## Chapter 5

Synchronous Sequential Logic

## Introduction

- Circuits require memory to store intermediate data
- Sequential circuits use a periodic signal to determine when to store values.
- A clock signal can determine storage times
- Clock signals are periodic
- Single bit storage element is a flip flop
- A basic type of flip flop is a atch
- Latches are made from logic gates
- NAND, NOR, AND, OR, Inverter


## Synchronous vs. Asynchronous

- Synchronous sequential circuit is a system whose behavior can be defined from the knowledge of its signal at discrete instants of time
- Asynchronous sequential circuit is a system whose behavior can be defined from the knowledge of its signals at any point of time


## Synchronous Sequential Circuit



## Cross Coupled Inverters



State 1


State 2

What if there is an inverter in the feedback path?

## SR Latch using NOR


${ }^{\circ}$ S-R latch made from cross-coupled NORs
${ }^{\circ}$ If $\mathbf{Q}=1$, set state
${ }^{\circ}$ If $\mathbf{Q}=0$, reset state
${ }^{\circ}$ Usually $\mathrm{S}=0$ and $\mathrm{R}=0$
${ }^{\circ} \mathrm{S}=1$ and $\mathrm{R}=1$ generates Smple $_{\text {mper }}$

## SR latch using NAND



| S | R | Q | $\mathrm{Q}^{\prime}$ |  |
| :--- | :--- | :---: | :--- | :--- |
| 0 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | Disallowed |
| 0 | 1 | $\mathbf{1}$ | $\mathbf{0}$ | Set |
| 1 | 0 | $\mathbf{0}$ | $\mathbf{1}$ | Reset |
| 1 | 1 | $\mathbf{0}$ | $\mathbf{1}$ | Store |

- Latch made from cross-coupled NANDs
- Sometimes called S'-R' latch
- Usually $\mathrm{S}=1$ and $\mathrm{R}=1$
- $\mathrm{S}=0$ and $\mathrm{R}=0$ generates unpredictable results


## SR Latch



(b) Function table

$$
\begin{array}{ll|lll}
S & R & Q & \bar{Q} & \\
\hline 0 & 1 & 1 & 0 & \text { Set state } \\
1 & 1 & 1 & 0 & \\
\hline 1 & 0 & 0 & 1 & \\
1 & 1 & 0 & 1 & \text { Reset state } \\
\hline 0 & 0 & 1 & 1 & \text { Undefined } \\
\hline
\end{array}
$$

(b) Function table


Fig. 5-5 SR Latch with Control Input
${ }^{\circ}$ Occasionally, desirable to avoid latch changes
${ }^{\circ} \mathrm{C}=0$ disables all latch state changes
${ }^{\circ}$ Control signal enables data change when $\mathrm{C}=1$
${ }^{\circ}$ Right side of circuit same as ordinary S-R latch.

## SR Latch with Control Input



## D Latch



## D Latch

- One wat to avoid the indeterminate state in the SR latch when both S and R are This is done by inverting $S$ and having the inverted S as R .
- As long as $C=0$, no change. If $C=1 D$ is delivered to Q and D' to Q'

$$
\begin{array}{cccc}
\mathrm{C} & \mathrm{D} & \mathrm{Q} & \mathrm{Q}^{\prime} \\
0 & \mathrm{X} & \mathrm{Q} & \mathrm{Q}^{\prime} \\
1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1
\end{array}
$$

## Symbols



## Flip-Flops

- The state of the latch or flip-flop is switched by a change in the control input, this is called a trigger
- The D latch with pulses in its control input. As long as the pulse input is 1 , any changes in the D input is transferred to the output.
- When the state of the latch changes, if the control pulse is still at logic 1 , any changes to the input (possible a feedback from the output) will change the output.
- Because of this, the output of a latch can not be applied directly to the input of this or other latches


## Flip-Flop

- A better way is if the flip-flop is triggered during the transition of the control input.
- A clock pulse goes through 2 transitions, $1 \rightarrow 0$ and $0 \rightarrow 1$.
- The first is called -ve edge, the second is positive edge.
- There are two ways to implement this, either by using master-salve or by special design of the circuit.


