Exercise #7
Due: March 6, 2008

7. Consider an asynchronous deterministic shared-memory system, where the n processes have unique ids in the range $\{1, 2, \ldots, n\}$ and any number of halting failures may occur.

(a) Consider the object type `Arithmetic` which stores a natural number and provides two types of operations:

- FETCH&ADD(x) adds x to the value stored in the object and returns the previous value stored (before the addition).

- FETCH&MULT(x) multiplies the value stored in the object by x and returns the previous value stored (before the multiplication).

Show that `Arithmetic` objects and registers can be used to solve consensus among any number of processes.

Hint: You can solve this problem by proving the following lemma: If there is a consensus algorithm for $n-1$ processes that uses k registers and j `Arithmetic` objects, then there is a consensus algorithm for n processes that uses $k+2$ registers and $j+1$ `Arithmetic` objects.

(b) Consider the `Multiplication` object type, which stores a natural number and provides only the FETCH&MULT(x) operation (as described above). Show that registers and `Multiplication` objects can solve consensus among 2 processes, but not among 3 processes.