Non-blocking Binary Search Trees

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State of the Art

The Java standard library has several non-blocking data structures, but no search trees.

"You might wonder why this doesn't use some kind of search tree instead . . . . The reason is that there are no known efficient lock-free insertion and deletion algorithms for search trees."

Doug Lea in java.util.concurrent.ConcurrentSkipListMap

Non-Blocking Data Structures

Non-blocking: some operation makes progress.

- Studied for 20+ years
- Universal constructions [1988–present]
  Disadvantage: inefficient
- Array-based structures [1990–2005]
  snapshots, stacks, queues
- List-based structures [1995–2005]
  singly-linked lists, stacks, queues, skip lists
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Prior Work on Concurrent Search Trees

- Many lock-based implementations [1978–present]
- Valois outlined how his linked lists might generalize to BSTs [1995]
  - complicated and lacks detail
- Non-blocking BST [Fraser 2003]
  - uses 8-word CAS
- Bender et al. outlined how their lock-based cache-oblivious B-trees might be made non-blocking [2005]
  - lacks details and proof of correctness

New Result

A non-blocking implementation of BSTs from single-word CAS.

Some properties:

- Conceptually simple
- Fast searches
- Concurrent updates to different parts of tree do not conflict
- Technique seems generalizable
- Experiments show good performance

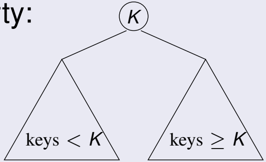
Model

- Asynchronous
- Crash failures allowed
- Shared memory with single-word compare-and-swap
- Linearizable

Leaf-oriented BST

Definition

- One leaf for each key in set
- Internal nodes used only for routing
- Each internal node has exactly 2 children
- BST property:

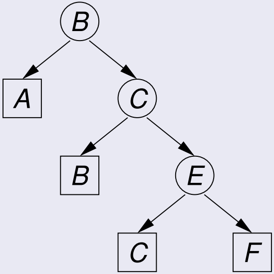


Advantages of Leaf-Oriented Trees

- Deletions much simpler
- Average depth only slightly higher

Example

Leaf-oriented BST storing key set $\{A, B, C, F\}$



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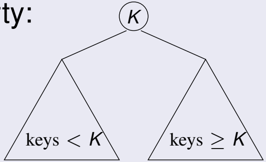

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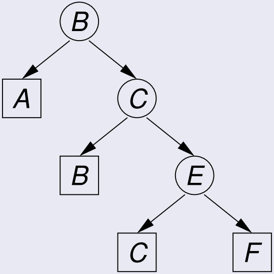

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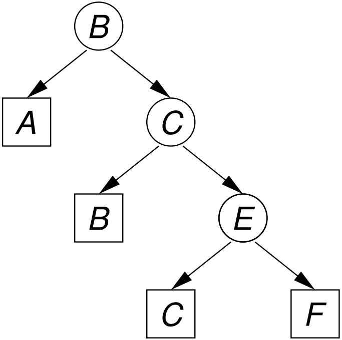

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4. Swing pointer

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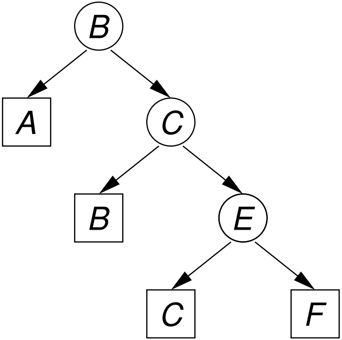

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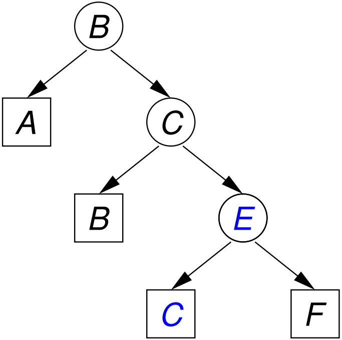