## Edge-Triggered Flip-Flop <br> (Master-Slave)



Fig. 5-9 Master-Slave $D$ Flip-Flop

Any changes in the input can affect only Y as long as CLK=1; After CLK=0, Y propagates to Q, byt the master is locked.

## D-Type Positive-Edge-triggered Flip-Flop



## Edge-triggered Flip-Flop


(a) Positive-edge

(a) Negative-edge

Fig. 5-11 Graphic Symbol for Edge-Triggered $D$ Flip-Flop


Lo-Hi edge

## J-K Flip-Flop


(a) Circuit diagram

(b) Graphic symbol
${ }^{\circ}$ Created from D flop
${ }^{\circ} \mathrm{J}$ sets
${ }^{\circ} \mathrm{K}$ resets
${ }^{\circ} \mathrm{J}=\mathrm{K}=1$-> invert output

## Fig. 5-12 JK Flip-Flop

|  |  |  |  |  |
| ---: | :--- | :--- | :--- | :--- |
| 0 | K | CLK | Q | $Q^{\prime}$ |
| 0 | 0 | $\uparrow$ | $\mathrm{Q}_{0}$ | $\mathrm{Q}_{0}{ }^{\prime}$ |
| 0 | 1 | $\uparrow$ | 0 | 1 |
| 1 | 0 | $\uparrow$ | 1 | 0 |
| 1 | 1 | $\uparrow$ | TOGGLE |  |
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## Positive Edge Triggered T Flip-Flop


(a) From $J K$ flip-flop

(b) From $D$ flip-flop

(c) Graphic symbol

Fig. 5-13 T Flip-Flop
${ }^{\circ}$ Created from D flop
${ }^{\circ} \mathrm{T}=0$-> keep current
${ }^{\circ} \mathrm{K}$ resets
${ }^{\circ} \mathrm{T}=1$-> invert current

| T | C | Q | $\mathrm{Q}^{\prime}$ |
| :---: | :---: | :---: | :---: |
| 0 | $\uparrow$ | $\mathrm{Q}_{0}$ | $\mathrm{Q}_{0}{ }^{\prime}$ |
| 1 | $\uparrow$ | TOGGLE |  |

## Characteristic Tables

- Characteristic tables Describes the operation of the flip-flop in a tabular form JK Flip Flop

| D Flip Flop |  |  | F Flip Flop |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
|  | D | $\mathrm{Q}(\mathrm{t}+1)$ |  | T | $\mathrm{Q}(\mathrm{t}+1)$ |
| 0 | 0 |  | 0 | $\mathrm{Q}(\mathrm{t})$ |  |
| 1 | 1 |  | 1 | $\mathrm{Q}^{\prime}(\mathrm{t})$ |  |

10
11 Q'(t)

## Characteristic Equation

- For D Flip-Flop

$$
Q(t+1)=D
$$

- For JK Flip-Flop

$$
Q(t+1)=J Q^{\prime}+K^{\prime} Q
$$

- For T Flip-Flop

$$
Q(t+1)=T \oplus Q=T Q^{\prime}+T^{\prime} Q
$$

## Analysis of Clocked Sequential Circuits

- State equation, state table
- State diagram: states are represented by circuits, transitions by arcs labeled I/O
- Flip-Flop Equations:


$$
\begin{aligned}
& \text { Output equation } \\
& \qquad \begin{array}{l}
y(t)=x(t) Q_{1}(t) Q_{0}(t) \\
Q_{0}(t+1)= \\
Q_{1}(t)=x(t) Q_{1}(t) \\
\text { State equations }(t+1)=D_{1}(t)=x(t)+Q_{0} \\
\\
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\end{array}
\end{aligned}
$$

## State Table

- Sequence of outputs, inputs, and flip flop states enumerated in state table
- Present state indicates current value of flip flops
- Next state indicates state after next rising clock edge
- Output is output value on current clock edge

