CSE3213 Computer Network I

Chapter 3.3-3.6
Digital Transmission Fundamentals

Course page:

http://www.cse.yorku.ca/course/3213

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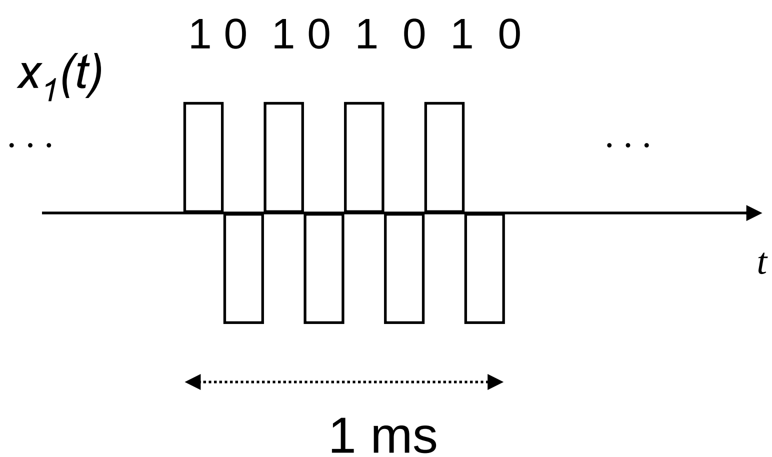
Digital Representation of Analog Signals

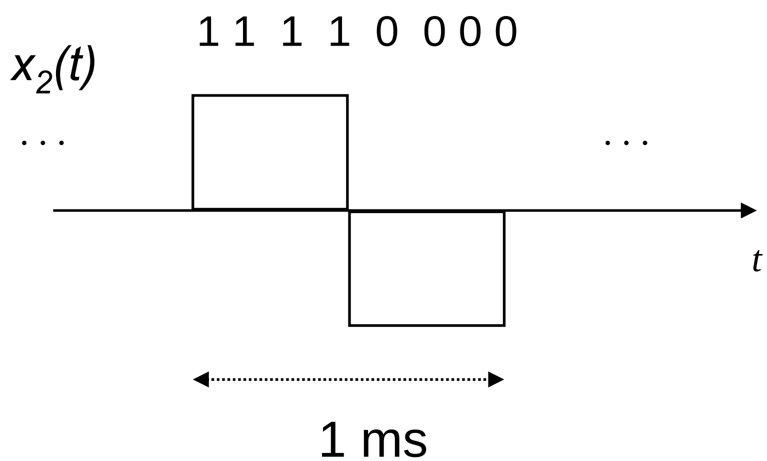
Digitization of Analog Signals

1. Sampling: obtain samples of $x(t)$ at uniformly spaced time intervals
2. Quantization: map each sample into an approximation value of finite precision
   - Pulse Code Modulation: telephone speech
   - CD audio
3. Compression: to lower bit rate further, apply additional compression method
   - Differential coding: cellular telephone speech
   - Subband coding: MP3 audio
   - Compression discussed in Chapter 12

Sampling Rate and Bandwidth

- A signal that varies faster needs to be sampled more frequently
- *Bandwidth* measures how fast a signal varies





- What is the bandwidth of a signal?
- How is bandwidth related to sampling rate?

Periodic Signals

- A periodic signal with period T can be represented as sum of sinusoids using Fourier Series:

$$x(t) = a_0 + a_1\cos(2\pi f_0 t + \phi_1) \quad + \quad a_2\cos(2\pi 2 f_0 t + \phi_2) \quad + \ldots$$
$$+ a_k\cos(2\pi k f_0 t + \phi_k) + \ldots$$

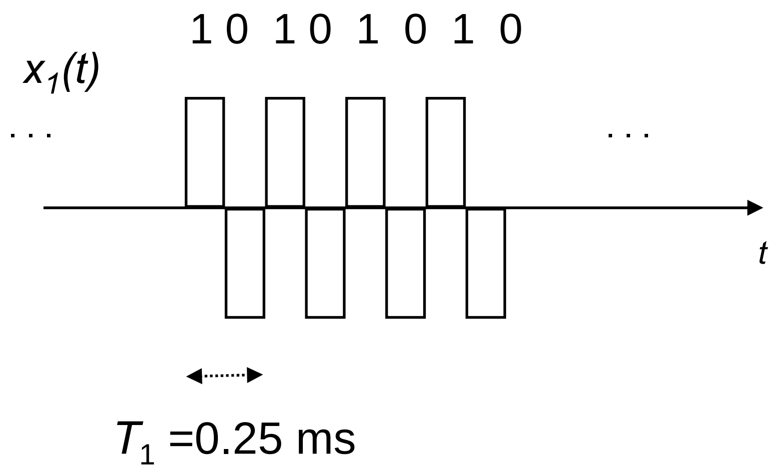
"DC"
long-term
average

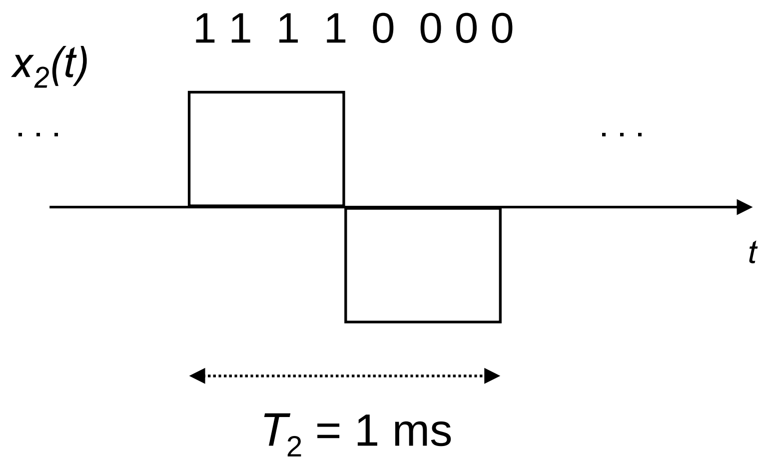
fundamental
frequency $f_0=1/T$
first harmonic

kth harmonic

- $|a_k|$ determines amount of power in kth harmonic
- Amplitude specturm $|a_0|$, $|a_1|$, $|a_2|$, …

Example Fourier Series





$$x_1(t) = 0 + \frac{4}{\pi}\cos(2\pi 4000t) + \frac{4}{3\pi}\cos(2\pi 3(4000)t) + \frac{4}{5\pi}\cos(2\pi 5(4000)t) + \ldots$$

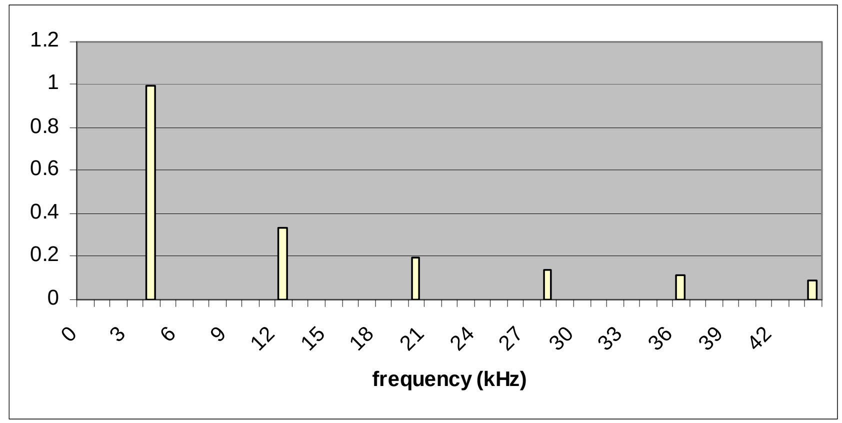
$$x_2(t) = 0 + \frac{4}{\pi}\cos(2\pi 1000t) + \frac{4}{3\pi}\cos(2\pi 3(1000)t) + \frac{4}{5\pi}\cos(2\pi 5(1000)t) + \ldots$$