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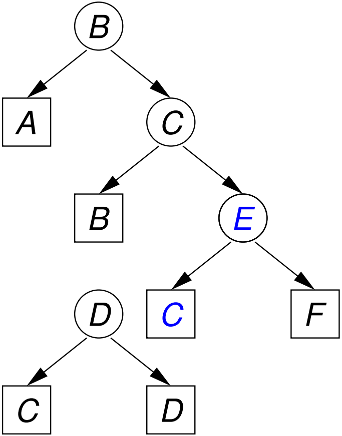

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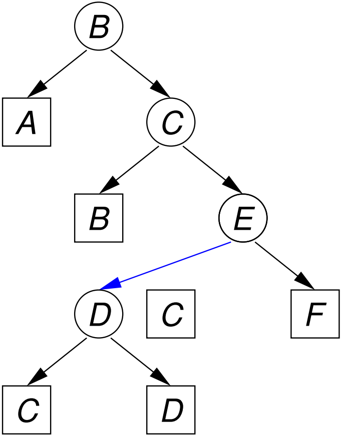

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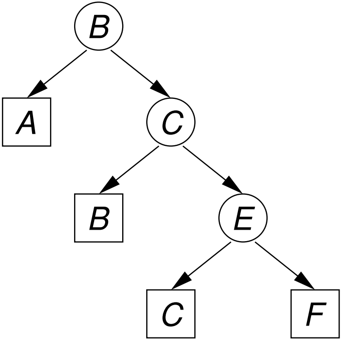

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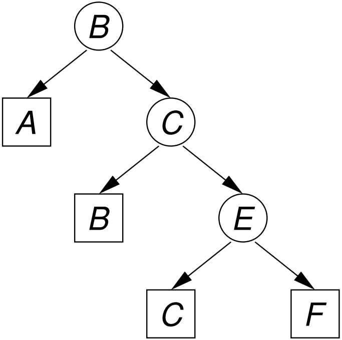

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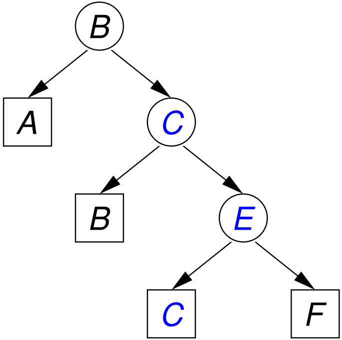

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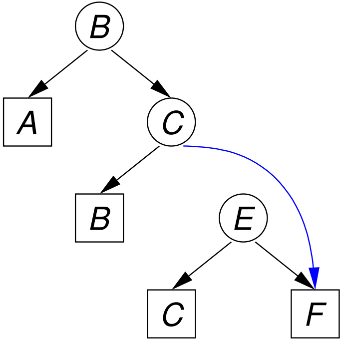

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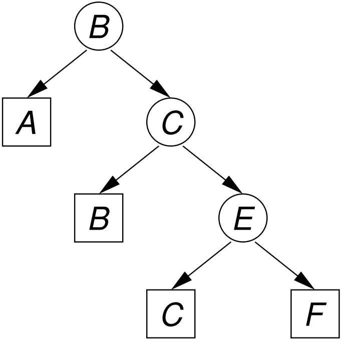

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$\Rightarrow$ C is still reachable from the root!

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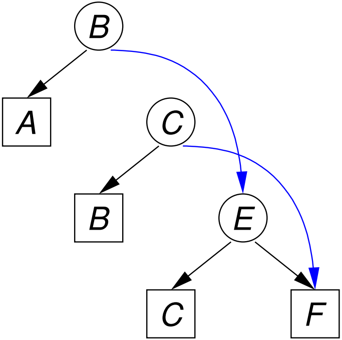

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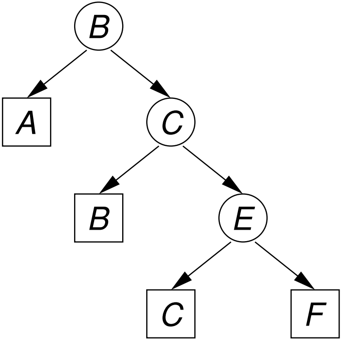

Concurrent Delete(C) and Insert(D).

