Depth-First Search

CSE 2011
Fall 2009

Depth-First Search (DFS)

- DFS is another popular graph search strategy
  - Idea is similar to pre-order traversal (visit node, then visit children recursively)
- DFS will continue to visit neighbors in a recursive pattern
  - Whenever we visit v from u, we recursively visit all unvisited neighbors of v. Then we backtrack (return) to u.

DFS Algorithm

```
Algorithm DFS(s)
1.   for each vertex v
2.       do flag[v] := false;
3.   RDFS(s);

Algorithm RDFS(v)
1.   flag[v] := true;
2.   for each neighbor w of v
3.       do if flag[w] = false
4.           then RDFS(w);
```

Flag all vertices as not visited

Flag yourself as visited

For unvisited neighbors, call *RDFS*(*w*) recursively

We can also record the paths using *prev*[ ].
Where do we insert the code for *prev*[ ]?

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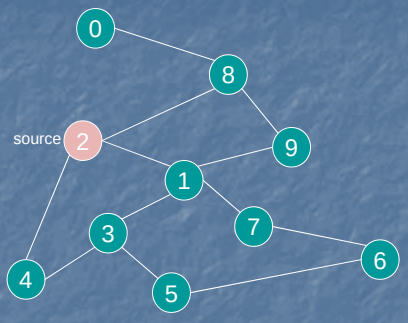
Example



Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

0	F
1	F
2	F
3	F
4	F
5	F
6	F
7	F
8	F
9	F

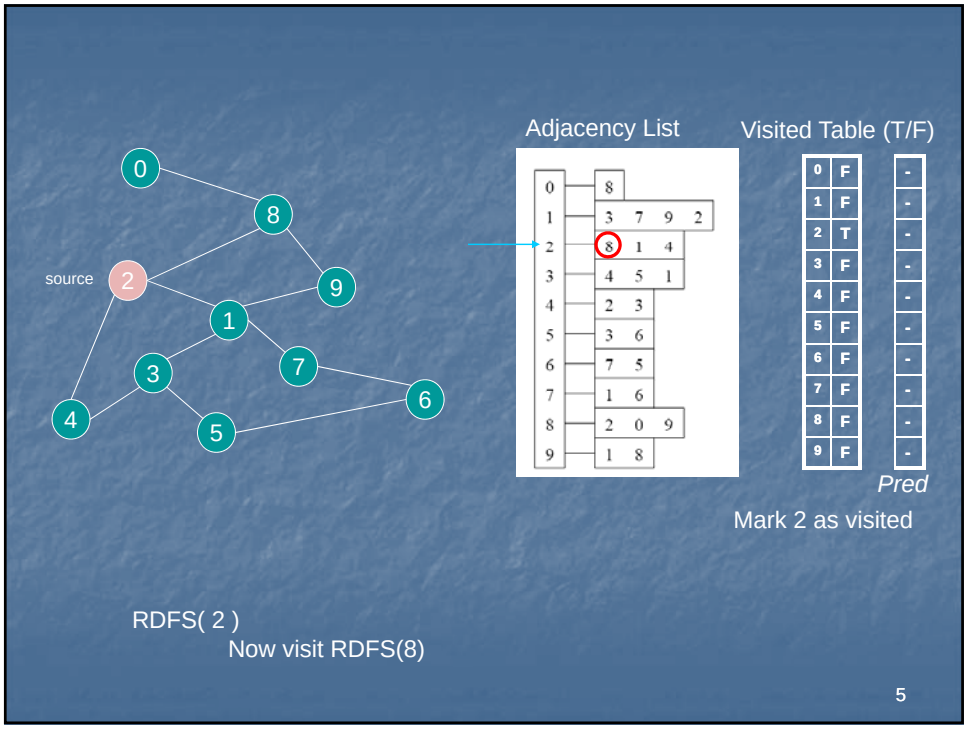
-
-
-
-
-
-
-
-
-
-

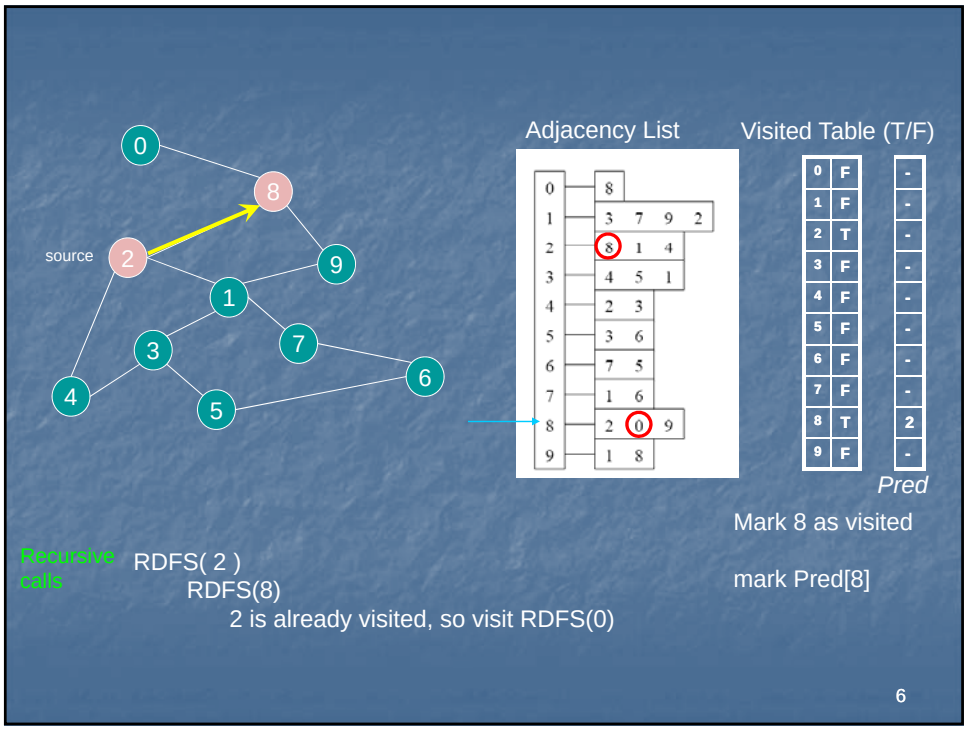
Pred

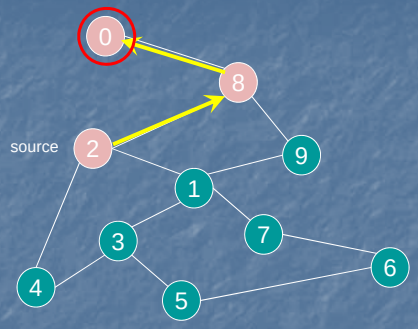
Initialize visited table (all False)

