Homework Assignment #13
Due: November 12, 9:30 a.m.

For each of the following, explain *briefly* why your answer is correct. (In particular, say when you are using the product rule, sum rule, principle of inclusion-exclusion, pigeonhole principle, etc.)

1. Frederick owns 6 three-piece suits. Each three-piece suit consists of a matching vest, jacket and pair of pants. Each suit is a different colour. All of these 18 items of clothing are mixed up in his closet, and the light bulb in the closet has burned out, so it's impossible to tell which items match. Frederick goes into the closet in the morning to find a suit. He brings out a collection of items and then chooses from among them a matching vest, jacket and pair of pants. When he is choosing items to bring out, he can tell the difference between different kinds of items (e.g. he can distinguish a vest from a jacket) by the way it feels, but he cannot tell what colour any item is. What is the *minimum* number of items of clothing Frederick must bring out of the closet so that he is guaranteed to have a matching suit. How many pairs of pants, how many vests and how many jackets should he bring out of the closet to achieve this?

2. It turns out that Frederick is a spider, so he has 8 feet. He also has a big, unsorted pile of socks in his closet. There are 5 different colours of socks: chartreuse, vermillion, lavender, fuchsia and puce. (Frederick has an idiosyncratic sense of style, even for a spider.) Assume he has at least 8 socks of each colour.

   (a) If Frederick cannot tell the difference between socks of different colours inside his still darkened closet, what is the minimum number of socks he must pull out of the closet so that he is guaranteed to get a matching set of 8 socks of the same colour?

   (b) Frederick fixes the light bulb in his closet so that he can see all of his socks. How many different ways are there for him to choose socks for his 8 feet? Two socks of the same colour are indistinguishable but his 8 feet are distinguishable. For example, one way of choosing socks would be to choose puce ones for his 4 left feet, a lavender one for his foremost right foot and vermillion ones for his other 3 feet.

   (c) How many of the ways in (b) do not have all 8 socks matching?

   (d) How many of the ways in (b) include at least one chartreuse sock?

   (e) How many of the ways in (b) include at least one chartreuse sock but no vermillion sock?

   (f) Frederick abhors mixing vermillion and chartreuse. He feels that they clash terribly. How many of the ways in (b) do not include *both* vermillion and chartreuse socks?