The Problem 2

- We are interest in obtaining a parts list for a bicycle.

  [rear_ fork , front_ fork , handle_bar , tire , inner_tube , rim , spokes , tire , inner_tube , rim , spokes]

  - **We have two wheels so there are two tires, inner_tubes, rims and spokes.**

- Using accumulators we can avoid wasteful re-computation as in the case for the ordinary recursion definition of the fibonacci series

Parts Explosion – Accumulator 1

- ◊ partsof (X ,P) – **P** is the list of parts for item **X**
- ◊ partsacc (X , A , P) – parts_of (**X**) || **A** = **P**.

|| is catenate (math append)

```
partsof ( X , P )  :-  partsacc ( X , [] , P ).
```

> **Basic part – parts list contains the part**

```
partsacc ( X , A , [ X | A ] )  :-  basicPart ( X ).
```

> **Not a basic part – find the components of the part**

```
partsacc ( X , A , P )  :-  assembly ( X , Subparts ) ,
```

> **parsacclist – parts_of (Subparts) || A = P**

```
                              partsacclist ( Subparts , A , P ).
```

Parts Explosion – Accumulator 2

◊ parsacclist (ListOfParts , AccParts , P)

- parts_of (**ListOfParts**) || **AccParts** = **P**

    > **No parts ⇒ no change in accumulator**

    partsacclist ([] , A , A).

    partsacclist ([Head | Tail] , A , Total) :-

    > **Get the parts for the first on the list**

    partsacc (Head , A , HeadParts)

    > **And catenate with the parts obtained from the rest of the ListOfParts**

    , partsacclist (Tail , HeadParts , Total).

Difference Lists and Holes

- The accumulator in the parts explosion program is a stack
  - **Items are stored in the reverse order in which they are found**
- How do we store accumulated items in the same order in which they are formed?
  - **Use a queue**
- Difference lists with holes are equivalent to a queue

Examples for Holes

- Consider the following list

  [a , b , c , d | X]

  - **X is a variable indicating the tail of the list. It is like a hole that can be filled in once a value for X is obtained**

- For example

  Res = [a , b , c , d | X] , X = [e , f].

  - **Yields**

  Res = [a , b , c , d , e , f]

Examples for Holes – 2

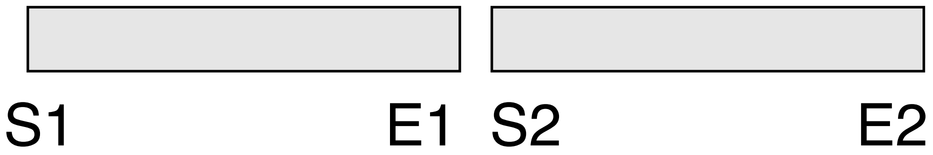
- ◊ Or could have the following with the hole going down the list

  Res = [a , b , c , d | X]

  - > **more goal searching gives X = [e , f | Y]**
  - > **more goal searching gives Y = [h , i , j]**
  - > **Back substitution Yields**

    Res = [a , b , c , d , e , f , h , i , j]

Difference Lists



(1) concat(S1 – E1 , S2 – E2 , S1 – E2) with E1 = S2

L1 = [A , B , C] = [A , B , C | R1] – R1
L2 = [D , E] = [D , E | R2] – R2

Pattern match (1) with (2)

(2) concat([A , B , C | R1] – R1 , [D , E | R2] – R2 , CL)
Using E1 = S2 we get

R1 = [D , E | R2]
CL = [A , B , C , D , E | R2] – R2

Parts Explosion – Difference List 1

- partsofd (X , P) – **P** is the list of parts for item **X**
- partsdiff (X , Hole , P) – parts_of (**X**) || **Hole** = **P**
  - **Hole and P are reversed compared to Clocksin & Mellish (v5) to better compare with accumulator version.**

partsofd (X , P) :- partsdiff (X , [] , P).

  - **Base case we have a basic part, then the parts list contains the part**

partsdiff (X , Hole , [X | Hole]) :- basicPart (X).

Parts Explosion – Difference List 2

> Not a base part, so we find the components of the part

partsdiff (X , Hole , P) :- assembly (X , Subparts)

> parsdifflistd – parts_of (Subparts) II Hole = P

, partsdifflist (Subparts , Hole , P).

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Parts Explosion – Difference Lists 3

◊ parsdifflist (ListOfParts , Hole , P)
  - parts_of (**ListOfParts**) || **Hole** = **P**

partsdifflist ([] , Hole , Hole).

partsdifflist ([Head | Tail] , Hole , Total) :-

> **Get the parts for the first on the list**

partsdiff (Head , Hole1 , Total)

> **And catenate with the parts obtained from the rest of the ListOfParts**

, partsdifflist (Tail , Hole , Hole1).

Compare Accumulator with Hole

```
partsof   ( X ,  P )  :-  partsacc   ( X , [] ,  P ).     Accumulator
partsofd ( X , P )   :-  partsdiff  ( X ,  [] ,  P ).     Difference/Hole


partsacc   ( X ,   A     , [ X | A ] )      :-  basicPart ( X ).
partsdiff ( X , Hole , [ X | Hole ] )   :-  basicPart ( X ).


partsacc   ( X ,  A      , P )  :-  assembly (  X ,  Subparts )
                                  ,  partsacclist  ( Subparts ,  A     , P ).

partsdiff ( X , Hole , P )  :-   assembly (  X ,  Subparts )
                                ,  partsdifflist ( Subparts , Hole , P ).
```

Compare Accumulator with Hole – 2

```
partsacclist  ( [] ,     A    ,  A    ).
partsdifflist (  []  ,  Hole , Hole ).


partsacclist   ( [ Head | Tail ] , A    , Total )
                    :-  partsacc  ( Head ,   A    , HeadParts )
                     ,  partsacclist  ( Tail , HeadParts , Total ).

partsdifflist ( [ Head | Tail ] , Hole , Total )
                   :-  partsdiff ( Head ,  Hole1 ,    Total      )
                   ,   partsdifflist ( Tail  ,   Hole   ,  Hole1 ).
```