Initialize Pred to -1

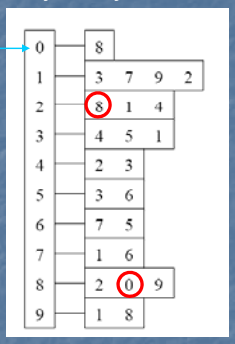
4


Adjacency List
Visited Table (T/F)
source
Pred
Mark 2 as visited
RDFS(2)
Now visit RDFS(8)
5


Adjacency List
Visited Table (T/F)
source
Pred
Mark 8 as visited
mark Pred[8]
Recursive calls
RDFS(2)
RDFS(8)
2 is already visited, so visit RDFS(0)
6



Adjacency List



Visited Table (T/F)

0	T
1	F
2	T
3	F
4	F
5	F
6	F
7	F
8	T
9	F

8
-
-
-
-
-
-
-
2
-

Pred

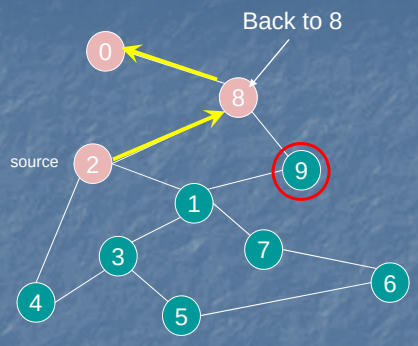
Mark 0 as visited

Mark Pred[0]

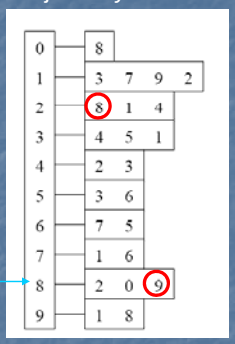
Recursive calls

RDFS(2)
RDFS(8)
RDFS(0) -> no unvisited neighbors, return to call RDFS(8)

7



Adjacency List



Visited Table (T/F)

