Solutions for Assignment 1

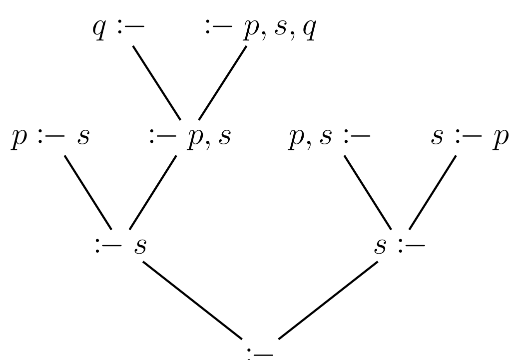
Question 1 (20 points - 10 points each part)**.** Which of the following sets of clauses are inconsistent ? Justify your answers by either giving a resolution refutation tree of by defining a suitable interpretation.

(a) $S_1 = \{$

$$
\begin{aligned}
&:\!-\ p, s, q \\
q\ &:\!- \\
q, s\ &:\!- \\
p, q\ &:\!- \\
p, s\ &:\!- \\
p\ &:\!-\ s \\
s\ &:\!-\ p
\end{aligned}
$$

$\}$

Answer: inconsistent. A possible resolution refutation tree is as follows:



(b) $S_2 = \{$

$$
\begin{aligned}
&:\!-\ p, q, s \\
p\ &:\!-\ t \\
q, t\ &:\!-\ r \\
&:\!-\ t \\
s\ &:\!-\ p, t
\end{aligned}
$$

$\}$

Answer: consistent. A possible interpretation is $p = q = r = s = t = 0$.

Question 2 (20 points)**.** Show that every logic program P is consistent. (*Hint:* if you cannot understand the question, review the definitions of *logic program* and *consistent*).

Answer: Since any logic program P consists of rule and fact clauses only, :– cannot be derived from P using resolution (resolution of facts and rules can only produce facts and rules), and, therefore, by Robinson's Theorem (Theorem 6, Lecture 7 (Sep 25)), P is consistent.

Question 3 (30 points - 10 points each part)**.** Execute Robinson's unification algorithm on each of the following three sets of terms. Show every step of the algorithm, the way we did this in the class (you can omit the first column, but if you do, make sure to write out the unifier explicitly by composing the appropriate substitutions).

(a) $U_1 = \{p(a, X, f(g(Y))),\quad p(T, h(T, Z), f(Z))\}$

Answer:

t_1	t_2	new subst.
$p(a, X, f(g(Y)))$	$p(T, h(T, Z), f(Z))$	$[T/a]$
$p(a, X, f(g(Y)))$	$p(a, h(a, Z), f(Z))$	$[X/h(a, Z)]$
$p(a, h(a, Z), f(g(Y)))$	$p(a, h(a, Z), f(Z))$	$[Z/g(Y)]$
$p(a, h(a, g(Y)), f(g(Y)))$	$p(a, h(a, g(Y)), f(g(Y)))$	done

The terms are unifiable, and the m.g.u. is

$$[T/a] \circ [X/h(a, Z)] \circ [Z/g(Y)] = [T/a, X/h(a, g(Y)), Z/g(Y)]$$

(b) $U_2 = \{p(X, X, V),\quad p(f(Y), W, g(Z))\}$

Answer:

t_1	t_2	new subst.
$p(X, X, V)$	$p(f(Y), W, g(Z))$	$[X/f(Y)]$
$p(f(Y), f(Y), V)$	$p(f(Y), W, g(Z))$	$[W/f(Y)]$
$p(f(Y), f(Y), V)$	$p(f(Y), f(Y), g(Z))$	$[V/g(Z)]$
$p(f(Y), f(Y), g(Z))$	$p(f(Y), f(Y), g(Z))$	done

The terms are unifiable, and the m.g.u. is

$$[X/f(Y)] \circ [W/f(Y)] \circ [V/g(Z)] = [X/f(Y), W/f(Y), V/g(Z)]$$

(c) $U_3 = \{q(g(b, T), h(c), h(T)),\quad q(X, h(Z), h(X))\}$

Answer:

t_1	t_2	new subst.
$q(g(b, T), h(c), h(T))$	$q(X, h(Z), h(X))$	$[X/g(b, T)]$
$q(g(b, T), h(c), h(T))$	$q(g(b, T), h(Z), h(g(b, T)))$	$[Z/c]$
$q(g(b, T), h(c), h(T))$	$q(g(b, T), h(c), h(g(b, T)))$	occurs-check fails

The terms are not unifiable.

Question 4 (30 points - 10 points part (a), 20 points part (b))**.** In this question we will consider a simplified model of the "People you may know" application in Facebook. The basic idea is that if two people have a common friend, then they may know each other. We will also take into account the fact in Facebook the relation "friend" is symmetric, that is, if X is a friend of Y, then Y is a friend of X.
Thus, as a problem description P, we have the following two predicate logic sentences:

$$\forall X \forall Y [(\exists Z (friend(X,Z) \wedge friend(Z,Y))) \leftrightarrow may\_know(X,Y)]$$
$$\forall X \forall Y [friend(X,Y) \rightarrow friend(Y,X)]$$

Our goal is to show, using resolution refutation, that the following sentence g is implied by P:

$$\forall X \forall Y [may\_know(X,Y) \rightarrow may\_know(Y,X)].$$

In other words, in Facebook, the relation "may know" is also symmetric.

(a) Represent the problem description and the goal in the form suitable for resolution refutation, that is, as a set of logic programming clauses which is inconsistent if and only if $P \rightarrow g$.

Answer: Since resolution is only capable of proving inconsistency, we need to do proof by contradiction, i.e. we need to show that the set $P \cup \{\neg g\}$ is inconsistent. The logic programming clauses for this set are (I skipped the derivation, but you should not have):

$$\begin{aligned}
C_1 =& mn(X,Y) :- \ f(X,T), f(T,Y) \\
C_2 =& f(X, g(X,Y)) :- \ mn(X,Y) \\
C_3 =& f(g(X,Y), Y) :- \ mn(X,Y) \\
C_4 =& f(Y,X) :- \ f(X,Y) \\
C_5 =& mn(a,b) :- \\
C_6 =& :- \ mn(b,a)
\end{aligned}$$

where $f(X,Y)$ stands for $friend(X,Y)$, and $mn(X,Y)$ stands for $may\_know(X,Y)$. Note that depending on the way you moved quantifiers, you may have gotten $g(X,Y,Z)$ instead of $g(X,Y)$ in clauses C_2 and C_3.

(b) Find the resolution refutation of the set of logic programming clauses you have constructed in part (a). *Hint:* linear refutation is likely to be long and confusing; think first how would you prove what you need to prove, and then construct the refutation that mimics your proof.

Answer: One of the possible refutations is as follows:

1. $f(a, g(a, b))$:– (from C_2 and C_5 with $[X/a, Y/b]$)
2. $f(g(a, b), a)$:– (from 1. and C_4 with $[X/a, Y/g(a, b)]$)
3. $f(g(a, b), b)$:– (from C_3 and C_5 with $[X/a, Y/b]$)
4. $f(b, g(a, b))$:– (from 3. and C_4 with $[X/g(a, b), Y/b]$)
5. :– $f(b, T), f(T, a)$ (from C_1 and C_6 with $[X/b, Y/a]$)
6. :– $f(g(a, b), a)$ (from 4. and 5. with $[T/g(a, b)]$)
7. :– (from 2. and 6.)