Recursion and Logarithms

CSE 2011

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Recursion

- In some problems, it may be natural to define the problem in terms of the problem itself.
- Recursion is useful for problems that can be represented by a simpler version of the same problem.
- Example: the factorial function

  6! = 6 * 5 * 4 * 3 * 2 * 1

  We could write:

  6! = 6 * 5!

Recursion (2)

- Recursion is one way to decompose a task into smaller subtasks. At least one of the subtasks is a smaller example of the same task.
- The smallest example of the same task has a non-recursive solution.
- Example: the factorial function

  `n! = n*(n-1)! and 1! = 1`

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Example: Factorial Function

- In general, we can express the factorial function as follows:

  `n! = n*(n-1)!`

  Is this correct? Well… almost.

- The factorial function is only defined for *positive* integers. So we should be more precise:

```
f(n) = 1          if n = 1
     = n*f(n-1)   if n > 1
```

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Factorial Function: Pseudo-code

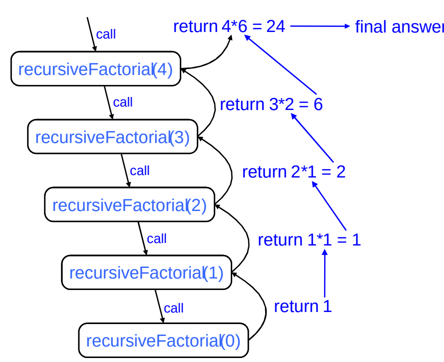
```
int recFactorial(int n){
   if(n == 0)
      return 1;
   else
      return n * recFactorial(n-1);
}
```

***recursion* means that a function calls itself**

Visualizing Recursion

- Recursion trace
- A box for each recursive call
- An arrow from each caller to callee
- An arrow from each callee to caller showing return value

Example recursion trace:



Recursive vs. Iterative Solutions

- For certain problems (such as the factorial function), a recursive solution often leads to short and elegant code. Compare the recursive solution with the iterative solution:

```
int fac(int numb) {
  if (numb == 0)
    return 1;
  else
    return
  (numb*fac(numb-1));
}
```

```
int fac(int numb){
  int product=1;
  while(numb>1){
    product *= numb;
    numb--;
  }
  return product;
}
```

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A Word of Caution

- To trace recursion, function calls operate as a stack – the new function is put on top of the caller.
- We have to pay a price for recursion:
  - calling a function consumes more time and memory than adjusting a loop counter.
  - high performance applications (graphic action games, simulations of nuclear explosions) hardly ever use recursion.
- In less demanding applications, recursion is an attractive alternative for iteration (for the right problems!)

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Infinite Loops

If we use iteration, we must be careful not to create an infinite loop by accident.

```
for (int incr=1; incr!=10; incr+=2)
  ...

int result = 1;
while(result > 0){
  ...
  result++;
}
```

Oops!

Oops!

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Infinite Recursion

Similarly, if we use recursion, we must be careful not to create an infinite chain of function calls.

```
int fac(int numb){
   return numb * fac(numb-1);
}

int fac(int numb){
   if (numb==0)
      return 1;
   else
      return numb * fac(numb+1);
}
```

Oops!
No termination condition

Oops!

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Tips

We must always make sure that the recursion *bottoms out*:

- A recursive function must contain at least one non-recursive branch.

- The recursive calls must eventually lead to a non-recursive branch.

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General Form of Recursion

- How to write recursively?

```
int recur_fn(parameters){
  if (stopping_condition)
      return stopping_value;
  // other stopping conditions if needed
  return function of recur_fn(revised_parameters)
}
```

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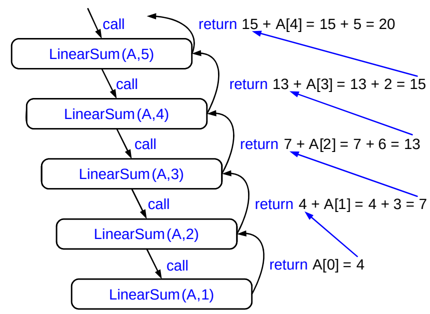
Example: Sum of an Array

Algorithm LinearSum(*A*, *n*):
Input:
A integer array *A* and an integer *n* = 1, such that *A* has at least *n* elements
Output:
The sum of the first *n* integers in *A*
if *n* = 1 **then**
  return *A*[0]
else
  return LinearSum(*A*, *n* - 1) + *A*[*n* - 1]

Example recursion trace:



Example: Reversing an Array

Algorithm ReverseArray(*A*, *i*, *j*):
Input: An array *A* and nonnegative integer indices *i* and *j*
Output: The reversal of the elements in *A* starting at index *i* and ending at *j*
if *i* < *j* **then**
  Swap *A*[*i*] and *A*[*j*]
  ReverseArray(*A*, *i* + 1, *j* - 1)
return

Defining Arguments for Recursion

- In creating recursive methods, it is important to define the methods in ways that facilitate recursion.
- This sometimes requires we define additional paramaters that are passed to the method.
- For example, we defined the array reversal method as ReverseArray(*A, i, j*), not ReverseArray(*A*).