0	T
1	F
2	T
3	F
4	F
5	F
6	F
7	F
8	T
9	F

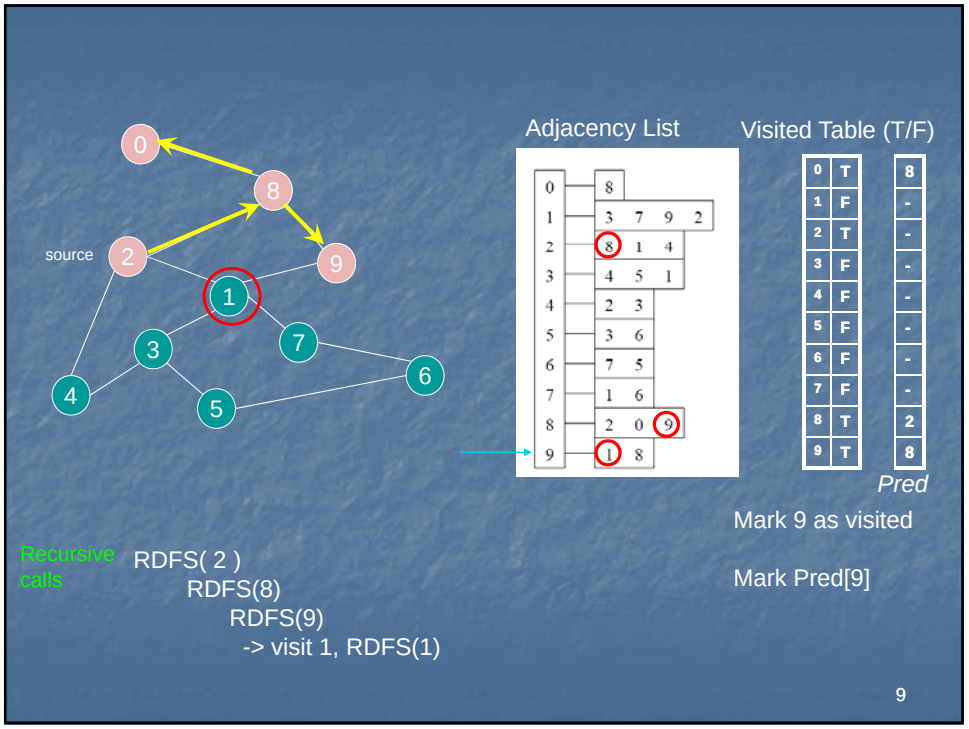
8
-
-
-
-
-
-
-
2
-

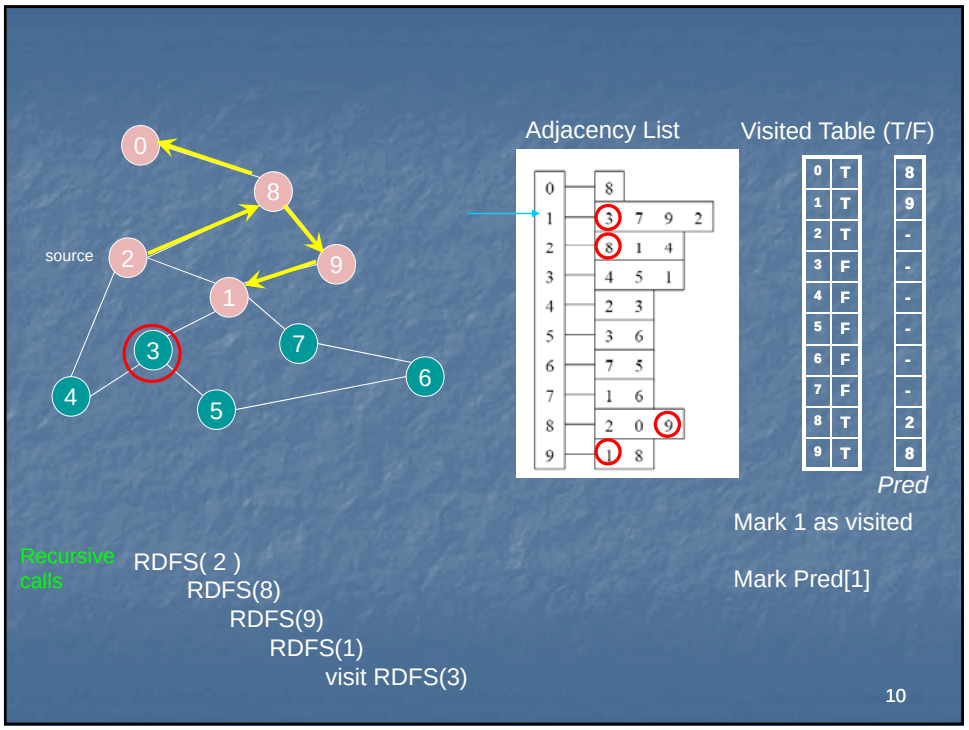
Pred

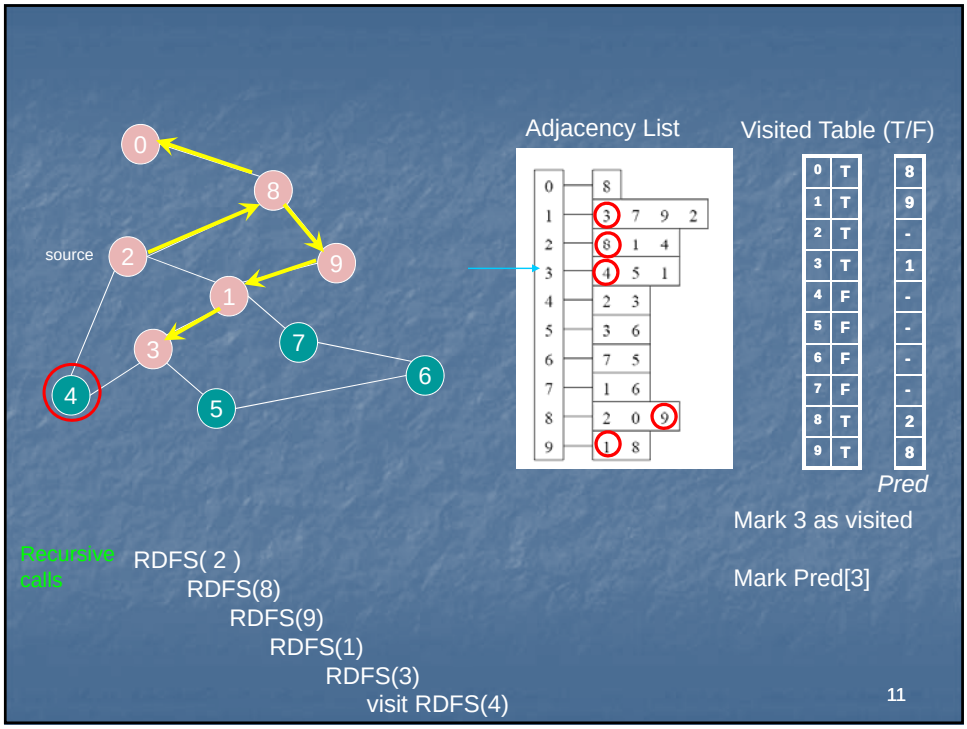
Recursive calls

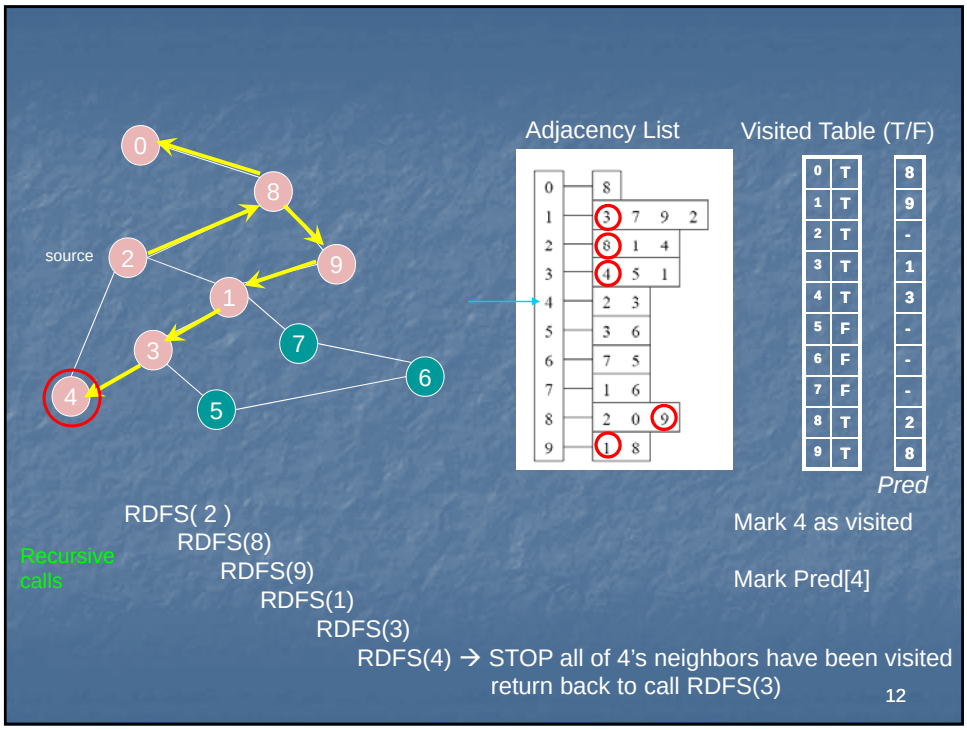
RDFS(2)
RDFS(8)
Now visit 9 -> RDFS(9)

