CSE4210 Architecture and Hardware for DSP

Lecture 4

Division and CORDIC

Division

- Division can be implemented as shift and subtract (shift and add for mul.)
- In division, the result of one subtraction determine the next operation (unlike multiplication).
- More difficult, and more time consuming than multiplication.

Subtract and Shift Division

- $X/D=Q \Rightarrow X = Q \bullet D + R, \quad R < D$
- For simplicity, we assume there is no overflow (more bits to represent the quotient than the register size).
- Restoring and non-restoring division

Restoring Division

- Assume that Q contains the divident (by the end of the calculation, it will contain the quotient).
- A Contains zero, by the end it will contain the remainder
- M contains the divisor

Restoring Division

- Algorithm
- Do n times (number of bits in Q)
  - Left shift A|Q
  - $A \leftarrow A-M$
  - If $A < 0$, $q_0=0$, $A \leftarrow A+M$ (restore)
  - Else $q_0=1$

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000
A=A+M	0011	

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000
A=A+M	0011	
	0010	1000
Left shift A|Q	0101	000?

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000
A=A+M	0011	
	0010	1000
Left shift A|Q	0101	000?
A=A-M	1101	

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000
A=A+M	0011	
	0010	1000
Left shift A|Q	0101	000?
A=A-M	1101	
A>0	0010	0001

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000
A=A+M	0011	
	0010	1000
Left shift A|Q	0101	000?
A=A-M	1101	
A>0	0010	0001
Left shift A!Q	0100	001?

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000
A=A+M	0011	
	0010	1000
Left shift A|Q	0101	000?
A=A-M	1101	
A>0	0010	0001
Left shift A!Q	0100	001?
A=A-M	1101	

Restoring Division

	A	Q
	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M	1101	
A < 0	1110	0100
A=A+M	0011	
	0001	0100
Left shift A|Q	0010	100?
A=A-M	1101	
A < 0	1111	1000
A=A+M	0011	
	0010	1000
Left shift A|Q	0101	000?
A=A-M	1101	
A>0	0010	0001
Left shift A!Q	0100	001?
A=A-M	1101	
A>0	0001	0011
	R=1	Q=3

Non Restoring Division

- Do n times {
  - Left shift A|Q
  - If previous A $\geq$ 0 then A=A $\leftarrow$ A-M
  - Else A $\leftarrow$ A+M
  - If current A $\geq$ 0) then $q_0 \leftarrow$ 1
  - Else $q_0 \leftarrow$ 0
  - }
- If A < 0 then A $\leftarrow$ A+M

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	
A < 0	1111	1000

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	
A < 0	1111	1000
Left shift A|Q	1111	000?

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	
A < 0	1111	1000
Left shift A|Q	1111	000?
A=A+M prev -	0011	

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	
A < 0	1111	1000
Left shift A|Q	1111	000?
A=A+M prev -	0011	
A > 0	0010	0001

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	
A < 0	1111	1000
Left shift A|Q	1111	000?
A=A+M prev -	0011	
A > 0	0010	0001
Left shift A|Q	0100	001?

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	
A < 0	1111	1000
Left shift A|Q	1111	000?
A=A+M prev -	0011	
A > 0	0010	0001
Left shift A|Q	0100	001?
A = A-M prev +	1101	

York University CSE

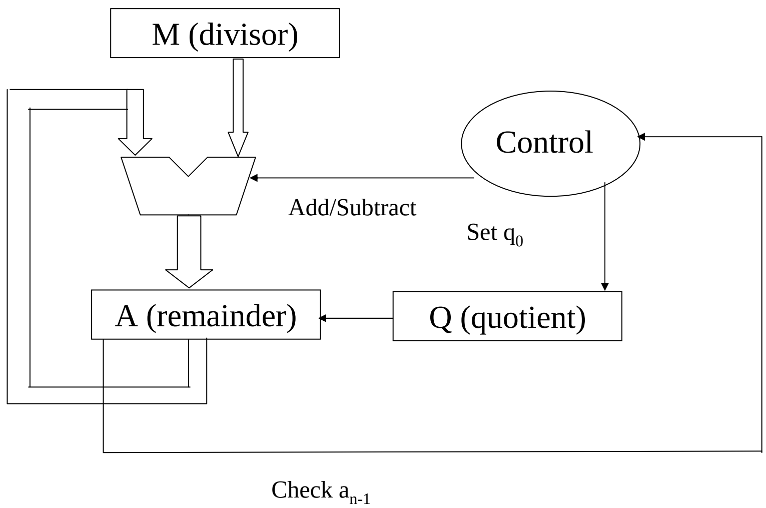
Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M prev -	0011	
A < 0	1111	1000
Left shift A|Q	1111	000?
A=A+M prev -	0011	
A > 0	0010	0001
Left shift A|Q	0100	001?
A = A-M prev +	1101	
A > 0	0001	0011

York University CSE

Restoring Division

	[M]=0011	
	[A]=0000	[Q]=1010
Left shift A|Q	0001	010?
A=A-M prev +	1101	
A < 0	1110	0100
Left shift A|Q	1100	100?
A=A+M	0011	
A < 0	1111	1000
Left shift A|Q	1111	000?
A=A+M	0011	
A > 0	0010	0001
Left shift A|Q	0100	001?
A = A-M	1101	
A > 0	0001	0011
A > 0 NO CORRECTION		
	R	Q



CORDIC

- CORDIC (COordinate Rotation DIgital Computer)
- Used to calculate trigonometric functions (and many other)
- Reduces calculation to a series of shift and add operations
- Delay and hardware cost comparable to division.

