

Homework Assignment #11

Due: October 31, 9:30 a.m.

Remember: whenever you prove something by induction, I want you to clearly identify the crucial parts of the proof. In particular, you should explicitly state the claim that you are proving by induction, your induction hypothesis, etc.

1. What is the last digit of the number $3^{182637168365231}$? Prove your answer is correct.
(Hint: don't try to compute the number, since you only have 5 days to complete this assignment. Instead look for patterns in numbers of the form 3^i . Once you correctly guess a pattern, you can prove that it really is a pattern.)
2. Define a function $f : \mathbb{N} \rightarrow \mathbb{N}$ by

$$\begin{aligned}f(0) &= 0, \\f(1) &= 1, \text{ and} \\f(n) &= f(n-1) + 2f(n-2) \text{ for } n \geq 2.\end{aligned}$$

- (a) Write down the values of $f(2), f(3), f(4), f(5), f(6), f(7), f(8)$.
- (b) Prove that $f(n)$ is $O(2^n)$.