Problem Session 2

Problem 1

Design a DFA that recognizes the following language:
\[ L = \{ w \mid w \text{ starts with 0 and has odd length, or starts with 1 and has even length} \} \]

Solution:

![DFA Diagram]

The meaning of the states:
- \( q_0 \) - initial state
- \( q_1 \) - the string the DFA read so far starts with 0 and has an odd length
- \( q_2 \) - the string the DFA read so far starts with 0 and has an even length
- \( q_3 \) - the string the DFA read so far starts with 1 and has an odd length
- \( q_4 \) - the string the DFA read so far starts with 1 and has an even length

As it was correctly pointed out, this DFA can be simplified by combining the states \( q_1 \) and \( q_4 \), as well as \( q_2 \) and \( q_3 \):

![Simplified DFA Diagram]

Problem 2

Design a DFA that recognizes the following language:
\[ L = \{ w \mid w \text{ is non-empty and has 1 on every odd position} \} \]

Solution:

\(^1\)In the class I didn’t require non-emptiness, which makes the definition of \( L \) ambiguous.
The meaning of the states:

- $q_0$ - initial state
- $q_1$ - the string the DFA read so far has odd length and has 1 on every odd position
- $q_2$ - the string the DFA read so far has even length and has 1 on every odd position
- $q_3$ - trap state (the string the DFA read so far has 0 on some odd)

**Problem 3**

What is the language recognized by the following DFA:

![DFA Diagram]

**Solution:** $L = \{0^m1^n \mid m \geq 0, n \geq 0\}$.  
Meaning of the states:  
The meaning of the states:  

- $q_0$ - the string the DFA read so far is a sequence of 0’s  
- $q_1$ - the string the DFA read so far is a sequence of 0’s followed by the sequence of 1’s  
- $q_2$ - trap state

Alternative answer: $L_2 = \{w \mid w$ does not contain 10 as a subsequence $\}$. One of the ways to convince yourself that $L_2$ is the language recognized by this DFA, is to construct a DFA for $\overline{L_2} = \{w \mid w$ contains 10 as a subsequence $\}$, and then switch the accepting/rejecting states, just like we did in the class.
Problem 4

What is the language recognized by the following DFA:

Solution: $L = \Sigma^* \{01\}$, that is all binary strings that end with 01. To see this, notice that we get to $q_2$ only if we read a sequence 01. At which point, if no symbols are read we accept, otherwise we continue waiting for 01.