State Table

|  | Present <br> State |  | Next State |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  | x=0 | $\mathrm{x}=1$ | $\mathrm{x}=0$ | $\mathrm{x}=1$ |

## State Table

- All possible input combinations enumerated
- All possible state combinations enumerated
- Separate columns for each output value.
- Sometimes easier to designate a symbol for each state.

|  |  |  | Next State |  | Output |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Let: | State | $\mathrm{x}=0$ | $\mathrm{x}=1$ | $\mathrm{x}=0$ | $\mathrm{x}=1$ |  |
|  | $\mathrm{~s}_{0}$ | $\mathrm{~s}_{0}$ | $\mathrm{~s}_{2}$ | 0 | 0 |  |
| $\mathrm{~s}_{0}=00$ | $\mathrm{~s}_{1}$ | $\mathrm{~s}_{2}$ | $\mathrm{~s}_{2}$ | 0 | 0 |  |
| $\mathrm{~s}_{1}=01$ | $\mathrm{~s}_{2}$ | $\mathrm{~s}_{0}$ | $\mathrm{~s}_{3}$ | 0 | 0 |  |
| $\mathrm{~s}_{2}=10$ | $\mathrm{~s}_{3}$ | $\mathrm{~s}_{2}$ | $\mathrm{~s}_{3}$ | 0 | 1 |  |

## State Diagram

| Present <br> State | Next State |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{x}=0$ | $\mathrm{x}=1$ | $\mathrm{x}=0$ | $\mathrm{x}=1$ |
| 0 | 00 | 10 | 0 | 0 |
| 0 | 10 | 10 | 0 | 0 |
| 10 | 00 | 11 | 0 | 0 |
| 11 | 10 | 11 | 0 | 1 |



## State Diagram

- Each state has two arrows leaving
- One for $\mathrm{x}=0$ and one for $\mathrm{x}=1$
- Unlimited arrows can enter a state
- Note use of state names in this example
- Easier to identify



## Flip-Flop Input Equations

- Boolean expressions which indicate the input to the flip flops.

$D_{Q 0}=x Q_{1}$
$D_{Q 1}=x+Q_{0}$$\quad$ Format implies type of flop used


## Analysis with D Flip-Flop


(a) Circuit diagram

(b) State table



## J-K Flip Flop

| Present |  | Input |
| :---: | :---: | :---: |
| A | B | X |
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

## J-K Flip-Flop

1


## Mealy and Moore Models

- Mealy model: the output is a function of both the present state and the input.
- Moore model: The output is a function of the present state only.
- In a Moore model, the output is synchronized with the clock (since it changes only if the state changes).
- In a Mealy model the output may change if the input changes during the clock cycle.


## HDL For Sequential Circuits

## Behavioral Modeling

| iitial | initial |
| :--- | :---: |
| begin | begin |
| clock $=1$ 'b0; | clock $=1$ 'b0; |
| repeat( 30$)$ | $\# 300$ \$finish; |
| $\# 10$ clock $=\sim$ clock | end; |
| end | always |
|  | $\# 10$ clock=~clock; |

## Behavioral Modeling

- One ways to use always statement always @ (A or B or reset)
- What fellows will be done if any changes in A, B, or reset,
- Or, we can use

Always @ (posedge clock or negedge reset)

## Behavioral Modeling

- Blocking assignmnet
$\mathrm{B}=\mathrm{A}$;
C=B+1;
- A is assigned to $B$, and the new value is incremented
- Non-blocking assignment
$\mathrm{B}<=\mathrm{A} ; \mathrm{C}<=\mathrm{B}+1$;
- Incrementing old value of A, because assignment is done after all the values in the block are done


## Flip-Flops and Latches

```
//Description od D latch
module D_Latch (Q,D,control);
    output Q;
    input D, control;
    reg Q;
    always @ (control or D)
    if (control) Q=D;
endmodule
```