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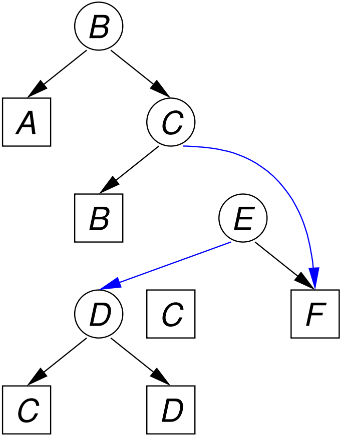

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Coordination Required

Crucial problem: A node's child pointer is changed while the node is being removed from the tree.

Solution: Updates to the same part of the tree must coordinate.

Desirable Properties of Coordination Scheme

- Avoid exclusive-access locks
- Maintain invariant that tree is always a BST
- Allow searches to pass unhindered
- Make updates as local as possible
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Flags and Marks

An internal node can be either flagged or marked (but not both). Status is changed using CAS.

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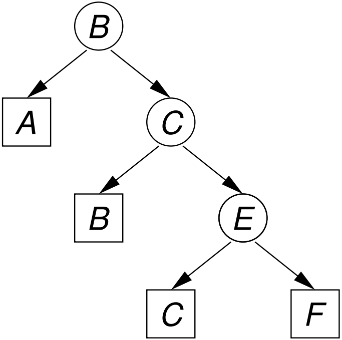

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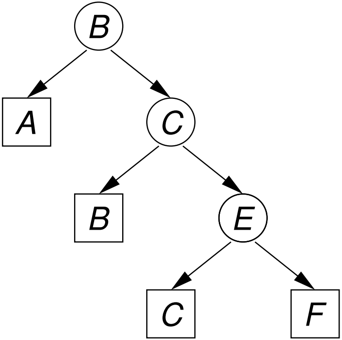

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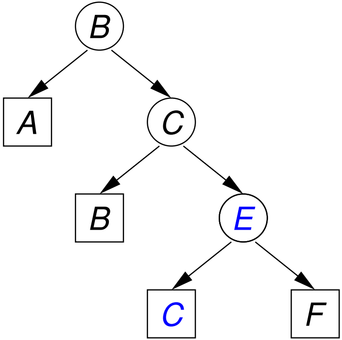

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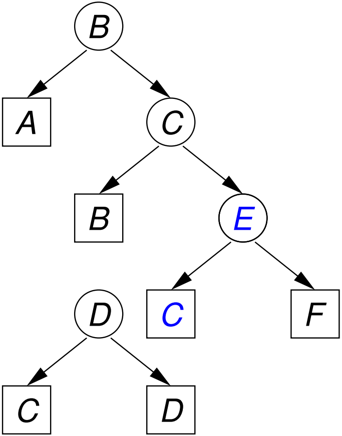

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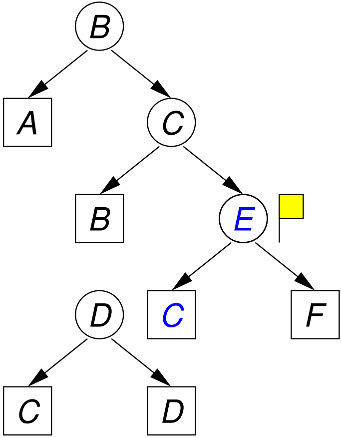

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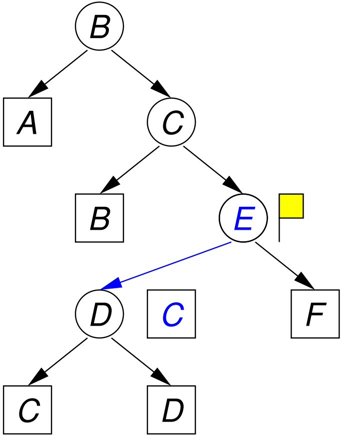