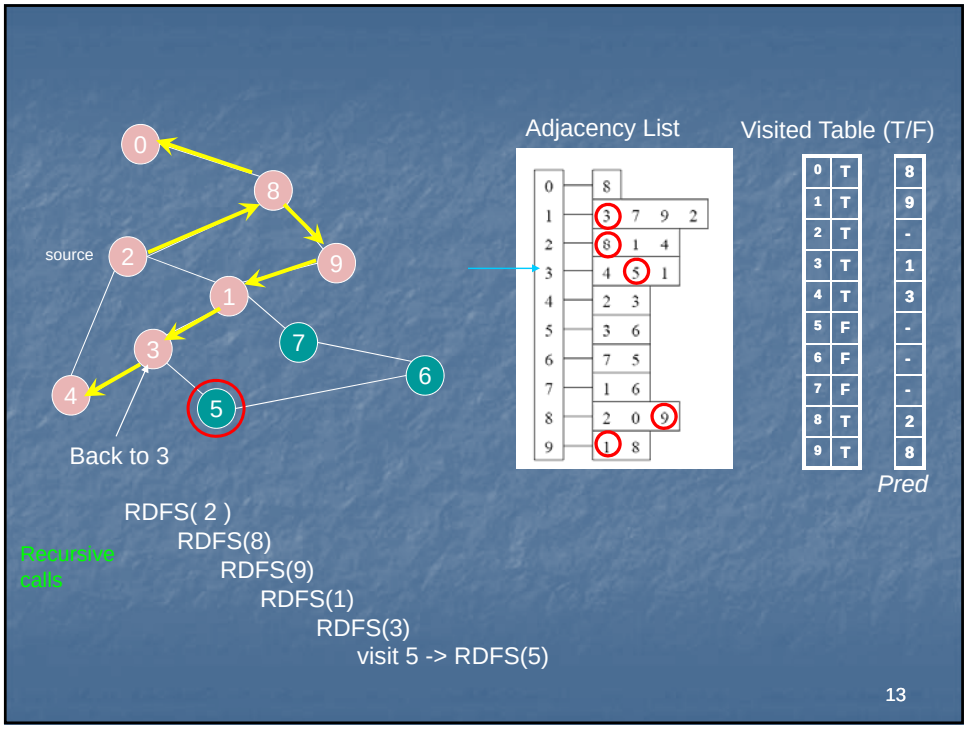
8

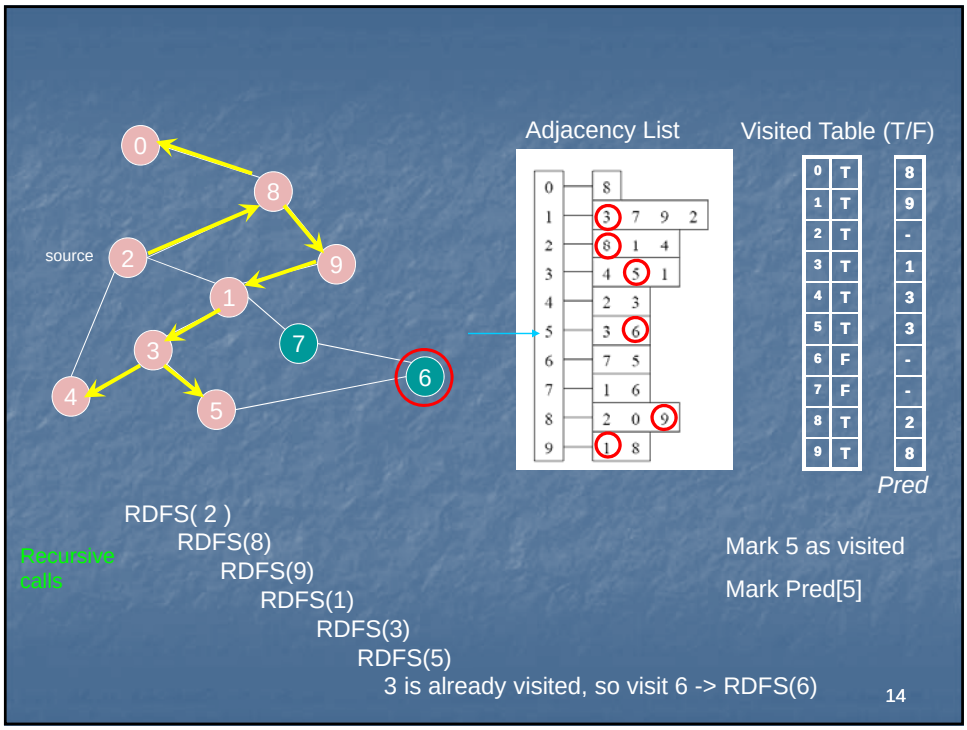

Adjacency List
Visited Table (T/F)
Pred
source
Mark 9 as visited
Mark Pred[9]
Recursive calls
RDFS(2)
RDFS(8)
RDFS(9)
-> visit 1, RDFS(1)
9

