Homework Assignment #7
Due: May 5, 2009

7. Recall the notion of a context-free grammar (CFG). It is defined by

- a finite alphabet V of variables,
- a finite alphabet T of terminals,
- a start symbol $S \in V$, and
- a finite set of rules R, where each rule is of the form $A \to \alpha$ with $A \in V$ and $\alpha \in (V \cup T)^*$.

We say that a string $x \in T^*$ is generated by the grammar if there is a sequence $\beta_0, \beta_1, \beta_2, \ldots, \beta_k$ such that

- for all i, $\beta_i \in (V \cup T)^*$,
- $\beta_0 = S$,
- $\beta_k = x$, and
- for all i, there exists a rule $A \to \alpha$ such that β_{i+1} can be obtained from β_i by replacing one occurrence of A in β_i by α.

(Notice that strings generated by the grammar contain only terminals and no variables.)

Prove that the problem of determining whether a given CFG generates at least one string is **P**-complete.