//D Flip-Flop
module D_FF(Q, D, CLK); output Q;
input $\mathrm{D}, \mathrm{CLK}$; reg Q ;
always @ (posedge CLK) Q=D;
endmodule

## Flip-Flops and Latches



## Other Types of Flip-Flop


//A Flip-Flop (using D)
Module TFF(Q,T,CLK,RST)
Output Q;
Input T,CLK,RST;
Wire DT;
Assign $\mathrm{DT}=\mathrm{Q}^{\wedge} \mathrm{T}$;
DFF TF1(Q,DT,CLK,RST);
endmodule
//JK Flip-Flop (using D)
Module JKFF(Q,T,CLK,RST);
Output Q;
Input JK,CLK,RST;
Wire JK;
Assign JK=(J\&~Q)|(~K\&Q);
DFF JK1(Q,JK,CLK,RST);
endmodule

## Other Types of Flip-Flop

//Functional Description of JK Flip-Flop
Module JK_FF(J,K,CLK, Q, Qnot);
output Q,Qnot;
input J,K,CLK;
reg Q;
assign Qnot=~Q
allways @ (posedge CLK)
case (\{J,K\})
2'b00: Q=Q;
2'b01: Q=1'b0;
2'b10: Q=1'b1;
2'b11: Q=~Q;
endcase
endmodule

## State Diagram


module Mealy Model(x,y,CLK,RST)
input $x, C L K, R S T$;
output y
reg y ;
parameter $\mathrm{S} 0=2^{\prime} \mathrm{b} 00$, $\mathrm{s} 1=2$ 'b01, s2=2'b10, $\mathrm{S} 3=2$ 'b11;
always @ (posedge CLK or negedge RST)
if( $\sim$ RST) Prstate = S0;
else Prstate=Nxtstate;
case(Prstate)
S0: if(x) Nxtstate = S1;
else Nxtstate $=\mathrm{S} 0$;
else Nxtstate $=$ S0;
endcase

## State Diagram

//Moore state diagram
module Mealy_Model(x,AB,CLK,RST);
input $x, C L K, R S T$;
output [:0]AB;
reg [1:0] State;
parameter $\mathrm{S} 0=2$ 'b00, $\mathrm{s} 1=2$ 'b01, $\mathrm{s} 2=2$ 'b10, $\mathrm{S} 3=2$ 'b11;
always @ (posedge CLK or negedge RST)
if( $\sim$ RST) Prstate = S0;
else Prstate=Nxtstate;
always @ (Prstate or x)
case(Prstate)

| S0: if( | $\begin{aligned} & \text { State = S1; } \\ & \text { else State = S0; } \end{aligned}$ |
| :---: | :---: |
| S0: if (x) | $\begin{aligned} & \text { State = s2; } \\ & \quad \text { else State }=\text { S3; } \end{aligned}$ |
| $\mathrm{S} 0 \text { : if }(\mathrm{x})$ | $\begin{aligned} & \text { State = S3; } \\ & \quad \text { else State }=\text { S2; } \end{aligned}$ |
| S0: if(x) | $\begin{aligned} & \text { State }=S 0 ; \\ & \quad \text { else State }=\text { S3; } \end{aligned}$ |

endcase
assign $A B=$ State;
endmodule

## Structural Description

//Figure 5-20 in the text book
module TCircuit(x,y,A,B,CLK,RST);
input $x$. CLK, RST;
output $\mathrm{y}, \mathrm{A}, \mathrm{B}$;
wire TA,TB;
//Flip-Flop input equation
assign $T B=x$,
TA=x \& B;
assign $y=A \& B ;$
// Instantiate 2 ff's
T_FF BF (B,TB,CLK,RST);
T_FF AF (A,TA,CLK,RST);
endmodule

## Structural Description



## State Reduction and Assignment

- In this part, we study some properties of the sequential circuits in order to reduce the number of gates or flip-flops.
- If we have $m$ flip-flops, we can have up tp $2^{m}$ states.
- Thus reducing the number of states, may result in reduction of the number of flip-flops.
- Sometimes, we care only about the output produced by a specific input sequence, while in others (counters) the states are important (considered as the output).