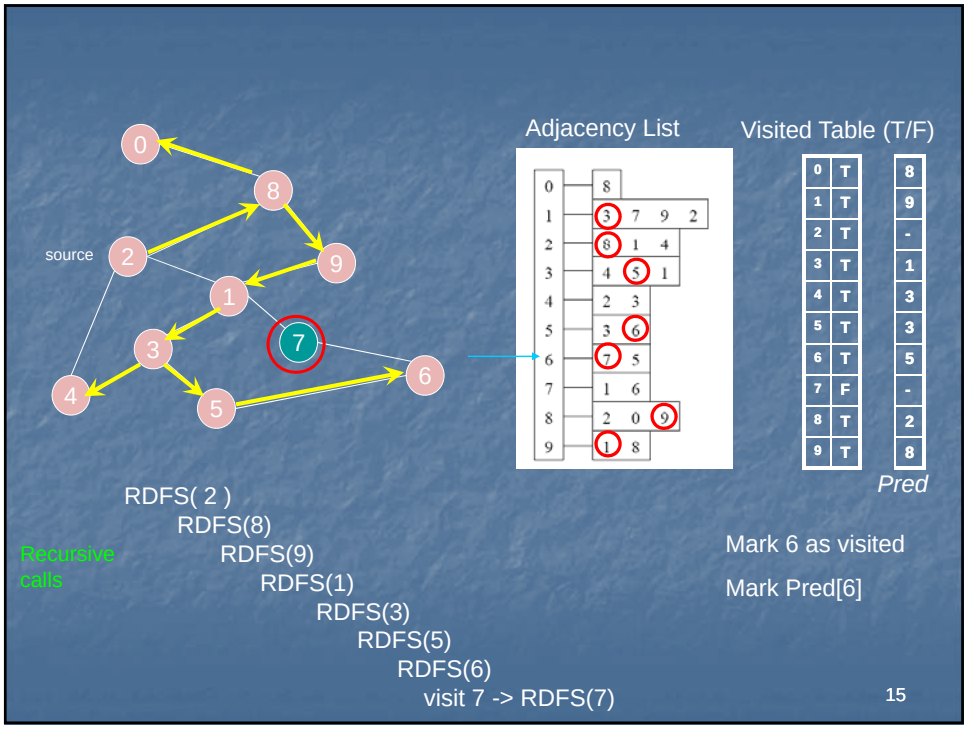

Adjacency List
Visited Table (T/F)
Pred
source
Mark 1 as visited
Mark Pred[1]
Recursive calls
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
visit RDFS(3)
10

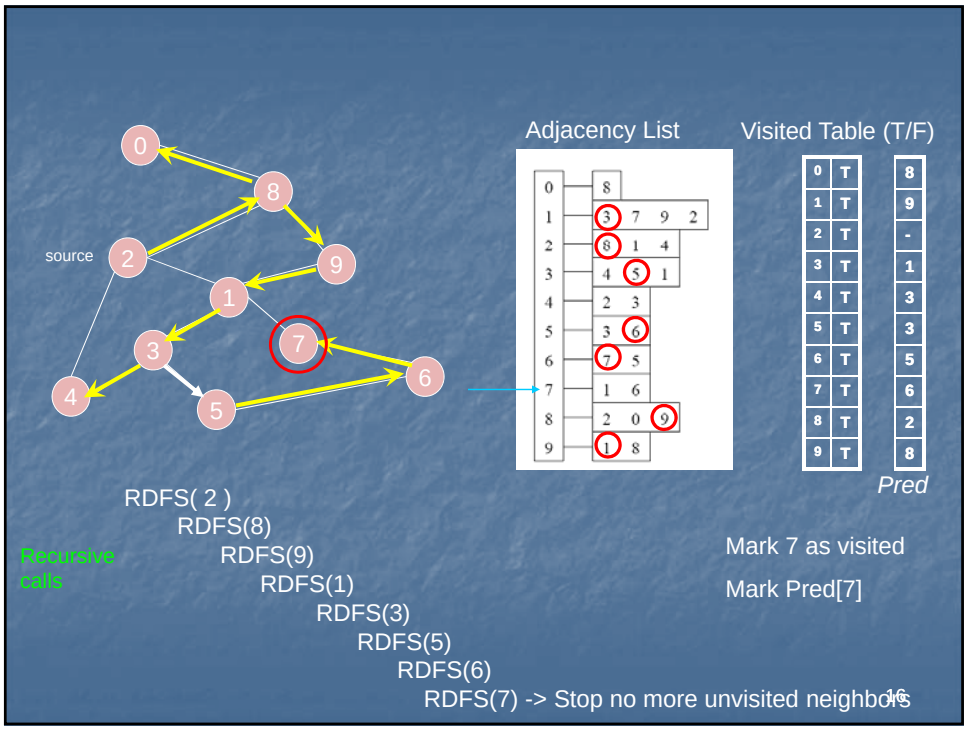

Adjacency List
Visited Table (T/F)
source
Pred
Mark 3 as visited
Mark Pred[3]
Recursive calls
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
visit RDFS(4)
11

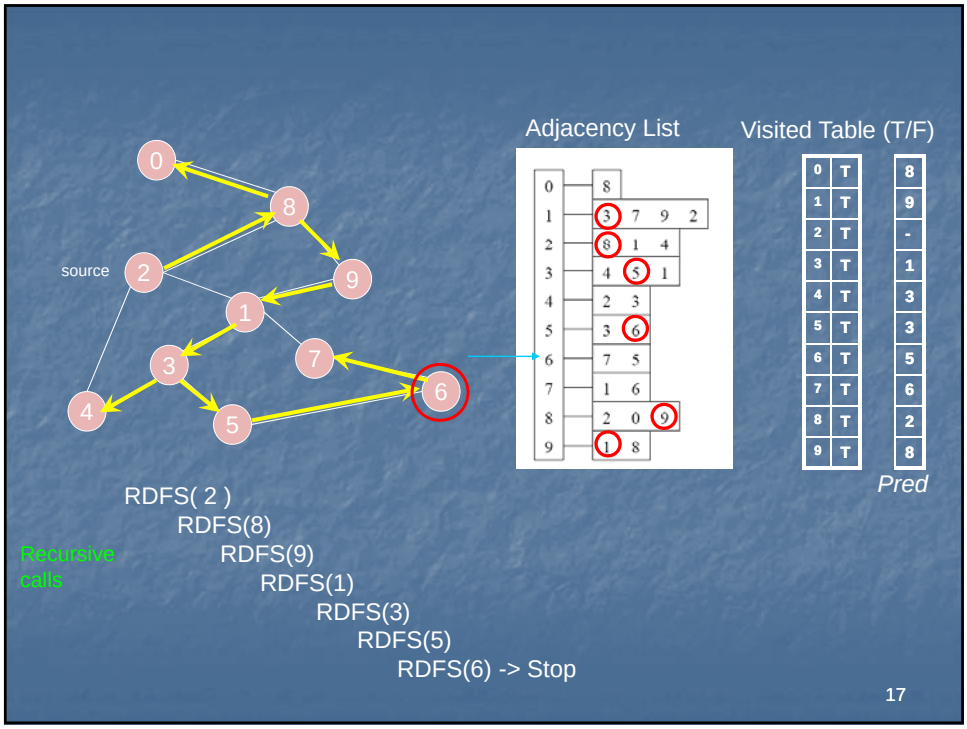

Adjacency List
Visited Table (T/F)
source
Pred
Mark 4 as visited
Mark Pred[4]
Recursive calls
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(4) → STOP all of 4's neighbors have been visited
return back to call RDFS(3)
12