Only odd harmonics have power

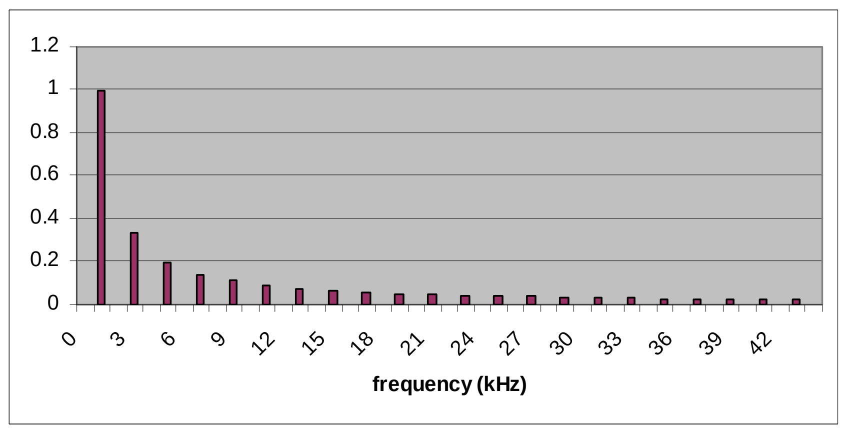
Spectra & Bandwidth

- Spectrum of a signal: magnitude of amplitudes as a function of frequency
- $x_1(t)$ varies faster in time & has more high frequency content than $x_2(t)$
- Bandwidth W_s is defined as range of frequencies where a signal has non-negligible power, e.g. range of band that contains 99% of total signal power

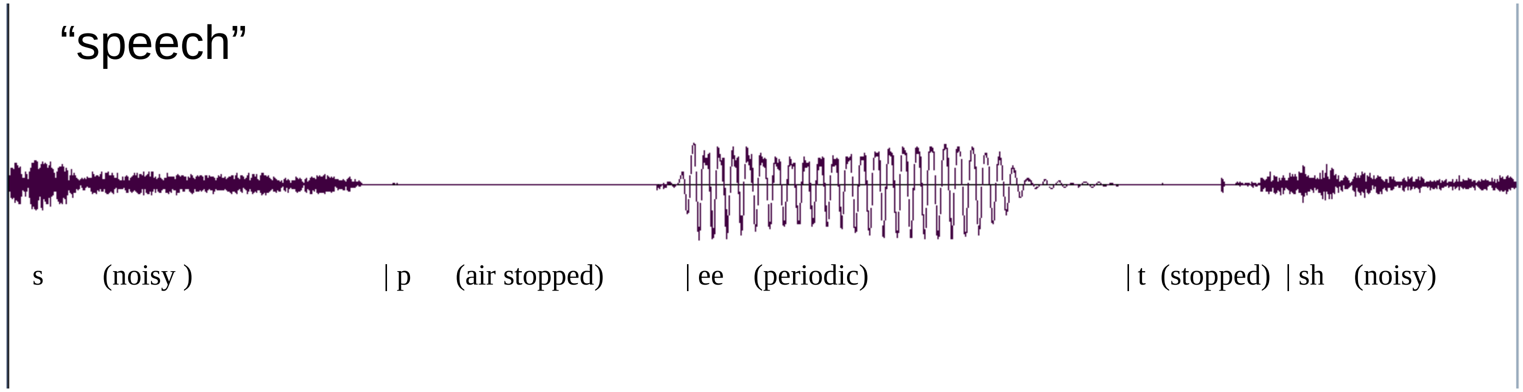
Spectrum of $x_1(t)$



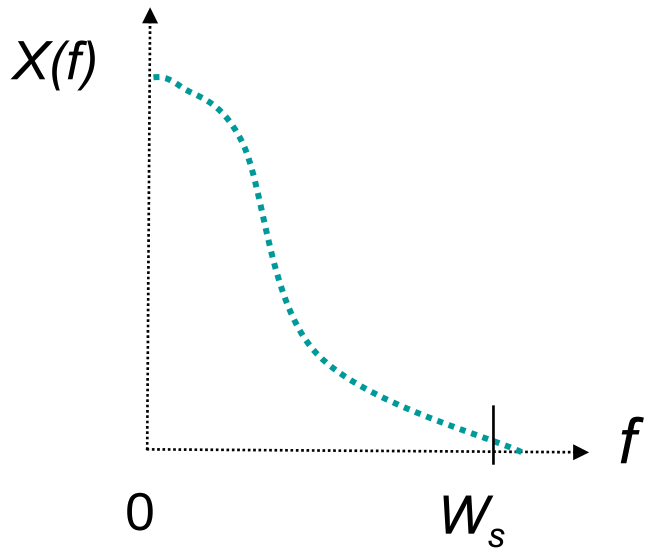
Spectrum of $x_2(t)$



Bandwidth of General Signals



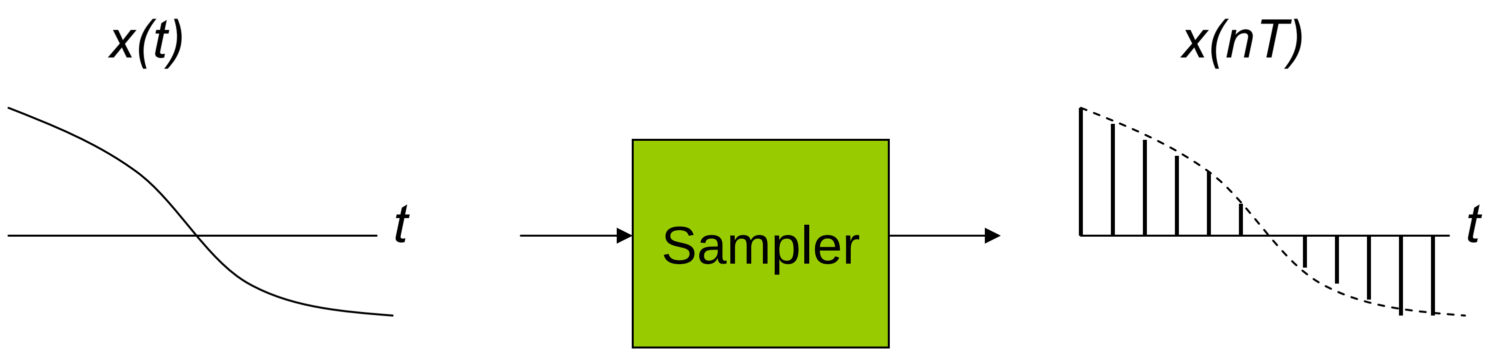
- Not all signals are periodic
  - E.g. voice signals varies according to sound
  - Vowels are periodic, "s" is noiselike
- Spectrum of long-term signal
  - Averages over many sounds, many speakers
  - Involves Fourier transform
- Telephone speech: 4 kHz
- CD Audio: 22 kHz



