Logic Quiz

3 questions

First name: ______________________________

Last name: ______________________________

Student ID: ______________________________

Notation: $\wedge$ is conjunction ("and"), $\vee$ is disjunction ("or"), $\rightarrow$ is implication, $\leftrightarrow$ is equivalence.

Question 1

Express the following English sentence in first-order predicate logic:

Two people are siblings if they have the same mother or the same father.

Make the domain of your variables to be the set of all objects. For example, "All humans are mortal" should be expressed as

$$\forall X \ (human(X) \rightarrow mortal(X)).$$

Answer:

For this question I asked to make the domain to be the set of all objects, so technically, all your formulas should have $person(X) \wedge person(Y)$ somewhere, but since there was only one person that did it, I will put the answers *assuming that the domain is the set of all people*. So, there you go:

$$\forall X \forall Y ((mother(X) = mother(Y) \vee father(X) = father(Y)) \rightarrow siblings(X, Y))$$

Note that this assumes that $mother(X)$ and $father(X)$ are functions, and not predicates – if you don't understand what this means, **read the 1019 book** !!! This is a version that uses predicates only:

$$\forall X \forall Y [(\exists Z (mother(X, Z) \wedge mother(Y, Z)) \vee \exists Z (father(X, Z) \wedge father(Y, Z))) \rightarrow siblings(X, Y)].$$

This works also:

$$\forall X \forall Y [(same\_mother(X,Y) \vee same\_father(X,Y)) \rightarrow siblings(X,Y)].$$

On the other hand:

$$\forall X \forall Y ((mother(X) \leftrightarrow mother(Y) \vee father(X) \leftrightarrow father(Y)) \rightarrow siblings(X,Y)),$$

doesn't work, because if $mother(X)$ is a function, then there's no such thing as logical equivalence ($\leftrightarrow$) between two objects. If $mother(X)$ is a predicate, then it means that X is a mother, and the whole sentence means something completely different from what is required.

Question 2

The reverse of question 1 – give an English sentence which is the closest to the following first-order predicate logic sentence (the domain of variables is the set of all natural numbers):

$$\forall N \ ((even(N) \wedge N > 2) \rightarrow \exists K, L \ (prime(K) \wedge prime(L) \wedge N = K + L))$$

Answer:

Every even natural number greater than 2 is a sum of two primes (aka *Goldbach's conjecture* – noone knows if its true or false, but it has been verified for all numbers up to 10^{18}).

Question 3

Push all negations inwards, i.e. so negations appear only in front of predicates:

$$\neg((\exists X \exists Y \neg p(X,Y)) \rightarrow (\forall X \forall Y q(X,Y)))$$

Answer:

Since $\neg(a \rightarrow b) \equiv \neg(\neg a \vee b) \equiv (a \wedge \neg b)$, the original sentence is equivalent to

$$(\exists X \exists Y \neg p(X,Y)) \wedge \neg(\forall X \forall Y q(X,Y)).$$

Now, since $\neg \forall X a(X) \equiv \exists X \neg a(X)$, we obtain:

$$(\exists X \exists Y \neg p(X,Y)) \wedge (\exists X \exists Y \neg q(X,Y)).$$