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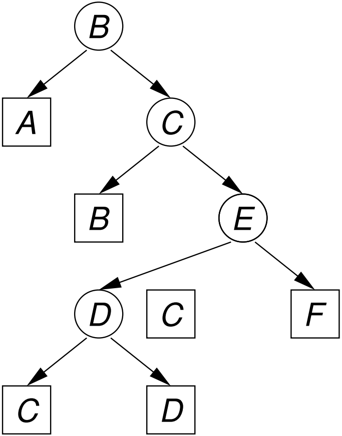

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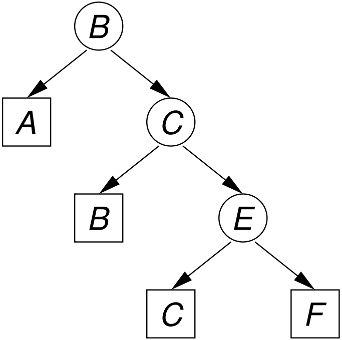

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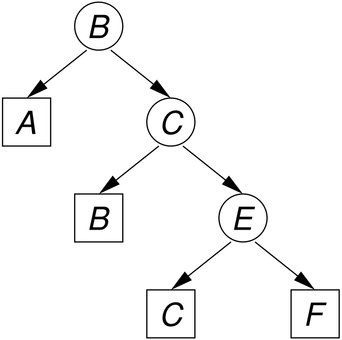

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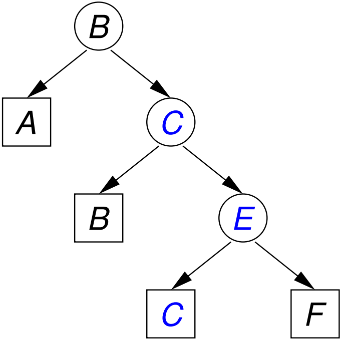

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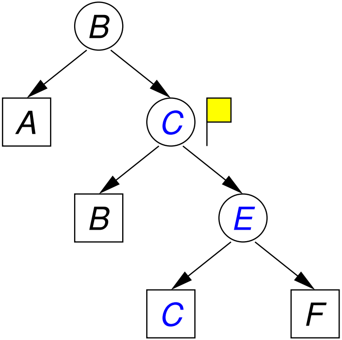

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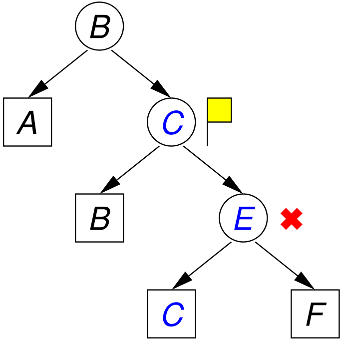

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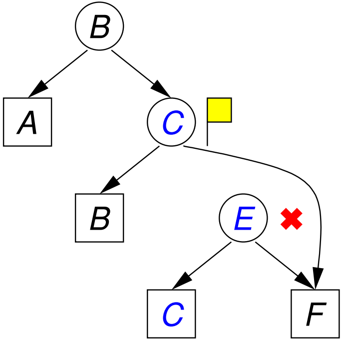

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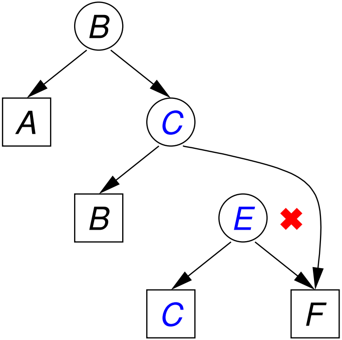

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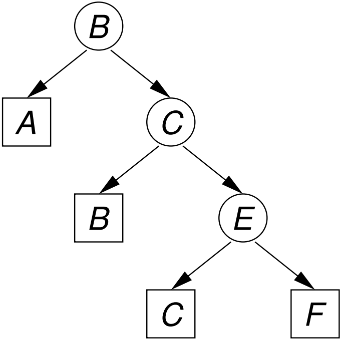

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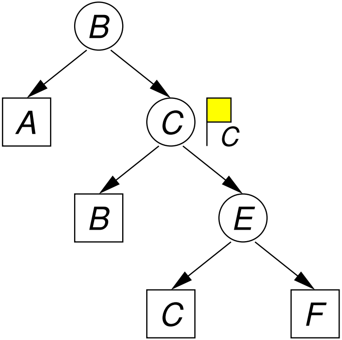

