Exercises

Exercise 1. We'll add a little bit to the car example we saw in the class: first, define the following additional variables:

$$c_{moves} \text{ – true if the car moves}$$
$$t_{ok} \text{ – true if the transmission is in working condition.}$$

Now, consider the following problem description P:

$$b_{ok} \wedge s_{on} \wedge e_{ok} \rightarrow e_{works} \text{ – if battery ok and switch on and engine ok, then engine works}$$
$$e_{works} \wedge t_{ok} \rightarrow c_{moves} \text{ – if engine works and transmission is ok, then car moves}$$
$$s_{on} \wedge \neg c_{moves} \text{ – the switch is on but car doesn't move}$$

And the following goal g:

$$(b_{ok} \wedge e_{ok}) \rightarrow \neg t_{ok} \text{ – if battery and engine are ok, then transmission must be broken}$$

Show that $P \rightarrow g$ using resolution.

Exercise 2. For each of the following formulas, first convert the formula to prenex normal form, then eliminate all existential quantifiers via skolemization:

(a) $\forall Y((\forall X p(X,Y)) \rightarrow \exists Z q(X,Z))$

(b) $(\exists X \forall Y p(X,Y)) \vee \neg \exists Y(q(Y) \rightarrow \forall Z r(Z))$

Exercise 3. How many different clauses can be constructed out of n propositional variables ? Assume that the same variable may not appear twice in a clause (neither with the same, nor with opposite polarity). For example for $n = 2$ all possible clauses are: :–, p_1, $\neg p_1$, p_2, $\neg p_2$, $p_1 \vee p_2$, $p_1 \vee \neg p_2$, $\neg p_1 \vee p_2$, $\neg p_1 \vee \neg p_2$, and the answer is 9.

Solutions are on the next page

Solutions

Exercise 1 The clause set for resolution is

$$
\begin{aligned}
e_{works} &:- b_{ok}, s_{on}, e_{ok} \\
c_{moves} &:- e_{works}, t_{ok} \\
s_{on} &:- \\
&:- c_{moves} \\
b_{ok} &:- \\
e_{ok} &:- \\
t_{ok} &:-
\end{aligned}
$$

There are many different resolution refutations – if you get an empty clause at the end, then its correct ;-) If in doubt – email it to me, I will check it. Try to come up with the linear refutation (i.e. start with $:- c_{moves}$).

Exercise 2

(a) Prenex normal form:

$$\forall Y \exists T \exists Z (\neg p(T, Y) \vee q(X, Z)).$$

Note that you may have a different variable instead of T. After skolemization:

$$\forall Y (\neg p(f(Y), Y) \vee q(X, h(Y))).$$

Again, you may have used other symbols instead of f and h.

(b) Prenex normal form:

$$\exists X \forall T \forall Y \exists Z (p(X, T) \vee (q(Y) \wedge \neg r(Z))).$$

After skolemization:

$$\forall T \forall Y (p(c, T) \vee (q(Y) \wedge \neg r(f(T, Y)))).$$

Again, you may have used different symbols in the above formulas.

Exercise 3
Send me your answer by email.