Linear Recursion

- The above 2 examples use **linear recursion.**

Linear Recursion (2)

- **Test for base cases.**
  - Begin by testing for a set of base cases (there should be at least one).
  - Every possible chain of recursive calls **must** eventually reach a base case, and the handling of each base case should not use recursion.
- ***Recur once.***
  - Perform a single recursive call. (This recursive step may involve a test that decides which of several possible recursive calls to make, but it should ultimately choose to make just one of these calls each time we perform this step.)
  - Define each possible recursive call so that it makes progress towards a base case.

Tail Recursion

- Tail recursion occurs when a linearly recursive method makes its recursive call as its last step.
- The array reversal method is an example.
- Such methods can be easily converted to non-recursive methods (which saves on some resources).
- Example:

Algorithm IterativeReverseArray(*A, i, j*):
 Input: An array *A* and nonnegative integer indices *i* and *j*
 Output: The reversal of the elements in *A* starting at index *i* and ending at *j*
 while *i* < *j* **do**
  Swap *A*[*i*] and *A*[*j*]
  i = *i* + 1
  j = *j* - 1
 return

Binary Recursion

- Binary recursion occurs whenever there are **two** recursive calls for each non-base case.
- Example: binary search

Example: Binary Search

- Search for an element in an *ordered* array
  - Sequential search
  - Binary search
- Binary search
  - Compare the search element with the middle element of the array
  - If not equal, then apply binary search to half of the array (if not empty) where the search element would be.

Binary Search with Recursion

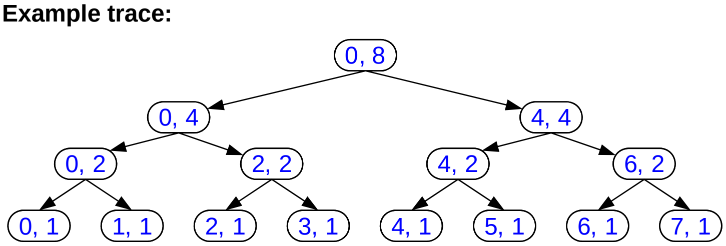
```
// Searches an ordered array of integers using recursion
int bsearchr(const int data[], // input: array
             int first,        // input: lower bound
             int last,        // input: upper bound
             int value         // input: value to find
        )// return index if found, otherwise return –1

{  int middle = (first + last) / 2;
   if (data[middle] == value)
      return middle;
   else if (first >= last)
      return -1;
   else if (value < data[middle])
      return bsearchr(data, first, middle-1, value);
   else
      return bsearchr(data, middle+1, last, value);
}
```


Another Binary Recusive Method

- Problem: add all the numbers in an integer array A:

Algorithm BinarySum(*A, i, n*):
__Input:__ An array *A* and integers *i* and *n*
__Output:__ The sum of the *n* integers in *A* starting at index *i*
if *n* = 1 **then**
return *A*[*i*]
return BinarySum(*A, i, n*/ 2) + BinarySum(*A, i* + *n*/ 2, *n*/ 2)

- **Example trace:**



Multiple Recursion

- Multiple recursion: makes potentially many recursive calls (not just one or two).
- Not covered in this course.

Running Time of Recursive Methods

- Could be just a hidden “for”/ “while” loop
  - See “Tail Recursion” slide
- Logarithmic (next)
  - Examples: binary search, exponentiation
- Solving a recurrence
  - Example: merge sort

Logarithms

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Logarithmic Running Time

- An algorithm is O(logN) if it takes constant (O(1)) time to cut the problem size by a fraction (e.g., ½).
- An algorithm is O(N) if constant time is required to merely reduce the problem by a constant amount (e.g., by 1).

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Binary Search

```
int binarySearch (int[] a, int x)
{
/*1*/    int low = 0, high = a.size() - 1;
/*2*/    while (low <= high)
         {
/*3*/      int mid = (low + high) / 2;
/*4*/      if (a[mid] < x)
/*5*/        low = mid + 1;
/*6*/      else if (x < a[mid])
/*7*/        high = mid - 1;
           else
/*8*/        return mid;  // found
         }
/*9*/    return NOT_FOUND
}
```

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Exponentiation x^n

```
long exp(long x, int n)
{
/*1*/    if (n==0)
/*2*/      return 1;
/*3*/    if (n==1)
/*4*/      return x;
/*5*/    if (isEven(n))
/*6*/      return exp(x*x, n/2);
         else
/*7*/      return exp(x*x, n/2)*x;
}
```

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Euclid's Algorithm

- Computing the greatest common divisor (GCD) of two integers

```
long gcd (long m, long n) // assuming m>=n
{
/*1*/   while (n!=0)
         {
/*2*/     long rem = m%n;
/*3*/     m = n;
/*4*/     n = rem;
        }
/*5*/   return m;
}
```

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Next time …

- Merge Sort
- Arrays, Lists (chapter 3)

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