York University CSE 3101 February 9, 2009

Homework Assignment #9
Due: February 18, 3:30 p.m.

1. In general, a graph can have many different minimum spanning trees. For an example, see Figure 23.1 on page 562 of the textbook.

In this question, you will prove that if the edges all have distinct weights, then the graph has only one minimum spanning tree.

Let $G = (V, E)$ be an undirected graph with weight function $w : E \to \mathbb{R}$. Assume the edge weights are all distinct. In other words, for any pair of distinct edges $e \neq e'$, we have $w(e) \neq w(e')$. Let r be any element of V.

(a) Suppose you run Prim's algorithm on G, starting with vertex r. Let T_i be the partial tree constructed after i iterations of the loop.

Prove the following claim by induction on i.
Claim: For every i and for *every* minimum spanning tree T of G, $T_i \subseteq T$.

Hint: To prove that MST-PRIM is correct, we proved that for every i there is *some* MST T^* of G such that $T_i \subseteq T^*$. Here our goal is different: instead of proving the algorithm is correct, we are showing that there is only one possible outcome. Nevertheless, the claim above is similar to the claim you would use to prove the algorithm is correct, *except* for the quantifier. So your proof might use ideas similar to the proof of correctness for Prim's algorithm, but it will have to have a different structure.

(b) Prove that G has only one minimum spanning tree.

2. Does algorithm MAYBE-MST-A on page 578 always produce a minimum spanning tree when run on a connected graph? Prove your answer is correct.