# Homework Assignment #4
# Due: November 19, 5:30 p.m.

**1.** Prove that the language $L = \{a^n b^m c^{mn} : m \geq 0, n \geq 0\}$ is not context-free.

**2.** If $A$ and $B$ are languages, define $A \diamond B = \{xy : x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$.

**(a)** Give two regular languages $A$ and $B$ such that $A \diamond B$ is not regular.

**(b)** Give two context-free languages $A$ and $B$ such that $A \diamond B$ is not context-free.

**(c)** Prove that, for all regular languages $A$ and $B$, $A \diamond B$ is context-free.

**3.** Give a high-level description of a Turing machine that decides the language

$$L = \{a^n b^m : n \geq 1, m = \lfloor \log_2 n \rfloor\}.$$

Do not give a formal description of the machine; instead, you should break an algorithm for deciding the language down into a sequence of very simple steps, each of which can easily be implemented on a Turing machine. The format of your answer might look something like the 5-step description of $M_3$ on page 147 of the text book.

**4.** Explain why, for all Turing-decidable languages $A$ and $B$, $A \diamond B$ is Turing-decidable. (For this question, you may use the Church-Turing Thesis.)