Homework Assignment #9
Due: May 21, 2009

9. Recall the definition of **ZPP** from class: a language L is in **ZPP** iff there is a polynomial p and a randomized Turing machine M that always halts within $p(|x|)$ steps on input x and outputs "yes", "no" or "I don't know" such that

- if $x \in L$, M outputs "yes" or "I don't know",
- if $x \notin L$, M outputs "no" or "I don't know", and
- for all x the probability that M outputs "I don't know" is at most $\frac{1}{2}$.

We define another class **EP** as follows. A language L is in **EP** iff there is a polynomial p and a randomized TM M such that

- if $x \in L$, M outputs "yes" if M terminates,
- if $x \notin L$, M outputs "no" if M terminates, and
- for all x, the expected running time of M on input x is at most $p(|x|)$.

Why stop there? We define yet another class $\mathbf{EP}^*$ as follows. A language L is in $\mathbf{EP}^*$ iff there is a polynomial p and a randomized TM M such that

- if $x \in L$, M outputs "yes" on input x,
- if $x \notin L$, M outputs "no" on input x, and
- for all x, the expected running time of M on input x is at most $p(|x|)$.

Prove that $\mathbf{ZPP} = \mathbf{EP} = \mathbf{EP}^*$.

Hint: If the expected running time of an algorithm is $p(n)$, how big can the probability that the running time exceeds $2p(n)$ be?