Sampling Theorem

Nyquist: Perfect reconstruction if sampling rate $1/T > 2W_s$

(a)



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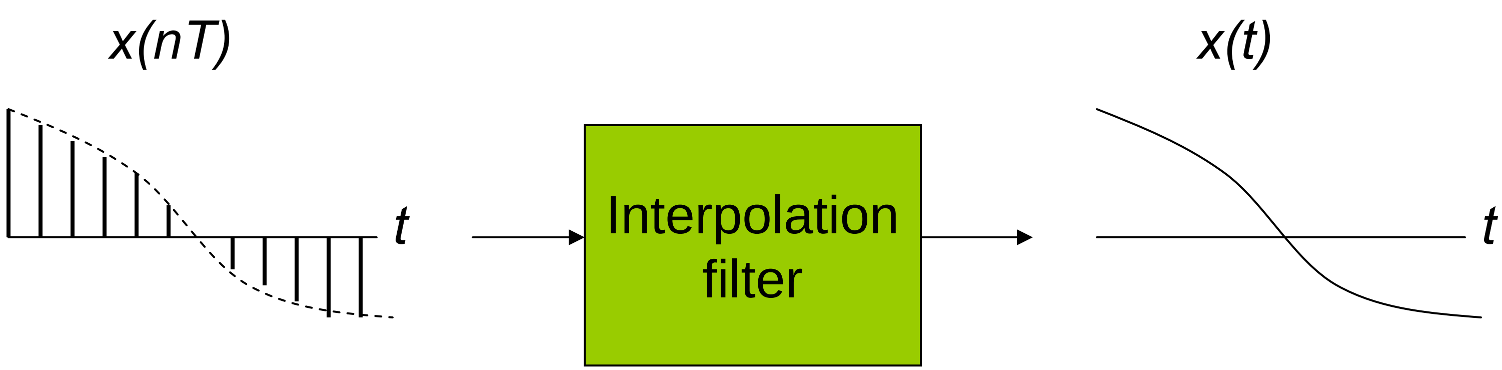
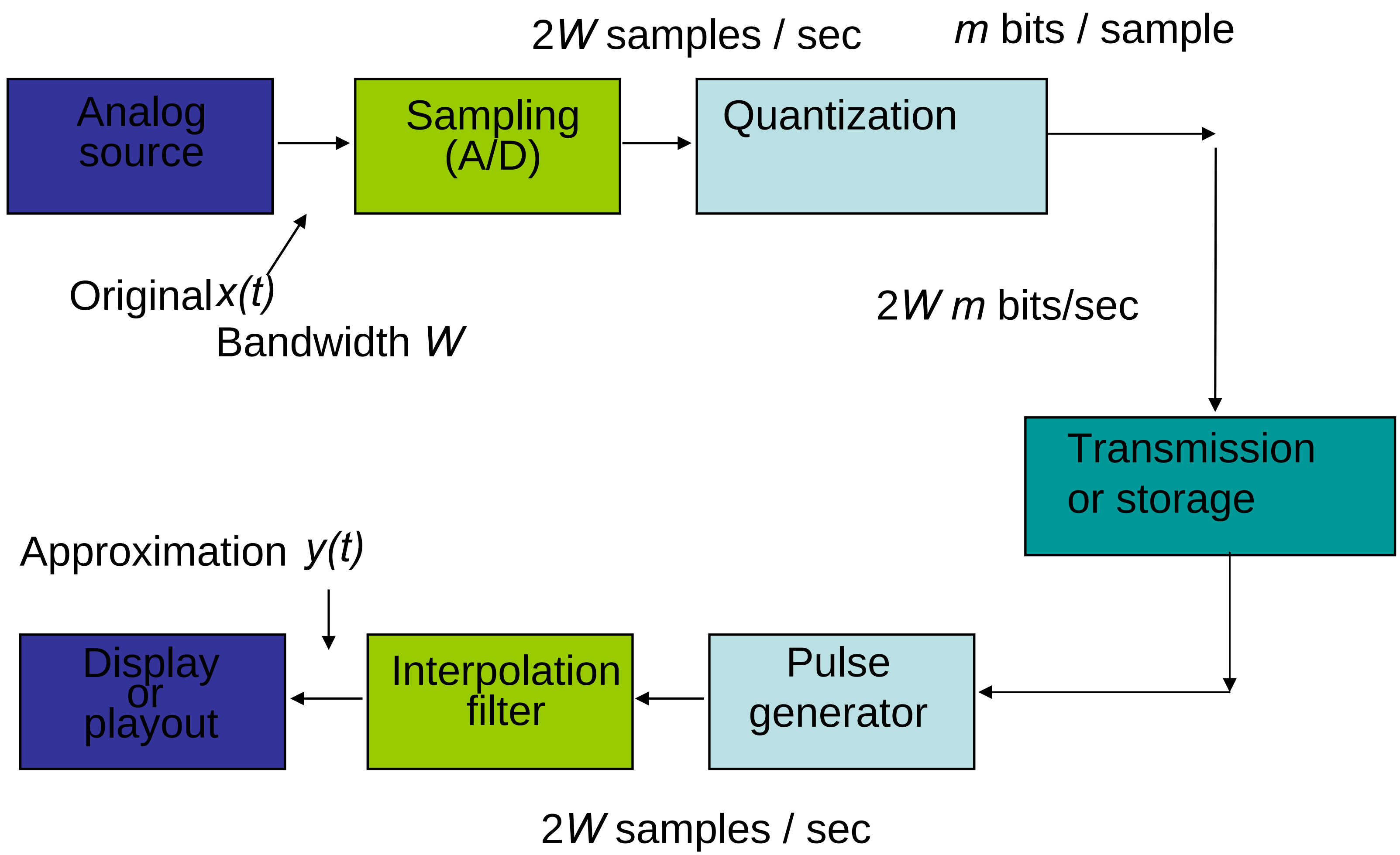
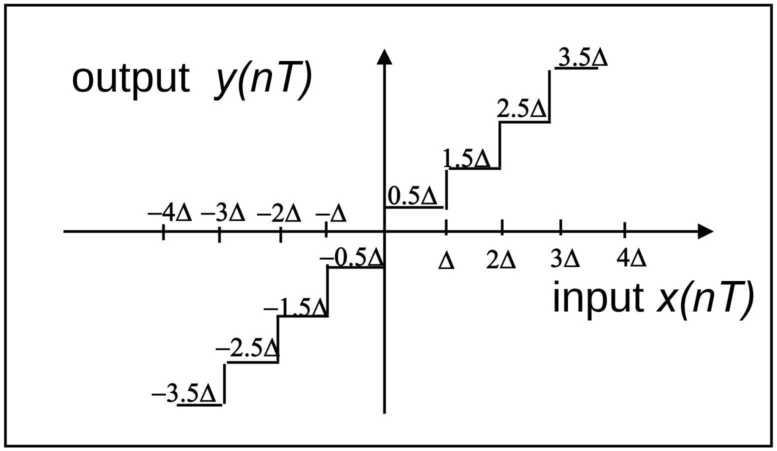