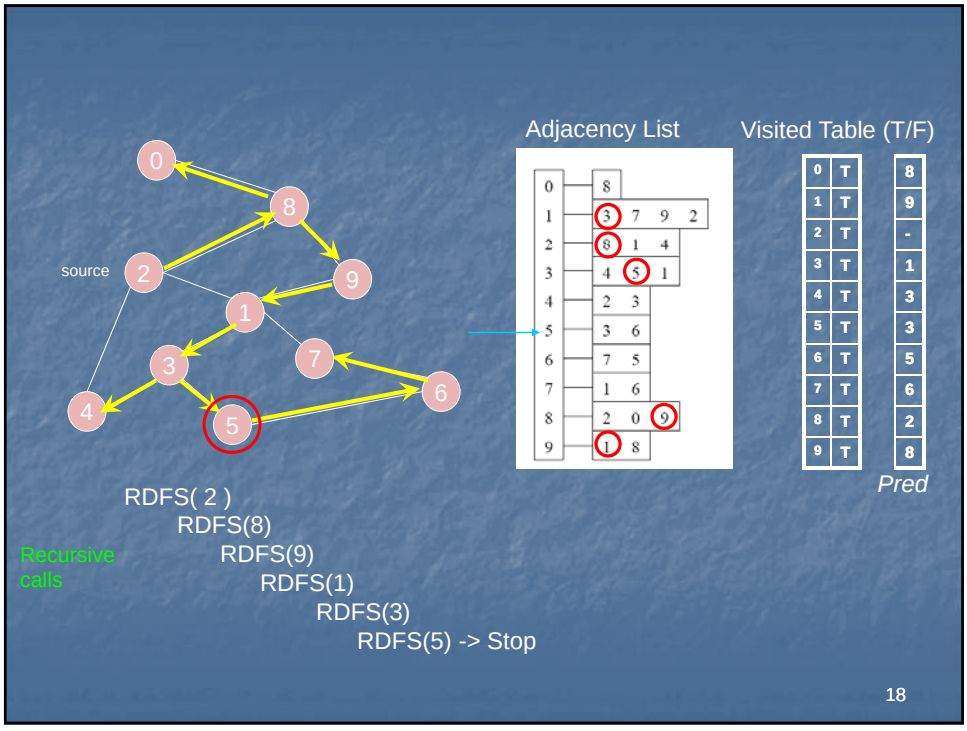

Adjacency List
Visited Table (T/F)
source
Back to 3
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
visit 5 -> RDFS(5)
Recursive calls
13

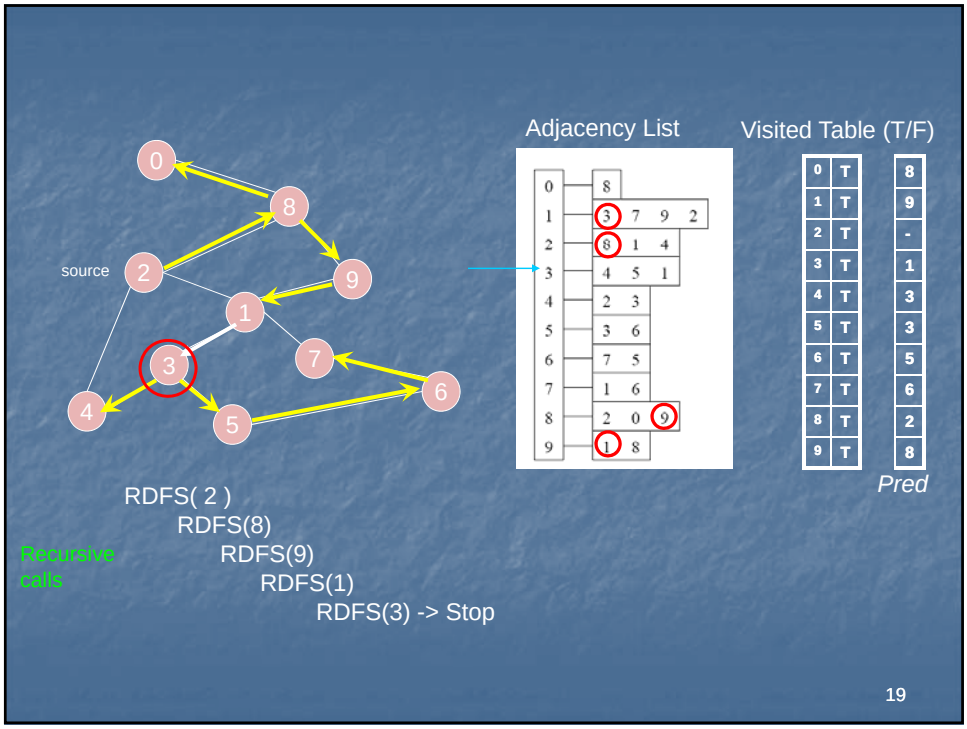

Adjacency List
Visited Table (T/F)
source
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
3 is already visited, so visit 6 -> RDFS(6)
Mark 5 as visited
Mark Pred[5]
Recursive calls
14

