

Homework Assignment #3

Due: October 31, 5:30 p.m.

1. Let $L = \{0^i 1^j 2^k : i = k \text{ or } j \geq 3i\}$.
 - (a) Give a context-free grammar for L .
 - (b) Prove that your grammar generates every string in L .
 - (c) Draw a PDA that recognizes L . Briefly explain why your PDA works.

2. Recall that $x^{\mathcal{R}}$ denotes the string x written backwards. Let B_i be the standard binary representation of positive integer i with no leading 0's. For example, $B_{13} = 1101$ and $B_{103} = 1100111$. Let $L = \{B_i \# B_{i+1}^{\mathcal{R}} : i \geq 1\}$. For example, $1100111 \# 0001011$ is in L because 1100111 is the binary representation of 103 and 0001011 is the reverse of the binary representation of $104 = 103 + 1$. Show that L is context-free.

3. Consider the grammar on page 101. Add the following rules:

$$\begin{aligned}
 \langle \text{SENTENCE} \rangle &\rightarrow \langle \text{VERB-PHRASE} \rangle \\
 \langle \text{CMLPX-NOUN} \rangle &\rightarrow \langle \text{NOUN} \rangle \\
 \langle \text{CMLPX-NOUN} \rangle &\rightarrow \langle \text{NOUN} \rangle \langle \text{NOUN} \rangle \\
 \langle \text{NOUN} \rangle &\rightarrow \text{fashion} \mid \text{wind} \mid \text{changes} \\
 \langle \text{VERB} \rangle &\rightarrow \text{fashion} \mid \text{wind} \mid \text{changes} \mid \text{like} \\
 \langle \text{PREP} \rangle &\rightarrow \text{like}
 \end{aligned}$$

Some comments on these new rules:

The first new rule allows sentences without a subject, like “Wind the clock” or “See the flower”.

The second new rule allows sentences where nouns are not preceded by an article. This happens frequently for plural nouns, abstract nouns or proper nouns. For example, “Puppies are cute” or “Time flies” or “John loves Peter”.

The third new rule allows for the creation of a noun by combining two nouns. Examples of this kind of construction include “fashion magazines” (magazines about fashion), “snow showers” (showers of snow), “snow fences” (fences to prevent snow from drifting), “snow pants” (pants to be worn in the snow).

The other new rules simply increase the grammar’s vocabulary.

Note that **fashion** can be used as a noun or verb: “Short trousers are in fashion” or “I fashion whistles out of blocks of wood”. So can **wind**: “The wind is blowing” or “I wind my clock”. So can **changes**: “Changes are coming soon” or “He changes his clothes daily”. The word **like** can be used as a verb or a preposition: “I like puppies” or “She runs like a deer”.

Words that have several different roles (like *fashion* or *changes*) are examples of *lexical ambiguity*. The possibility of different parse trees for the same sentence is called *syntactic ambiguity*. Both types of ambiguity are common in human languages. They cause major difficulties when building computer systems that are supposed to extract information from English sentences. Such a system requires a lot of additional knowledge beyond a context-free grammar for the language in order to find the intended interpretation of a sentence. In some cases, these ambiguities make sentences difficult even for humans to interpret: “I saw the man with the telescope” could mean “Using the telescope, I saw the man” or “I saw the man who was carrying a telescope”, depending on the context.

- (a) Draw all possible parse trees for the sentence “**fashion changes like the wind**”. Which one do you think represents the intended meaning of the sentence best?
 - (b) Give an example of a sentence that can be generated by the grammar, but would not be considered a grammatical English sentence. Your sentence should use only the vocabulary of the context-free grammar (**boy, girl, flower, touches, wind, etc.**). (Note: it is possible for an English sentence to be grammatical even if it does not make any sense, like “The colourless green ideas sleep furiously”, but I want you to give a generated sentence that is *grammatically wrong*, not just nonsensical.)
 - (c) Add some new rules to the grammar so that it will generate the sentence “**the small girl gives the boy the big red smelly flower**”. Try to make your rules general, so that they will be useful not just for this one sentence, but also for similar grammatical English sentences. Also, parse trees that use your new rules should reflect the grammatical structure of the sentence as well as possible.
4. Prove that the class of context-free languages is closed under star. In other words, show that whenever L is context-free, L^* is also context-free.