Digital Transmission of Analog Information

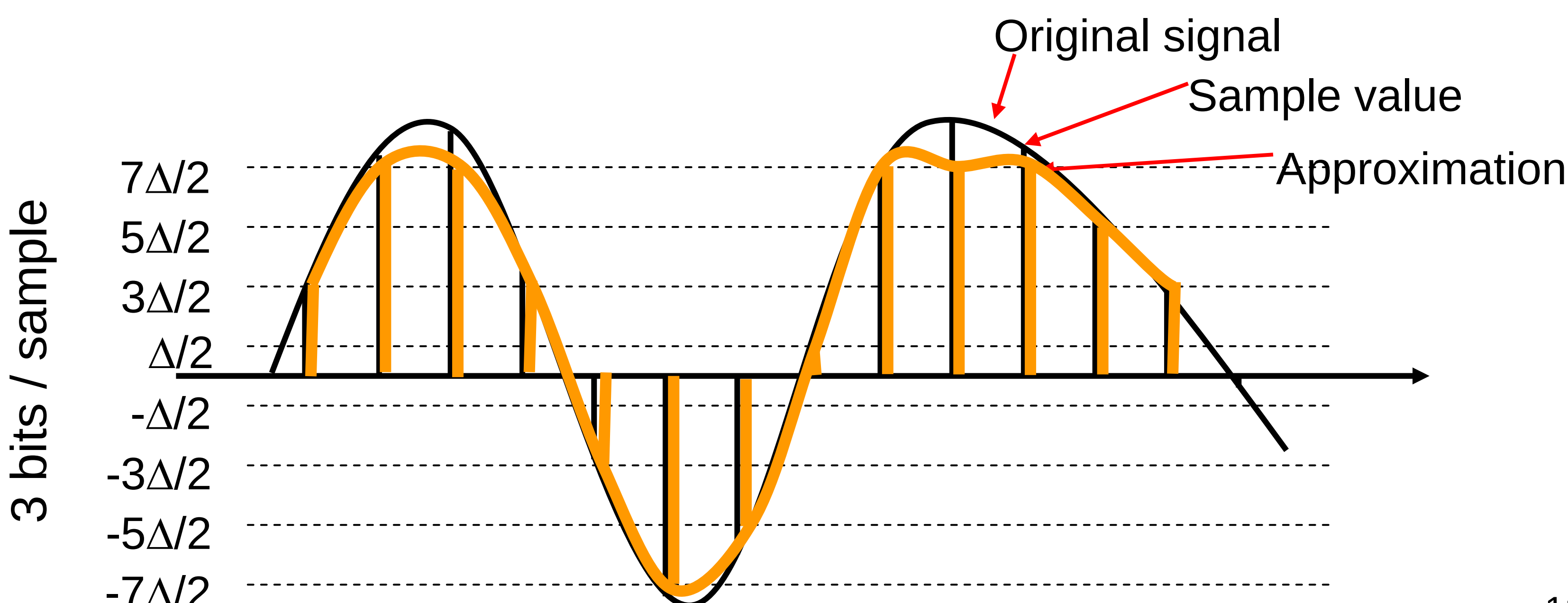


Quantization of Analog Samples



Quantizer maps input into closest of 2^m representation values

Quantization error:
"noise" = $x(nT) - y(nT)$



Example: Telephone Speech

W = 4KHz, so Nyquist sampling theorem

$\Rightarrow$ 2W = 8000 samples/second

PCM ("Pulse Code Modulation") Telephone Speech (8 bits/sample):

Bit rate= 8000 x 8 bits/sec= 64 kbps

Channel Characteristics

Transmission Impairments

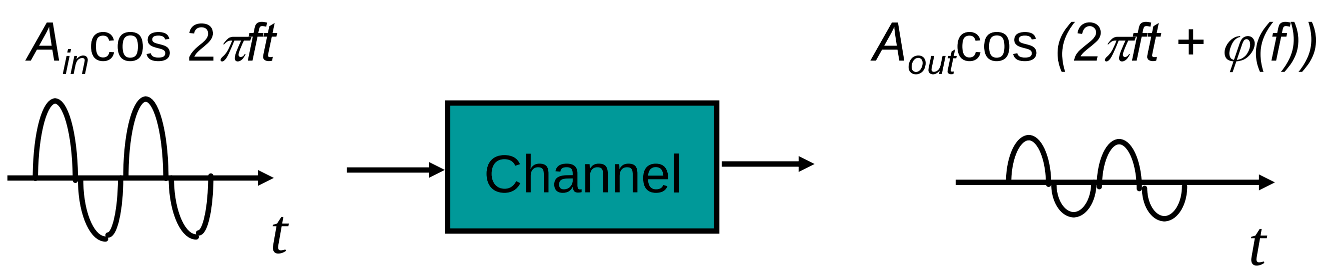
- Caused by imperfections of transmission media
- Analog signal: impairments degrade signal quality
- Digital signal: impairments cause bit errors
- Three main types of transmission impairments
  - Attenuation
  - Distortion
  - Noise

Attenuation

- Loss in power signal
  - A signal loses its energy while traveling through a medium
  - Loss in signal power as it is transferred across a system
- Overcome by boosting the signal
  - Analog → amplifiers
  - Digital → repeaters

Attenuation (cont.)

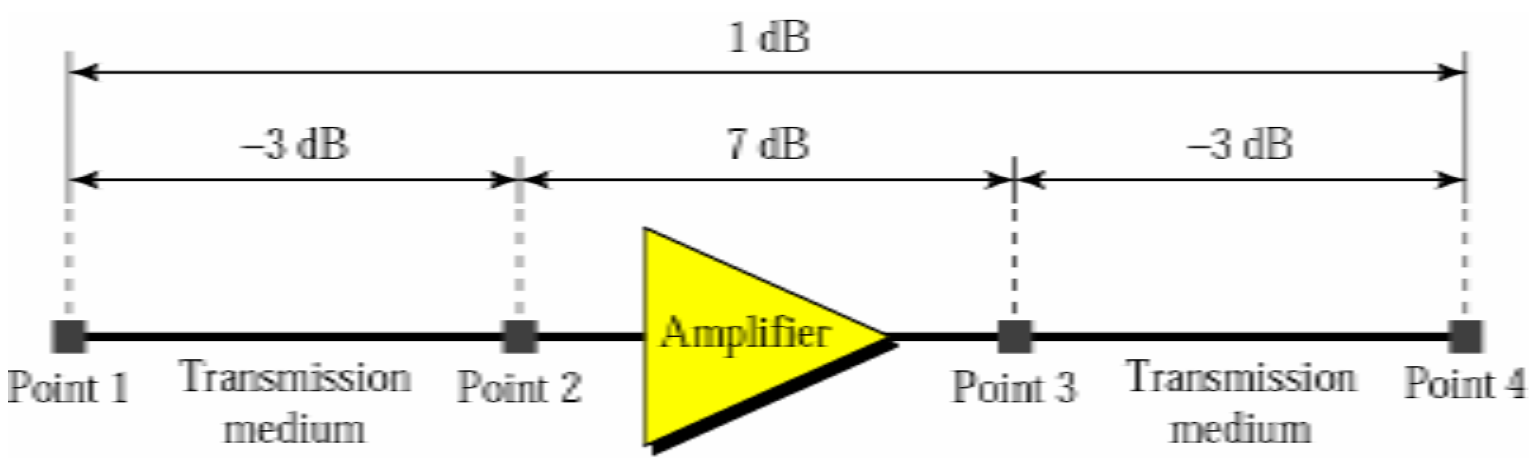
- Attenuation is usually expressed in decibel (dB)
- Atten.(f) =10 $\log_{10} P_{in}/P_{out}$ [dB]
- $P_{in}/P_{out} = A^2_{in}/A^2_{out} = 1/A^2$
- Atten.(f) =20 $\log_{10} 1/A^2$ [dB]



$$A(f) = \frac{A_{out}}{A_{in}}$$

Attenuation (cont.)

