York University | CSE 2001 | November 21, 2007

Homework Assignment #5
Due: December 3, 5:30 p.m.

You may use the Church-Turing Thesis throughout this assignment.

1. **(a)** Let G be a grammar for the context-free language L. Let p be the pumping length for L. (In other words, p is the number that satisfies Theorem 2.34 for the language L.) Prove that L is infinite if and only if it contains a string of length at least p.

(b) Prove that $L_1 = \{\langle G \rangle : G \text{ is a CFG and } L(G) \text{ is an infinite language}\}$ is decidable. Hint: Use the result of Problem 2.18(a), which is proved on page 133 of the text.

(c) Prove that $L_2 = \{\langle G \rangle : G \text{ is a CFG and } |L(G)| = 17\}$ is decidable.

2. Let $E_{PDA} = \{\langle M_1, M_2 \rangle : M_1 \text{ and } M_2 \text{ are PDAs and } L(M_1) = L(M_2)\}$.

(a) Prove that E_{PDA} is undecidable.

(b) Prove that $\overline{E_{PDA}}$ is recognizable.

(c) Prove that E_{PDA} is not recognizable.

3. Let $L_3 = \{\langle M \rangle : M \text{ is a Turing machine that accepts the string } 0011\}$. Prove that L_3 is undecidable. (Do not use Rice's Theorem.)