YORK UNIVERSITY

FACULTY OF SCIENCE AND ENGINEERING

2007 FALL TERM EXAMINATION

Course Number: CSE2001

Title: Introduction to Theory of Computation

Duration: 3 hours

No aids allowed.

- There should be 10 pages in the exam, including this page.
- Write all answers on the examination paper. If your answer does not fit in the space provided, you can continue your answer on the back of a page or on page 10, indicating clearly that you have done so.
- You may use Church's Thesis in your answer to any question, except Question 12.
- Write legibly.

Name _____

(Please underline your family name.)

Student Number _____

1. _____/8
2. _____/4
3. _____/3
4. _____/3
5. _____/3
6. _____/4
7. _____/5
8. _____/1
9. _____/1
10. _____/4
11. _____/4
12. _____/4
13. _____/3
14. _____/3

Total: _____/50

CONT'D. . .

1. [8 marks] For each of the following languages, you must determine whether the language is regular, context-free, decidable, recognizable or not recognizable. For each language, circle the *leftmost* correct answer. For example, if a language is both recognizable and decidable, but not context-free, circle decidable.

(a) $\{ww : w \in \{0,1\}^*\}$

regular context-free decidable recognizable not recognizable

(b) $\{\langle M, w\rangle : M \text{ is a DFA and } w \in L(M)\}$

regular context-free decidable recognizable not recognizable

(c) $\{0^n 1^m : m \leq n \leq 2n\}$

regular context-free decidable recognizable not recognizable

(d) $\{\langle M\rangle : M \text{ is a PDA and } L(M) \text{ is not finite}\}$

regular context-free decidable recognizable not recognizable

(e) $\{\langle M\rangle : M \text{ is a Turing machine and } L(M) \text{ does not contain } \varepsilon\}$

regular context-free decidable recognizable not recognizable

(f) $\{\langle M\rangle : M \text{ is a Turing machine and } L(M) \text{ is not recognizable}\}$

regular context-free decidable recognizable not recognizable

(g) The set of all binary strings that contain an odd number of 0's.

regular context-free decidable recognizable not recognizable

(h) The set of all regular expressions (over the alphabet $\{0,1\}$).

regular context-free decidable recognizable not recognizable

CONT'D. . .

2. [4 marks]
Let $L_2 = \{x \in \{0,1\}^* : x$ either starts with 0 and has even length or starts with 1 and has odd length$\}$. Draw a deterministic finite automaton that accepts L_2.

3. [3 marks]
Let $L_3 = \{z \in \{0,1\}^* :$ some prefix of z contains more 0's than 1's$\}$. (Recall that string x is a prefix of string z if there exists a string y such that $z = xy$.) For example, $110010011101 \in L_3$, since its prefix 1100100 contains more 0's than 1's. Draw a pushdown automaton that accepts L_3.

CONT'D. . .

4. [3 marks] Consider the context-free grammar with variables S, A, B, terminals a, b, start symbol S and the following rules.

$$\begin{aligned} S &\rightarrow ASB \mid BSA \mid \varepsilon \\ A &\rightarrow aA \mid a \\ B &\rightarrow bB \mid \varepsilon \end{aligned}$$

Draw a parse tree for $aabba$.

5. [3 marks] Give a regular expression for the set of binary strings whose lengths are multiples of 5.

6. [4 marks]
For any language $L \subseteq \Sigma^*$, let $P(L) = \{x \in \Sigma^* : \text{ there exists a string } y \text{ such that } xy \in L\}$. In other words, $P(L)$ is the set of all prefixes of strings in L. Prove that whenever L is regular, $P(L)$ is regular too.

CONT'D. . .

7. [5 marks] Let B_i be the standard binary representation of positive integer i with no leading 0's. For example, $B_{13} = 1101$ and $B_{103} = 1100111$. Let $L_7 = \{B_i \# B_{2i} : i \geq 1\}$. For example, 11#110 is in L_7, because $B_3 = 11$ and $B_6 = 110$. Is L_7 context-free? Circle the correct answer and then prove your answer is correct.

YES NO

8. [1 mark] Explain what it means for a nondeterministic finite automaton to accept a string.

9. [1 mark] Explain the difference between recognizing a language and deciding a language.

10. [4 marks] Suppose L and L' are recognizable languages. Is $L \cap L'$ necessarily recognizable? Circle the correct answer and then prove your answer is correct.

YES NO

CONT'D. . .

11. [4 marks] Prove that the language $\{\langle G_1, G_2\rangle : G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) = \overline{L(G_2)}\}$ is undecidable.

12. [4 marks] Describe, at a high level, a Turing machine that decides the language $\{0^n 1^m : n = m^2\}$.

CONT'D. . .

13. [3 marks] Give an algorithm that recognizes the language $\{\langle G \rangle : G \text{ is a context-free grammar and } L(G) \neq \Sigma^*\}$.

14. [3 marks] State the result that you think is the most interesting one that was proved in this course. Explain (in one or two sentences) why you think it is interesting. If you did not find any result interesting, say so and explain (in one or two sentences) why you feel this way.

CONT'D. . .

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