- Loss positive dB
- Gain negative dB
- Overall just sum them up



Channel Distortion

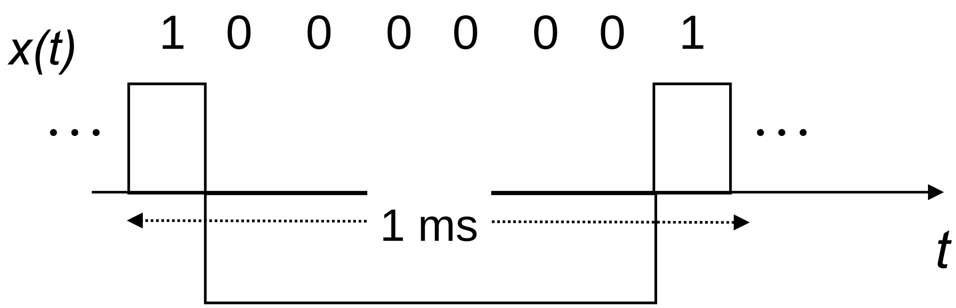
$$x(t) = \sum a_k \cos(2\pi f_k t + \theta_k) \longrightarrow \boxed{\text{Channel}} \longrightarrow y(t)$$

- Let *x(t)* corresponds to a digital signal bearing data information
- How well does *y(t)* follow *x(t)*?

$$y(t) = \sum A(f_k)\, a_k \cos(2\pi f_k t + \theta_k + \Phi(f_k))$$

- Channel has two effects:
  - If amplitude response is not flat, then different frequency components of *x(t)* will be transferred by different amounts
  - If phase response is not flat, then different frequency components of *x(t)* will be delayed by different amounts
- In either case, the shape of *x(t)* is altered

Amplitude Distortion

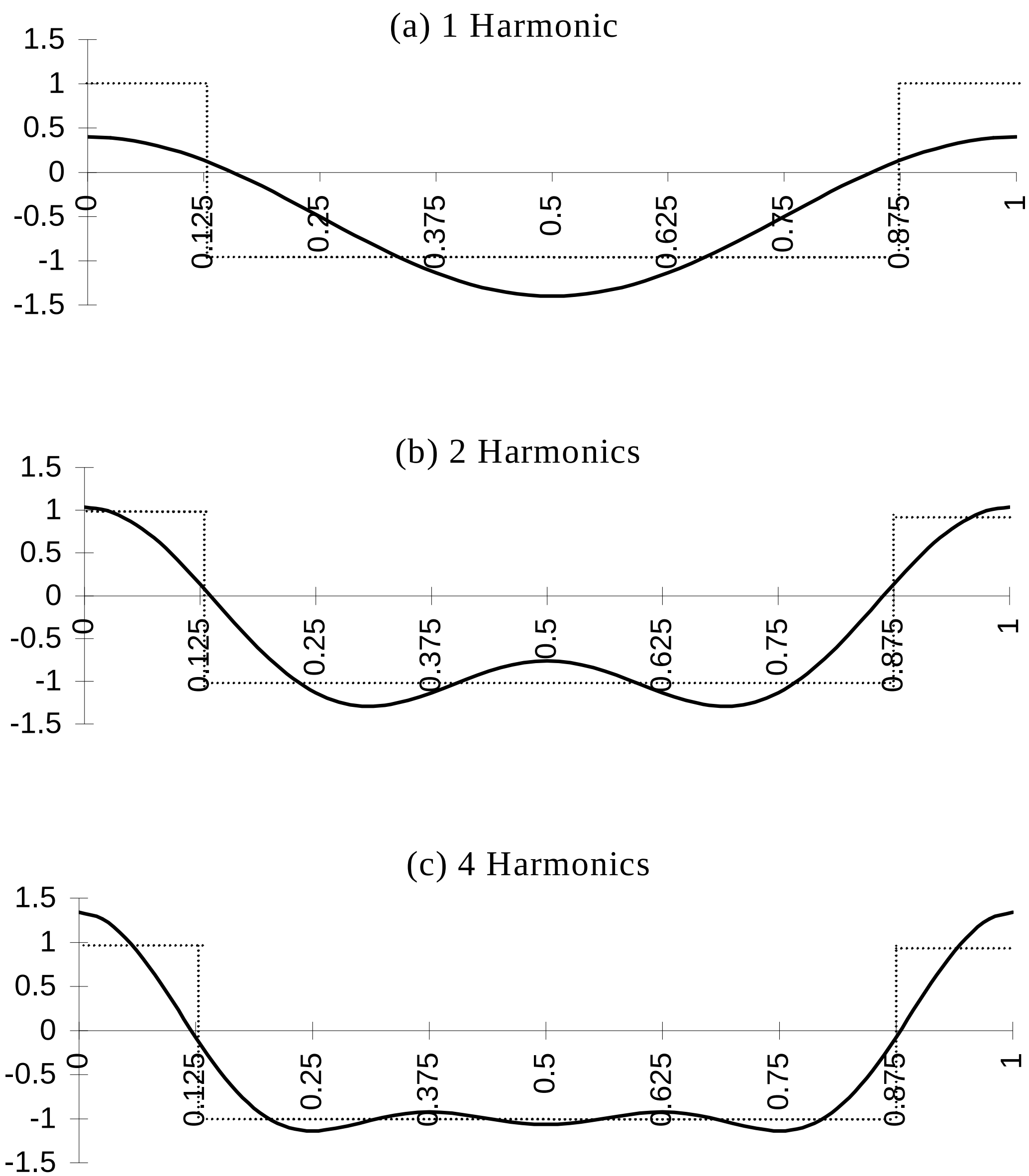


- Let *x(t)* input to ideal lowpass filter that has zero delay and W_c = 1.5 kHz, 2.5 kHz, or 4.5 kHz

$$x(t) = -0.5 + \frac{4}{\pi}\sin(\frac{\pi}{4})\cos(2\pi 1000t) + \frac{4}{\pi}\sin(\frac{2\pi}{4})\cos(2\pi 2000t) + \frac{4}{\pi}\sin(\frac{3\pi}{4})\cos(2\pi 3000t) + \dots$$

- W_c = 1.5 kHz passes only the first two terms
- W_c = 2.5 kHz passes the first three terms
- W_c = 4.5 kHz passes the first five terms

Amplitude Distortion (cont.)



