# Exercise #10
# Due: April 3, 2008

**10.** In class, we have been talking about population protocols that produce binary output values (or, more generally, outputs from a finite set). Here we consider what happens when the size of the output may depend on the size of the input (which is equal to the number of processes).

Recall that the inputs are distributed to the processes: each process gets one input character from the finite set $\Sigma$. We shall represent the output in a similar fashion: each process will have one piece of the output.

For example, suppose we want to compute the "divide-by-7" function: $f(x) = \lfloor x/7 \rfloor$. Suppose the input alphabet is $\Sigma = \{0, 1\}$ and the input $x$ is represented by giving $x$ of the processes in the system input value 1 and giving all other processes input value 0. The output of the protocol will also be distributed because it is impossible to represent the quotient $\lfloor x/7 \rfloor$ in a single process's state: recall that each process only has a constant amount of memory. We say that a protocol correctly computes this function $f$ if, eventually, exactly $\lfloor x/7 \rfloor$ of the processes output 1 and all others output 0.

**(a)** Design a population protocol to compute the function $f(x, y) = x - y$ where $x$ and $y$ are positive integers. Describe exactly what input/output conventions your algorithm uses. (For example, you should come up with some reasonable way of representing negative-valued outputs so that they can be distinguished from positive-valued outputs.) Explain why your algorithm stabilizes to a correct result in all fair runs.

**(b)** Repeat exercise (a) for the function $f(x) = \lfloor x/3 \rfloor + 2$.