CORDIC

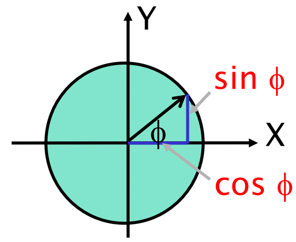
- The vector (1,0) is rotated by an angle ϕ

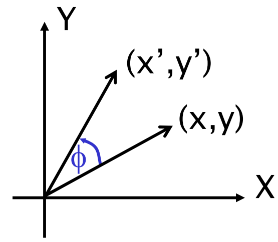
$$x' = x.\cos(\phi) - y.\sin(\phi)$$

$$y' = y.\cos(\phi) + x.\sin(\phi)$$

$$x' = \cos(\phi)\left[x - y\tan(\phi)\right]$$

$$y' = \cos(\phi)\left[x + y\tan(\phi)\right]$$





CORDIC

$$x' = \cos(\phi)\left[x - y\tan(\phi)\right]$$

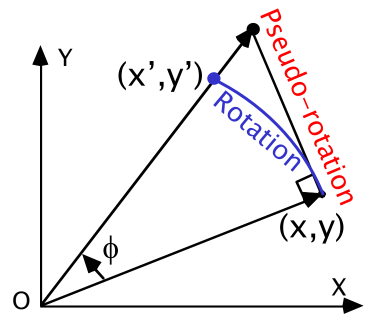
$$y' = \cos(\phi)\left[x + y\tan(\phi)\right]$$

If we do not multiply by $\cos(\phi) \rightarrow$ pseudo rotation

If we start by (1,0) then x' is $\cos(\phi)$

If we choose $\tan(\phi)$ to be 2^{-1}, then the multiplication is a shift.

Any angle can be the sum of many ϕ's with tan = 2^{-i}



CORDIC

i	α_i	$\tan(\alpha_i)$	
0	45.0°	1	$= 2^{-0}$
1	26.6°	0.5	$= 2^{-1}$
2	14.0°	0.25	$= 2^{-2}$
3	7.1°	0.125	$= 2^{-3}$
4	3.6°	0.0625	$= 2^{-4}$
5	1.8°	0.03125	$= 2^{-5}$
6	0.9°	0.015625	$= 2^{-6}$
7	0.4°	0.0078125	$= 2^{-7}$
8	0.2°	0.00390625	$= 2^{-8}$
9	0.1°	0.001953125	$= 2^{-9}$
10	0.055	0.000976562	$=2^{-10}$

- 37.2° =45-26.6+14.0+7.1-3.6+1.8-0.9+0.4 = 37.2°

York University CSE

CORDIC

$$x_{i+1} = \cos(\phi_i).[x_i - y_i.\sigma_i.\tan(\phi_i)]$$
$$y_{i+1} = \cos(\phi_i).[y_i + x_i.\sigma_i.\tan(\phi_i)]$$

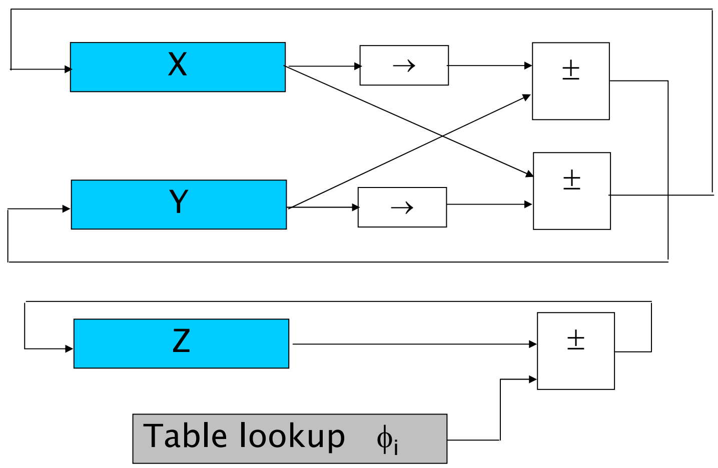
$$\sigma_i \in \{-1,1\}$$

$$x_{i+1} = \cos(\phi_i).[x_i - .\sigma_i.2^{-i}.y_i]$$
$$y_{i+1} = \cos(\phi_i).[y_i + .\sigma_i.2^{-i}x_i]$$
$$z_{i+1} = z_i - \sigma_i \tan^{-1}(2^{-i})$$

$\cos(\phi_i)$ is a constant K_i

York University CSE

CORDIC -- Hardware



York University CSE

CORDIC

$$x_{i+1} = K_i.[x_i - .\sigma_i.2^{-i}.y_i]$$
$$y_{i+1} = K_i.[y_i + .\sigma_i.2^{-i}x_i]$$

We can multiply by a constant K once a the end

$$K = \prod_{i=1}^{n} K_i \qquad n \to \infty, \qquad K = 0.607252935...$$

Or, we can start with (0.60725,0) instead of (1,0) and save one multiplication