- As the channel bandwidth increases, the output of the channel resembles the input more closely

Noise

- Unwanted signals that get inserted or generated somewhere between a transmitter and a receiver
- Types of noise
  - Thermal noise: result of random motion of electrons → depends on temperature
  - Intermodulation noise: generated during modulation and demodulation
  - Crosstalk: effect of one wire on the other
  - Impulse noise: irregular pulses or noise spikes i.e. electromagnetic disturbances

Data Rate Limit

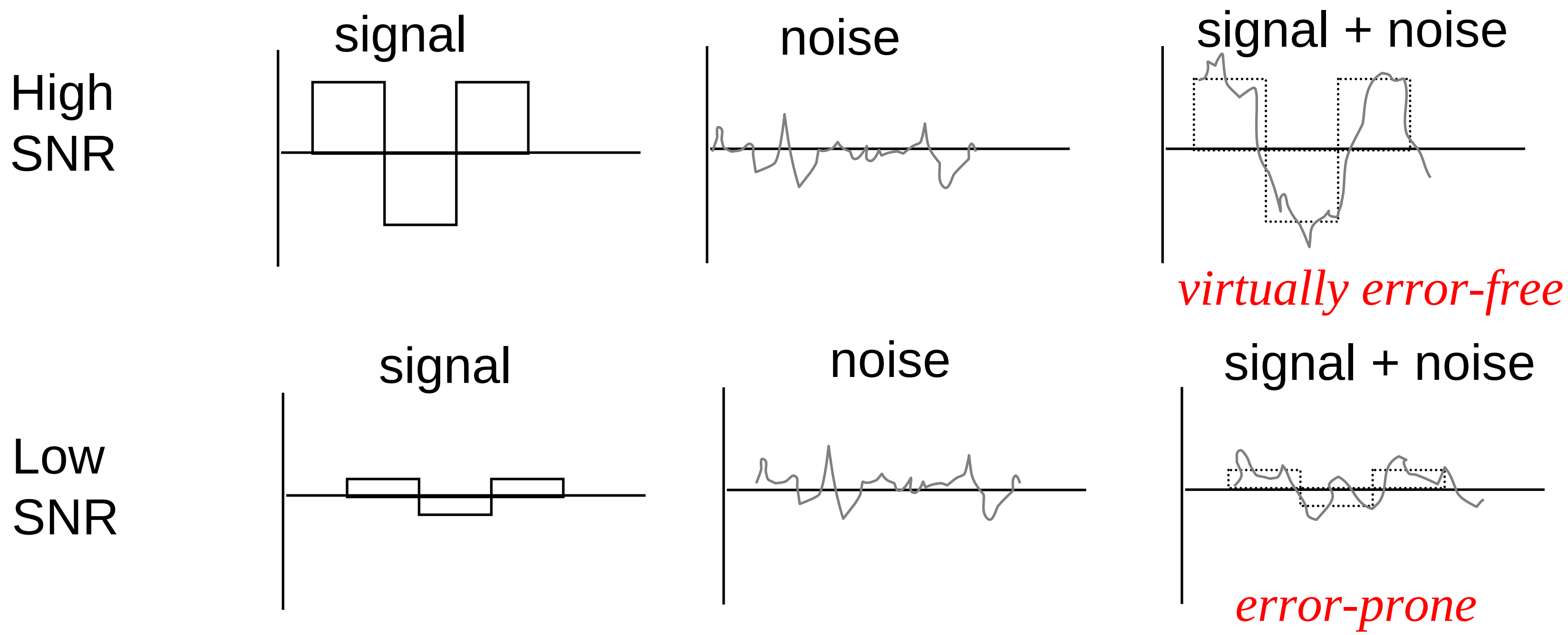
- Nyquist Theorem: maximum rate at which digital data can be transmitted over a channel of bandwidth B [Hz] is

$$C = 2 \times B \times \log_2 M \text{ [bps]}$$

M is a number of levels in digital signals

- Theoretical limit
- In practice we need to use both Nyquist and Shannon to find what data rate and signal levels are appropriate for each particular channel

Channel Noise affects Reliability



$$\text{SNR} = \frac{\text{Average Signal Power}}{\text{Average Noise Power}}$$

$$\text{SNR (dB)} = 10 \log_{10} \text{SNR}$$

Shannon Channel Capacity

- If transmitted power is limited, then as M increases spacing between levels decreases
- Presence of noise at receiver causes more frequent errors to occur as M is increased

Shannon Channel Capacity:

The maximum reliable transmission rate over an ideal channel with bandwidth W Hz, with Gaussian distributed noise, and with SNR S/N is

$$C = W \log_2 (1 + S/N) \text{ bits per second}$$

- Reliable means error rate can be made arbitrarily small by proper coding

Example

- Consider a 3 kHz channel with 8-level signaling. Compare bit rate to channel capacity at 20 dB SNR

- 3KHz telephone channel with 8 level signaling
  Bit rate = 2*3000 pulses/sec * 3 bits/pulse = 18 kbps

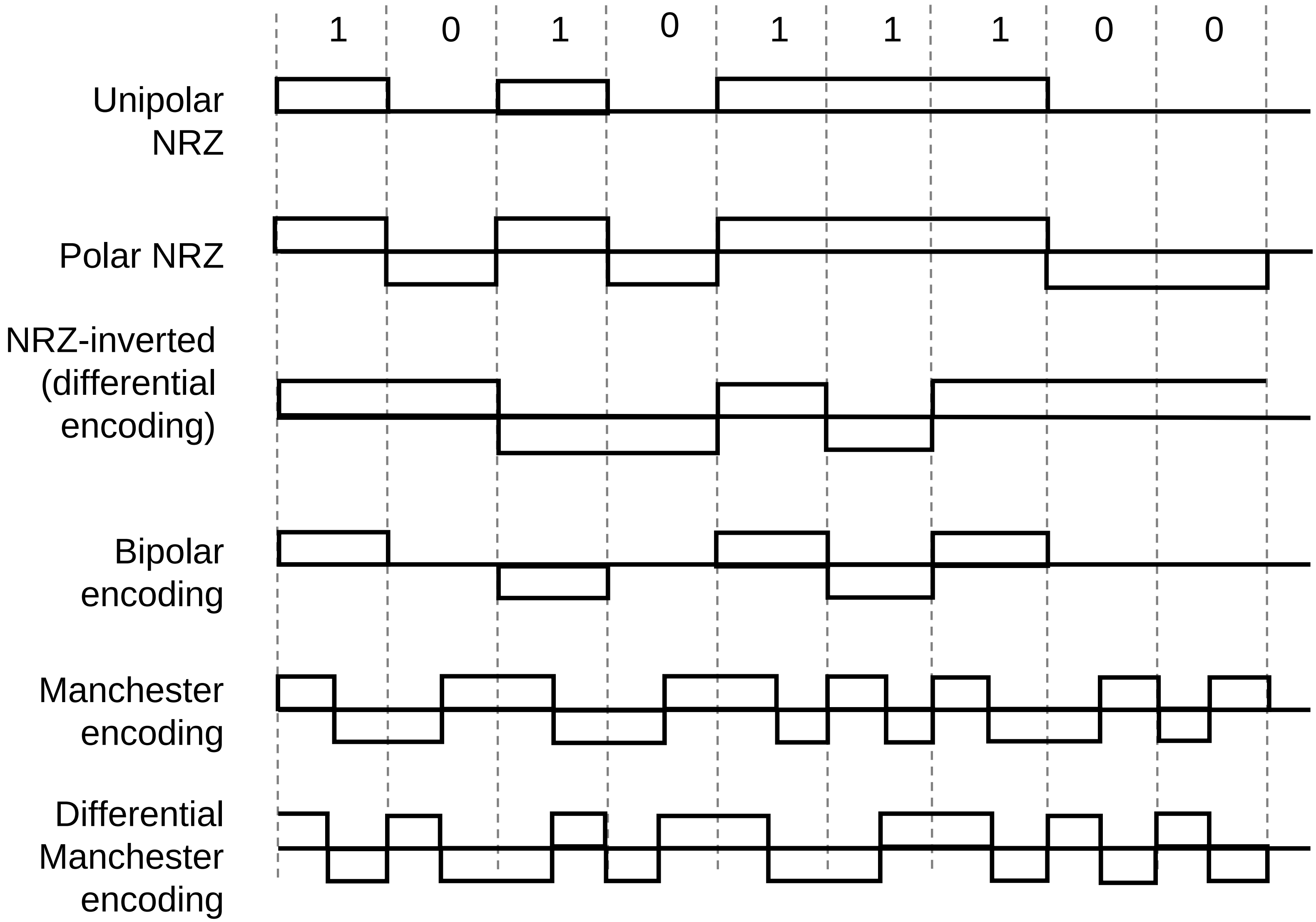
- 20 dB SNR means $10 \log_{10} S/N = 20$
  Implies $S/N = 100$
- Shannon Channel Capacity is then
  $C = 3000 \log (1 + 100) = 19,963$ bits/second

Line Coding

What is Line Coding?

