Homework Assignment #6
Due: October 31, 3:30 p.m.

1. Triin and Tiiu are twin sisters who go out trick-or-treating together on Hallowe'en. They agree that when they arrive home afterwards, they will combine all the candy they have received and then split it evenly. Of course, not all types of candy are equally valuable. So the sisters assign a value between 1 and 10 to each piece of candy they receive. Their goal is to split the pile of candy into two piles so that the total value of the first pile is as close as possible to the total value of the second pile. (It may not be possible to split them so that the two piles have *exactly* the same total value.)

When the sisters arrive home, their combined pile of candies contains n items. They enter the values of all the items into an array $A[1..n]$ in some arbitrary order. Then they ask for your help to compute an optimal division of the candies into two piles.

Let $M[i,j] = \left\{ \begin{array}{ll} 1 & \text{if some } \textit{subset} \text{ of the first } i \text{ elements of } A \text{ sum to } j \\ 0 & \text{otherwise} \end{array} \right\}$,
where $0 \leq i \leq n$ and $0 \leq j \leq$??.

(a) What should the range of possible values for j be in the definition above?

(b) What is $M[0,0]$?

(c) What is $M[i,0]$ for $i > 0$? Explain in one sentence why your answer is correct.

(d) What is $M[0,j]$ for $j > 0$? Explain in one sentence why your answer is correct.

(e) Give a recurrence that defines $M[i,j]$ when i and j are both positive. *Briefly* explain why your answer is correct. Your recurrence should be designed so that it can be used to fill in the array M efficiently.

(f) If you were computing the array M using your recurrence, in what order would you fill in the entries?

(g) How long would it take fill in all entries of the array M using your recurrence? (Give your answer using Θ notation in terms of n.)

(h) Suppose all entries of M have been computed. How would you determine how equitably the pile of candies can be divided?

(i) Suppose all entries of M have been computed. Give an algorithm that uses M to output the elements of the first pile of a division that is as equitable as possible. (Do not prove the algorithm is correct.)