

Bonus Homework Assignment**Due: December 3, 9:30 a.m.**

1. 40 friends are going to the opera. They have reserved the front row of the theatre, which contains 40 seats. Each seat is numbered, and each friend has a ticket with a seat number on it. When they arrive at the theatre, they line up to enter.

Albert is at the front of the line. Unfortunately, Albert drops his ticket right after entering the theatre, so he doesn't know which seat he is supposed to sit in. The orchestra is beginning to warm up, and the lights are beginning to dim. Albert panics. He picks a random seat in the front row.

Bertram is second in line. He looks at the seat indicated on his ticket. If that seat is not occupied by Albert, Bertram sits in it. However, if that seat is occupied by Albert, Bertram picks another seat in the front row randomly. (He doesn't want to bother Albert to move, and he just wants to sit quickly, since the show is about to begin.)

Each friend in the line follows the same procedure: if his own seat is unoccupied, he sits in it and, otherwise, he sits in a randomly chosen unoccupied seat in the front row.

- (a) How many different ways can the friends seat themselves? Prove your answer is correct.
- (b) Zebediah is last in the line. The overture is beginning as he rushes to sit in the only unoccupied seat in the front row. People glare at him icily for making noise as he rushes in while the music is already playing. As he sits down he compares the number on his seat and the number on his ticket. What are the chances that they are the same? Prove your answer is correct.

Hint: in thinking about this problem, it might help to label the seats in the order that their owners stand in the lineup so that you can talk about them more easily. (The seat on Albert's ticket is labelled 1, the seat on Bertram's ticket is labelled 2, ..., the seat on Zebediah's ticket is labelled 40.) You might first consider a similar situation where there are 3 or 4 or 5 friends instead of 40 to get some intuition about the problem.