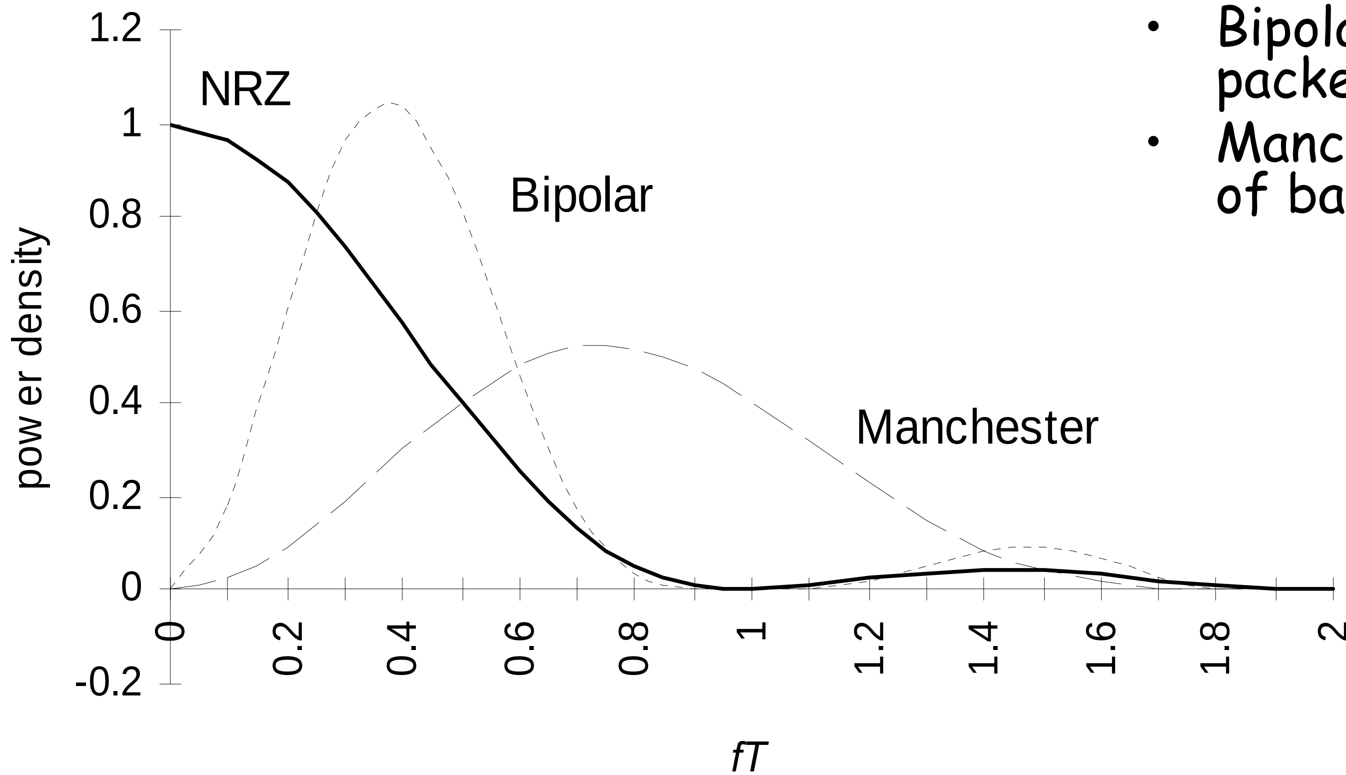
- Mapping of binary information sequence into the digital signal that enters the channel
  - Ex. "1" maps to +A square pulse; "0" to –A pulse
- Line code selected to meet system requirements:
  - *Transmitted power*: Power consumption = $
  - *Bit timing*: Transitions in signal help timing recovery
  - *Bandwidth efficiency*: Excessive transitions wastes bw
  - *Low frequency content*: Some channels block low frequencies
    - long periods of +A or of –A causes signal to "droop"
    - Waveform should not have low-frequency content
  - *Error detection*: Ability to detect errors helps
  - *Complexity/cost*: Is code implementable in chip at high speed?

Line coding examples



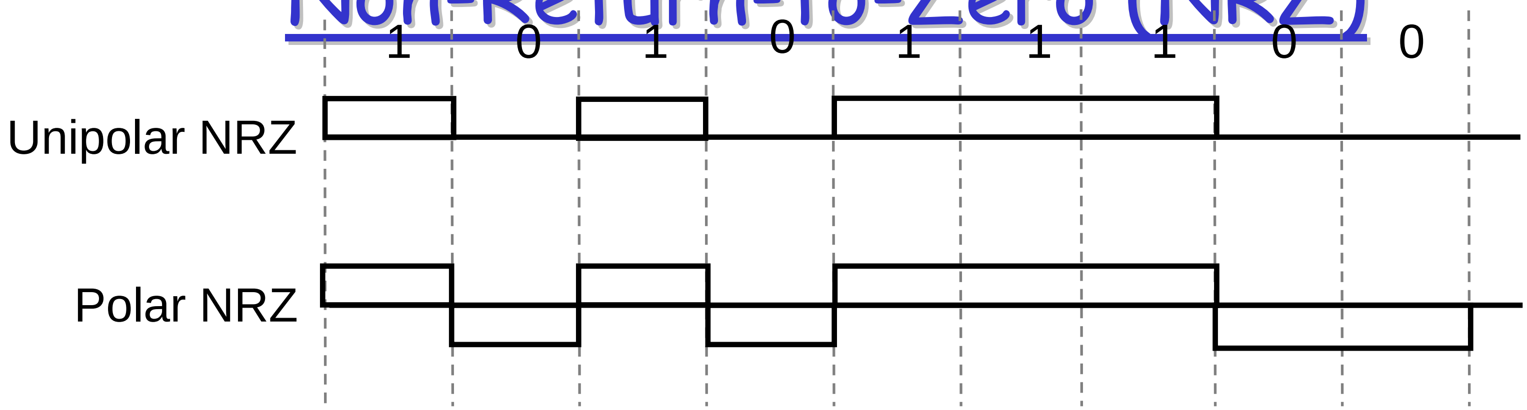
Spectrum of Line codes

- Assume 1s & 0s independent & equiprobable



- NRZ has high content at low frequencies
- Bipolar tightly packed around *T*/2
- Manchester wasteful of bandwidth

Unipolar & Polar Non-Return-to-Zero (NRZ)



