Homework Assignment #1
Due: September 24, 4:00 p.m.

Along with your solutions to this assignment, hand in a *separate* **sheet of paper containing your name, student number and the following declaration: "I have read and understood the policy on academic honesty on the CSE2001 course web page." Sign this paper and date it. Without this declaration, your solutions will not be marked.**

1. Let Σ be an alphabet. Given a string $x \in \Sigma^*$ define the reverse of x, denoted x^R, recursively as follows:

$$\begin{aligned} \varepsilon^R &= \varepsilon, \text{ and} \\ (wa)^R &= a(w^R), \text{ for } w \in \Sigma^*, a \in \Sigma. \end{aligned}$$

Define $A^R = \{x^R : x \in A\}$, where A is a language over the alphabet Σ.

Recall the recursive definition of string concatenation:

$$\begin{aligned} v\varepsilon &= v, \text{ for } v \in \Sigma^*, \text{ and} \\ v(wa) &= (vw)a, \text{ for } v, w \in \Sigma^* \text{ and } a \in \Sigma. \end{aligned}$$

Define $AB = \{vw : v \in A \text{ and } w \in B\}$, where A and B are languages over the alphabet Σ.

Let A and B be languages over the alphabet Σ. Use the above definitions to give careful proofs of the following statements.

(a) $\varepsilon x = x$ for all strings $x \in \Sigma^*$. (Hint: use induction.)

(b) $(x^R)(y^R) = (yx)^R$ for all strings $x, y \in \Sigma^*$. (Hint: use induction on the length of x.)

(c) $(AB)^R = (B^R)(A^R)$ for all languages $A, B \subseteq \Sigma^*$.