## State Reduction and Assignment

- There are infinite number of sequences that could be applied to the circuit, producing some output.

8. Two circuits (may have different states) will produce the same output for the same input are considered equivalent (from the input output point of view).

- We want to find a way to reduce the number of states and keeping the same input-output equivalence.


## State Reduction



## State reduction

|  | next state | Output |
| :--- | :--- | :--- |
| Present State | $\mathbf{x = 0} \quad \mathbf{x = 1}$ | $\mathbf{x = 0} \quad \mathbf{x = 1}$ |
| a | a | b |
| b | c | d |
| c | 0 | 0 |
| d | a | d |
| e | e | f |
| f | a | f |
| g | g | f |
| a | f | 0 |
| 1 |  |  |

look for any two state that go to the same next state and have the same output for both input combination (states $g$ and $e$ ), remove one and replace it by the other

## State reduction

| Present State | next state $x=0 x=1$ | Output <br> $\mathrm{x}=0 \mathrm{x}=1$ | $G$ is removed and every occurrence of $g$ |
| :---: | :---: | :---: | :---: |
| a | a b | 00 | is replaced by g in the |
| b | c d | 00 | emaining of the table |
| c | a d | 0 0 |  |
| d | e | $0 \quad 1$ |  |
| e | a | 0 |  |
| f | e f | $0 \quad 1$ | $\square$ |

States $d$ and $f$ are equivalent
f is removed and replaced by e

## State reduction



Fig. 5-23 Reduced State Diagram

## State Assignment

- In this stage, you have to map states to binary numbers. You can use any mapping scheme.
- binary
- Gray code
- one-hot (more flip-flops)


## Design Procedure

- From the word description Derive the state diagram
- Reduce the number of states if necessary
- Assign Binary values to states.
- Obtain the binary-coded state table
- Choose the type of flip-flop
- Derive the simplified flip-flop input and output equations
- Draw the logic diagram


## Design

- Design a circuit that detects three or more consecutive ones.



## State table for sequence detector



## Synthesis with D Flip-Flops

- Construct the state table

$$
\begin{aligned}
& A(t+1)=D_{A}(A, B, X)=\sum_{A B X}(3,5,7) \\
& B(t+1)=D_{B}(A, B, x)=\sum_{A B X}(1,5,7) \\
& y(A, B, x)=\sum_{A B x}(6,7) \\
& D_{A}=A x+B x \\
& D_{B}=A x+B^{\prime} x \\
& y=A B
\end{aligned}
$$

## Synthesis with D Flip-Flops



## Synthesis with J-K Flip-Flop

- With J-K flip-flop, it is not as easy as D, since the output is not the same as the previous input, we need an excitation table

| $\mathrm{Q}(\mathrm{t})$ | $\mathrm{Q}(\mathrm{t}+1)$ | J | K | $\mathrm{Q}(\mathrm{t})$ | $\mathrm{Q}(\mathrm{t}+1)$ | T |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | X | 0 | 0 | 0 |
| 0 | 1 | 1 | X | 0 | 1 | 1 |
| 1 | 0 | X | 1 | 1 | 0 | 1 |
| 1 | 1 | X | 0 | 1 | 1 | 0 |



## Synthesis with J-K Flip Flop



## Synthesis with J-K Flip-Flop



Fig. 5-28 Logic Diagram for Sequential Circuit with $J K$ Flip-Flops

## Example: Odd Number of 1's

Assert output whenever input bit stream has odd \# of 1's


State
Diagram

- Note: Present state and output are the same value


## Odd Number of 1's

Example: Odd Parity Checker
Next State/Output Functions
NS = PS xor PI; OUT = PS



## Vending Machine

-Deliver package of gum after 15 cents deposited
$\square$ Single coin slot for dimes, nickels
aNo change
-Design the FSM using combinational logic and flip flops