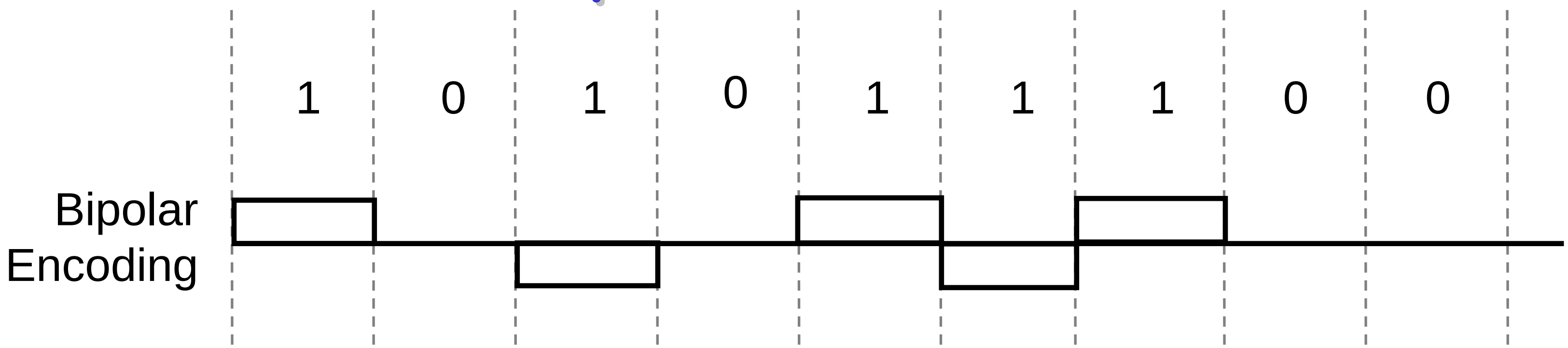
Unipolar NRZ

- "1" maps to +A pulse
- "0" maps to no pulse
- High Average Power
  $0.5*A^2 + 0.5*0^2 = A^2/2$
- Long strings of A or 0
  - Poor timing
  - Low-frequency content
- Simple

Polar NRZ

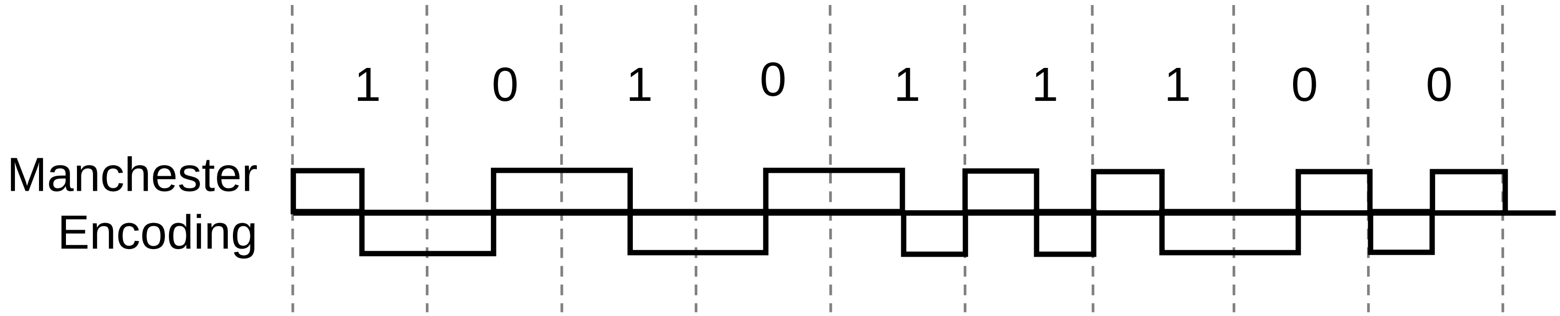
- "1" maps to +A/2 pulse
- "0" maps to –A/2 pulse
- Better Average Power
  $0.5*(A/2)^2 + 0.5*(-A/2)^2 = A^2/4$
- Long strings of +A/2 or –A/2
  - Poor timing
  - Low-frequency content
- Simple

Bipolar Code



- Three signal levels: {-A, 0, +A}
- "1" maps to +A or –A in alternation
- "0" maps to no pulse
  - Every +pulse matched by –pulse so little content at low frequencies
- String of 1s produces a square wave
  - Spectrum centered at $T/2$
- Long string of 0s causes receiver to lose synch
- Zero-substitution codes

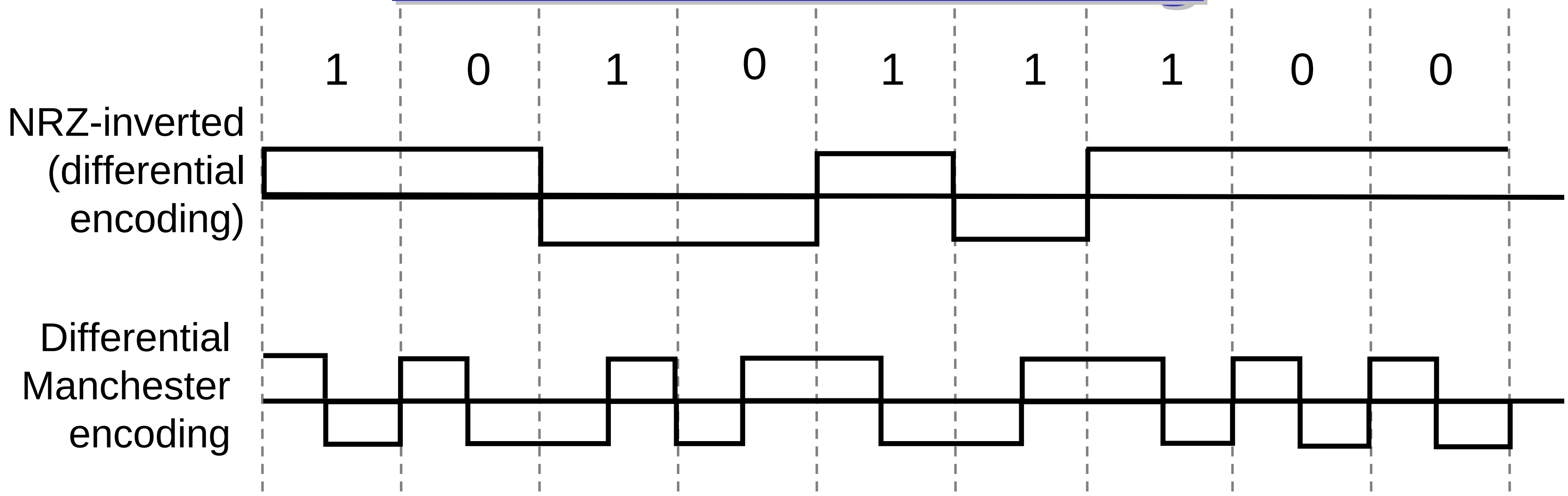
Manchester code & *m*B*n*B codes



- "1" maps into $A/2$ first $T/2$, $-A/2$ last $T/2$
- "0" maps into $-A/2$ first $T/2$, $A/2$ last $T/2$
- Every interval has transition in middle
  - Timing recovery easy
  - Uses double the minimum bandwidth
- Simple to implement
- Used in 10-Mbps Ethernet & other LAN standards

- *m*B*n*B line code
- Maps block of *m* bits into *n* bits
- Manchester code is 1B2B code
- 4B5B code used in FDDI LAN
- 8B10b code used in Gigabit Ethernet
- 64B66B code used in 10G Ethernet

Differential Coding



- Errors in some systems cause transposition in polarity, +A become –A and vice versa
  - All subsequent bits in Polar NRZ coding would be in error
- Differential line coding provides robustness to this type of error
- "1" mapped into transition in signal level
- "0" mapped into no transition in signal level
- Same spectrum as NRZ
- Errors occur in pairs
- Also used with Manchester coding