Homework Exercise #2
Due: January 29, 2008

2. Consider an asynchronous, failure-free distributed system. The n processes communicate by sending messages across bidirectional channels. The graph of channels between processes forms a connected graph with m edges. The processes each have a distinct id from the range $\{1, 2, \ldots, n\}$. Initially, agents know nothing about the network graph (not even the ids of adjacent processes).

At the beginning of an execution, process number i will receive an input bit v_i (for all i). Let $f : \{0,1\}^n \to \{0,1\}$ be a function. The goal of this question is for the processes to compute the value $f(v_1, v_2, \ldots, v_n)$. That is, every process must eventually output $f(v_1, v_2, \ldots, v_n)$.

(a) Describe an algorithm that uses $O(m)$ messages to solve the problem.

(b) The bit complexity of an algorithm is the maximum (over all possible executions) of the total number of bits in all messages in that execution. What is the bit complexity of your algorithm?

(c) If $f(v_1, \ldots, v_n) = \sum_{i=1}^{n} v_i$, could you modify your algorithm to have better bit complexity? How?

(d) If $f(v_1, \ldots, v_n) = \left(\sum_{i=1}^{n} v_i\right) \bmod 2$, could you modify your algorithm to have better bit complexity? How?