## Vending Machine




Symbolic State Table

## State encoding

| $\begin{aligned} & \text { Present State } \\ & \mathrm{Q}_{1} \mathrm{Q}_{0} \end{aligned}$ | Inputs |  | Next State |  | Output Open |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | D | N | $\mathrm{D}_{1}$ | $\mathrm{D}_{0}$ |  |
| 00 | 0 | 0 | 0 | 0 | 0 |
|  | 0 | 1 | 0 | 1 | 0 |
|  | 1 | 0 | 1 | 0 | 0 |
|  | 1 | 1 | X | X | X |
| 01 | 0 | 0 | 0 | 1 | 0 |
|  | 0 | 1 | 1 | 0 | 0 |
|  | 1 | 0 | 1 | 1 | 0 |
|  | 1 | 1 | X | X | X |
| 10 | 0 | 0 | 1 | 0 | 0 |
|  | 0 | 1 | 1 | 1 | 0 |
|  | 1 | 0 | 1 | 1 | 0 |
|  | 1 | 1 | X | X | X |
| 11 | 0 | 0 | 1 | 1 | 1 |
|  | 0 | 1 | 1 | 1 | 1 |
|  | 1 | 0 | 1 | 1 | 1 |
|  | 1 | 1 | X | X | X |

## Vending Machine


$\mathrm{D}_{1}=\mathrm{D}+\mathrm{Q}_{1}+\mathrm{NQ}_{0}$

$\mathrm{D}_{0}=\mathrm{NQ}_{1}+\mathrm{NQ}_{0}^{\prime}+\mathrm{Q}_{0} \mathrm{~N}^{\prime}+$ $\mathrm{DQ}_{1}$

$\mathrm{Q}_{1} \mathrm{Q}_{0}$


## 3-bit Binary Counter



Fig. 5-29 State Diagram of 3-Bit Binary Counter

## 3-Bit Binary Counter

| Present State | Next State | T-F Inputs |
| :---: | :---: | :---: |
| $\mathrm{A}_{2} \mathrm{~A}_{1} \mathrm{~A}_{0}$ | $\mathrm{A}_{2} \mathrm{~A}_{1} \mathrm{~A}_{0}$ | $\mathrm{T}_{\mathrm{A} 2} \mathrm{~T}_{\mathrm{A} 1} \mathrm{~T}_{\mathrm{A} 0}$ |
| 000 | $0 \quad 01$ | $0 \quad 0 \quad 1$ |
| $0 \quad 01$ | 0110 | 011 |
| 010 | $\begin{array}{lll}0 & 1 & 1\end{array}$ | $0 \quad 0 \quad 1$ |
| $\begin{array}{llll}0 & 1 & 1\end{array}$ | 100 | 111 |
| 100 | 1001 | $0 \quad 0 \quad 1$ |
| 101 | 110 | $0 \begin{array}{lll}0 & 1\end{array}$ |
| 110 | $\begin{array}{lll}1 & 1 & 1\end{array}$ | $0 \quad 0 \quad 1$ |
| 111 | $\begin{array}{lll} 0 & 0 & 0 \end{array}$ | 111 |

## 3-Bit Binary Counter



Fig. 5-30 Maps for 3-Bit Binary Counter

## 3-Bit Binary Counter



Fig. 5-31 Logic Diagram of 3-Bit Binary Counter

## Example (Mealy vs. Moore)

- Design a circuit that asserts its output for one cycle for every change in the input from 0 to 1
- We will try two different approaches.
- Compare between them


## Solution 1



| ZERO $\left\{\begin{array}{ll\|ll}\text { IN } & \text { PS } & \text { NS } & \text { OUT } \\ \hline 0 & 00 & 00 & 0 \\ 1 & 00 & 01 & 0\end{array}\right.$ |
| :---: |
| CHANGE $\left\{\begin{array}{llll}0 & 01 & 00 & 1 \\ 1 & 01 & 11 & 1\end{array}\right.$ |
| $\begin{array}{llll}0 & 11 & 00 & 0 \\ 1 & 11 & 11 & 0\end{array}$ |



## Solution 2





## Comparison