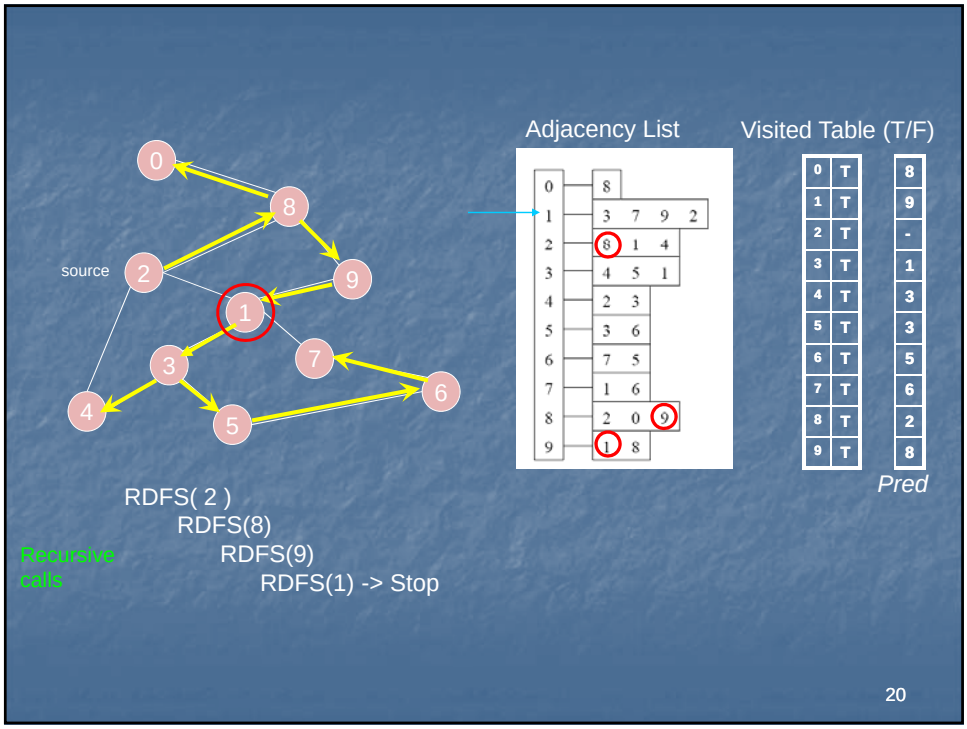

Adjacency List
Visited Table (T/F)
source
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6)
visit 7 -> RDFS(7)
Recursive calls
Mark 6 as visited
Mark Pred[6]
15

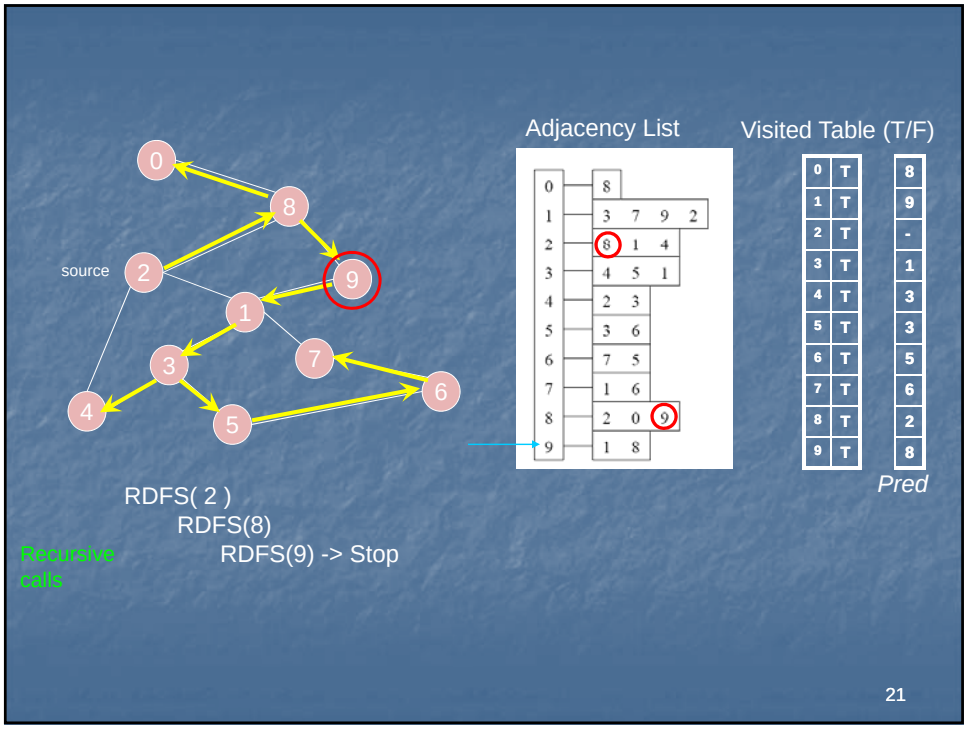

Adjacency List
Visited Table (T/F)
source
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6)
RDFS(7) -> Stop no more unvisited neighbors
Recursive calls
Mark 7 as visited
Mark Pred[7]
16

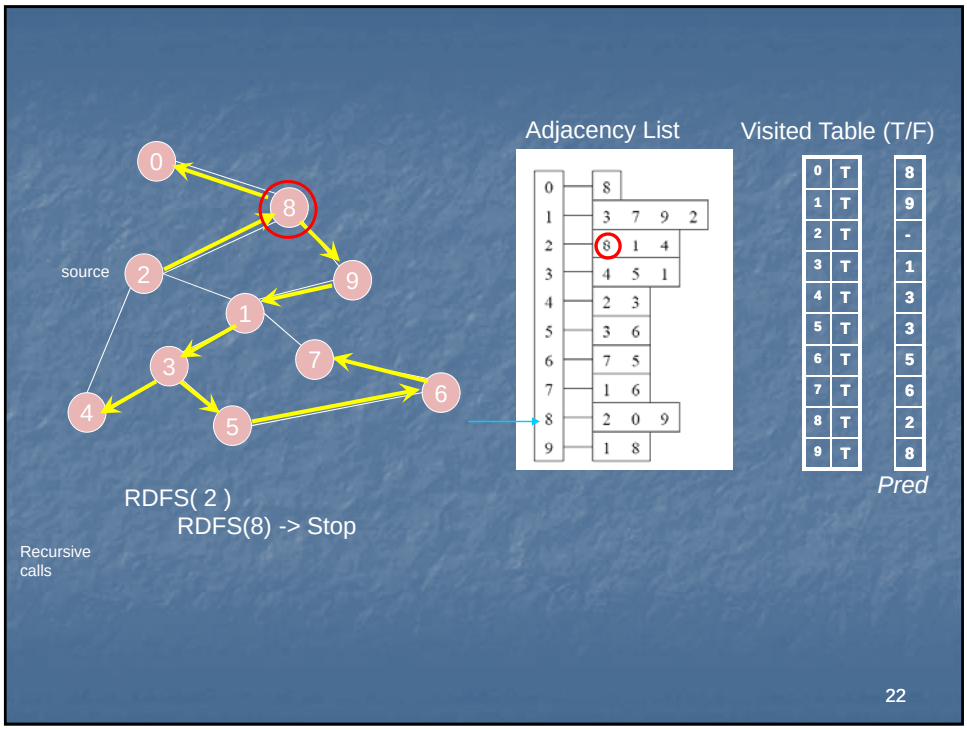

Adjacency List
Visited Table (T/F)
source
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6) -> Stop
Recursive calls
17