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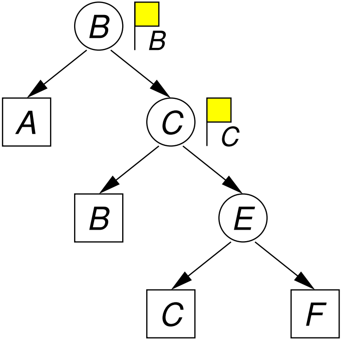

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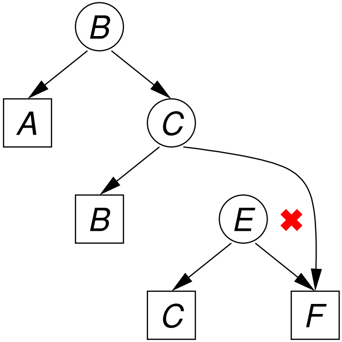

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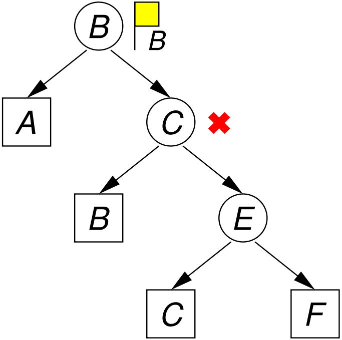

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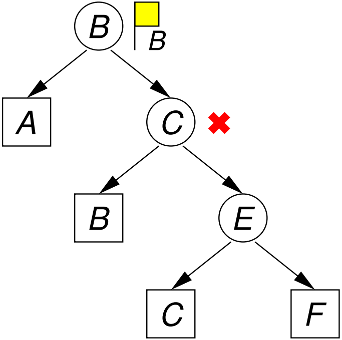

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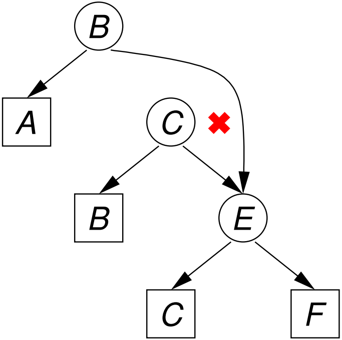

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Wait a second . . .

We want the data structure to be non-blocking!

Whenever "locking" a node, leave a key under the doormat.

A flag or mark is actually a pointer to a small record that tells a process how to help the original operation.

If an operation fails to acquire a lock, it helps complete the update that holds the lock before retrying.

Thus, locks are owned by operations, not processes.

Some similarities to Barnes's cooperative technique.

Wait a second . . .

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Searching

Searches just traverse edges of the BST until reaching a leaf.

They can ignore flags and marks.

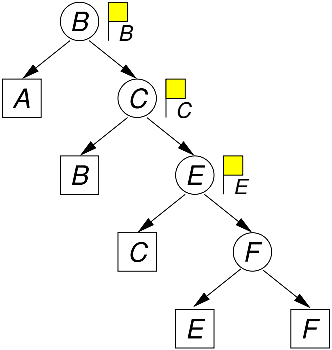
Can prove by induction that each node visited by a Search(K) was on the search path for K at some time during the Search.

Progress

Goal: Show data structure is non-blocking (some operation completes).

- If an Insert successfully flags, it finishes.
- If a Delete successfully flags and marks, it finishes.
- If updates stop happening, searches must finish.

One CAS fails only if another succeeds.
$\Rightarrow$ A successful CAS guarantees progress, except for a Delete's flag.



A Delete may flag, then fail to mark, then unflag to retry.

$\Rightarrow$ The Delete's changes may cause other CAS's to fail.

However, lowest Delete will make progress.

Some Details Omitted

The formal proof of correctness is surprisingly difficult (20 pages long).

See the Technical Report.

Further Work

- Balancing the tree
- Proving worst-case complexity bounds
- Can same approach yield (efficient) wait-free BSTs?
  (Or at least wait-free Finds?)
- Other data structures