Homework Assignment #3
Due: April 7, 2009

3. This question asks you to prove some closure properties of complexity classes.

(a) Let $f : \mathbb{N} \rightarrow \mathbb{N}$. Prove that $\mathbf{TIME}(f(n))$ is closed under intersection. That is, show that whenever L_1 and L_2 are in $\mathbf{TIME}(f(n))$, so is $L_1 \cap L_2$.

(b) The Kleene star operator is defined as follows: If L is a language, then $L^* = \{w_1 w_2 \ldots w_k : k \in \mathbb{N}$ and for $1 \leq i \leq k, w_i \in L\}$. In other words, L^* contains all strings that can be formed by concatenating strings in L. Note that k can be 0 in the above definition, so the empty string is in L^*.

Let $\mathbf{P}$ be the set of all languages that can be decided in polynomial time. Formally, $L \in \mathbf{P}$ iff there is a polynomial $p(n)$ such that $L \in \mathbf{TIME}(p(n))$.

Prove that $\mathbf{P}$ is closed under the Kleene star operator. In other words, show that if $L \in \mathbf{P}$, then $L^* \in \mathbf{P}$.