Homework Assignment #2
Due: October 15, 5:30 p.m.

1. For this problem, you will be handling strings that represent natural numbers in binary notation, but the order in which the characters appear will be the reverse of usual. Thus, the first bit is the 1's bit, the second is the 2's, the third is the 4's, etc. For example, the string 011001 represents $2 + 4 + 32 = 38$ and 1110110 represents $1 + 2 + 4 + 16 + 32 = 55$.

Read the informal definition of a finite state transducer in Problem 1.24 on page 87 of the textbook. Design a finite state transducer that outputs a value that is 1 larger than the input value (using the representation of numbers described above). For example, if the finite state transducer receives 00111 (which represents 28) as input then it should output 10111 (which represents 29).

The specification has one exception: if the input string is all 1's, the output should be all 0's (since the output will not be long enough to output the value with one added).

You do not have to give a proof of correctness.

2.

(a) Describe, in plain but precise English, the set of strings defined by the regular expression $(01^*)^*0$.

(b) Draw an NFA that recognizes the language described by the regular expression $(ab \cup ba)^*(abb \cup ab \cup bba)^*$. Try to make your NFA as simple as possible.

(c) Give a regular expression for the set of all binary strings that do not contain 011 as a substring.

3. The *reverse* of a string $x = a_1a_2a_3 \ldots a_n$ is $x^{\mathcal{R}} = a_na_{n-1}a_{n-2} \ldots a_1$ (*i.e.*, the characters of the original string written in reverse order). Consider the alphabet $\left\{\binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{1}{1}\right\}$. Let L be the language of all strings over this alphabet where the top row is *not* equal to the reverse of the bottom row.

For example, $\binom{0}{1}\binom{0}{0}\binom{1}{1}\binom{1}{0}$ is in the language because the top row is 0011 and the reverse of the bottom row is 0101 and these two binary strings are different. As another example, the string $\binom{0}{1}\binom{1}{0}\binom{1}{0}\binom{1}{1}\binom{0}{1}\binom{0}{1}\binom{1}{0}$ is not in the language because the top row is 0111001 and the reverse of the bottom row is also 0111001.

Prove that L is not regular.

4. Prove that if any language L is regular, then $L^{\mathcal{R}} = \{x^{\mathcal{R}} : x \in L\}$ is also regular.