Test 2

First Name: _____

Last Name: _____

Student Number: _____

This test lasts 75 minutes. *No aids allowed.*
Make sure your test has 5 pages, including this cover page.
Answer in the space provided. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*
Write legibly.

Question 1	/8
Question 2	/4
Question 3	/3
Question 4	/5
Question 5	/5
Total	/25

1. [8 marks] Let $L_1 = \{0^i 1^j : i \neq j\}$.

(a) Give a context-free grammar for L_1.

(b) Prove that every string of the form $0^i 1^j$, where $i > j$, is generated by your grammar.

2. [4 marks] Recall that $x^{\mathcal{R}}$ denotes the string x written backwards. Let B_i be the standard binary representation of positive integer i with no leading 0's. For example, $B_{13} = 1101$ and $B_{103} = 1100111$. Draw a PDA for the language $L_2 = \{B_i \# B_{2i}^{\mathcal{R}} : i \geq 1\}$. (For example, 11#011 is in the language, because $B_3 = 11$ and $B_6 = 110$).

3. [3 marks] Consider the context-free grammar with one variable, S, two terminals, a and b, and rules $S \to aS \mid Sb \mid a \mid b$. Write down a regular expression for the language described by this grammar.

4. [5 marks] Show that the language $L_4 = \{0^i 1^j 2^i 3^j : i, j \geq 0\}$ is not context-free.

5. [5 marks] A 2-dimensional Turing machine is like a Turing machine, except it uses a 2-dimensional array instead of a tape to store information. You can imagine a Turing machine moving around on an unbounded flat surface marked out with a square grid. Each grid square can be used to store one symbol from the finite alphabet Γ. In each step, the Turing machine can choose what to do next based on its own state and the symbol stored at its current location. In that step it can rewrite the symbol at its current location and move one of four directions (north, east, south or west) to an adjacent grid square.

(a) If you wanted to simulate a 2-dimensional Turing machine using a multi-tape Turing machine, what would you store on each tape of the multi-tape machine?

(b) *Briefly* describe how you would simulate one step of the 2-dimensional Turing machine using the multi-tape machine.