Adjacency List
Visited Table (T/F)
source
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5) -> Stop
Recursive calls
18

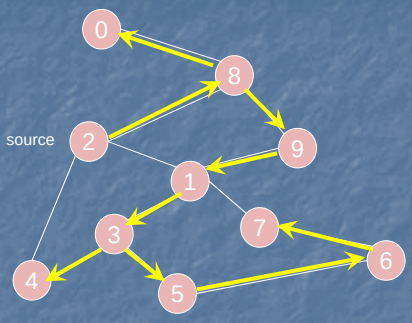

Adjacency List
Visited Table (T/F)
source
0
8
9
2
1
3
7
6
4
5
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3) -> Stop
Recursive calls
19


Adjacency List
Visited Table (T/F)
source
0
8
9
2
1
3
7
6
4
5
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1) -> Stop
Recursive calls
20

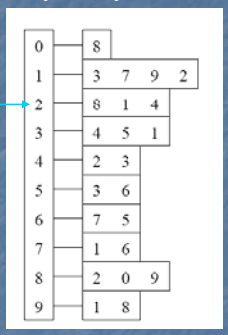

Adjacency List
Visited Table (T/F)
source
0
8
1
3 7 9 2
2
8 1 4
3
4 5 1
4
2 3
5
3 6
6
7 5
7
1 6
8
2 0 9
9
1 8
0 T
1 T
2 T
3 T
4 T
5 T
6 T
7 T
8 T
9 T
8
9
-
1
3
3
5
6
2
8
Pred
RDFS(2)
RDFS(8)
RDFS(9) -> Stop
Recursive calls
21


Adjacency List
Visited Table (T/F)
source
0
8
1
3 7 9 2
2
8 1 4
3
4 5 1
4
2 3
5
3 6
6
7 5
7
1 6
8
2 0 9
9
1 8
0 T
1 T
2 T
3 T
4 T
5 T
6 T
7 T
8 T
9 T
8
9
-
1
3
3
5
6
2
8
Pred
RDFS(2)
RDFS(8) -> Stop
Recursive calls
22

Example Finished



Adjacency List



Visited Table (T/F)

0	T
1	T
2	T
3	T
4	T
5	T
6	T
7	T
8	T
9	T

Pred
8
9
-
1
3
3
5
6
2
8

RDFS(2) -> Stop

Recursive calls finished

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Time Complexity of DFS

- We never visited a vertex more than once.
- We had to examine the adjacency lists of all vertices.
  - $\Sigma_{vertex\ v}$ degree(v) = 2E
- So, the running time of DFS is proportional to the number of edges and number of vertices (same as BFS)
  - O(V + E)

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Enhanced DFS Algorithm

- What if a graph is not connected (strongly connected)?
  - Use an enhanced version of DFS, which is similar to the enhanced BFS algorithm

```
BFSearch( G ) {
i = 1;     // component number
for every vertex v
    flag[v] = false;
for every vertex v
    if ( flag[v] == false ) {
        print ( "Component " +  i++ );
        BFS( v );
    }
}
```

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Applications of DFS

- Is there a path from source *s* to a vertex *v*?
- Is an undirected graph connected?
- Is a directed graph strongly connected?
- To output the contents (e.g., the vertices) of a graph
- To find the connected components of a graph
- To find out if a graph contains cycles and report cycles.
- To construct a DSF tree/forest from a graph

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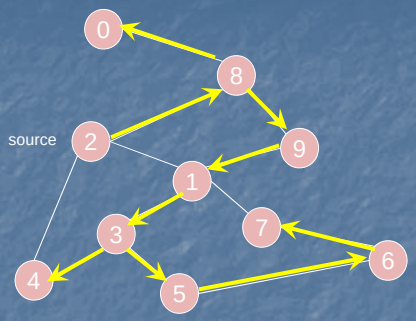
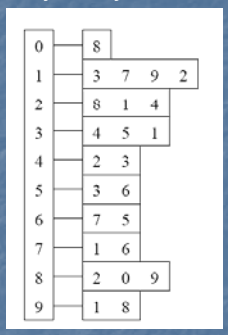
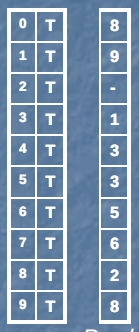
DFS Path Tracking



Adjacency List



Visited Table (T/F)



Pred

DFS find out path too

Algorithm *Path*(w)
1. **if** $pred[w] \neq -1$
2. **then**
3. *Path*$(pred[w])$;
4. output w

Try some examples.
Path(0) ->
Path(6) ->
Path(7) ->

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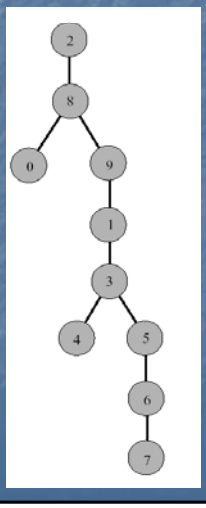
DFS Tree

Resulting DFS-tree.
Notice it is much "deeper" than the BFS tree.



Captures the structure of the recursive calls

- when we visit a neighbor w of v, we add w as child of v
- whenever DFS returns from a vertex v, we climb up in the tree from v to its parent

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Applications – DFS vs BFS

- What can BFS do and DFS can't?
  - Finding shortest paths (in unweighted graphs)
- What can DFS do and BFS can't?
  - Finding out if a connected undirected graph is *biconnected*
    - A connected undirected graph is biconnected if there are no vertices whose removal disconnects the rest of the graph

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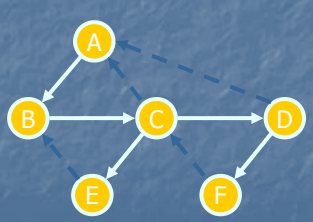
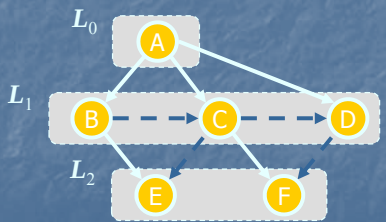
DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	√	√
Shortest paths		√
Biconnected components	√	



DFS



BFS

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Next time …

- Review — Dec. 8
- Final exam — Dec. 11

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