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Depth Image Coding & Processing

Part 2: Depth Image Coding

Outline

- Depth Image Coding
 - Graph Fourier Transform
 - Multi-resolution Graph Fourier Transform
 - Generalized Graph Fourier Transform
- Graph based Representation (GBR)

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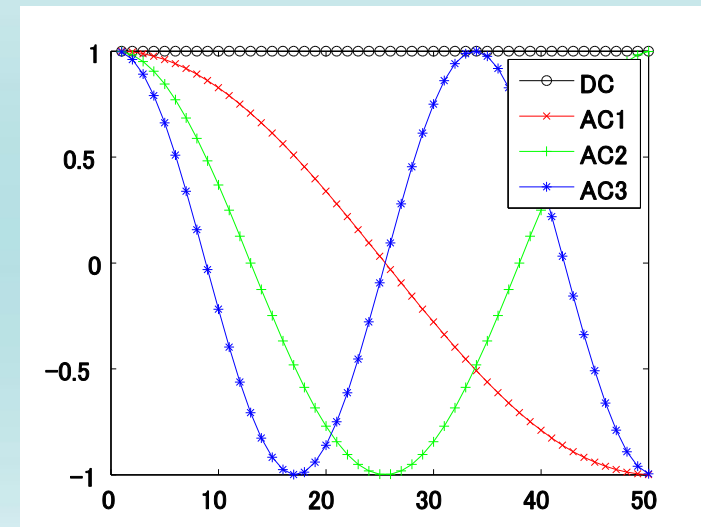
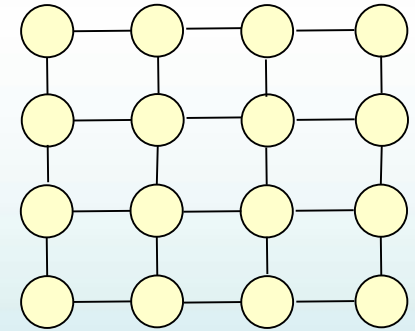
Graph Fourier Transform (GFT) for Graph-signals

Graph Fourier Transform:

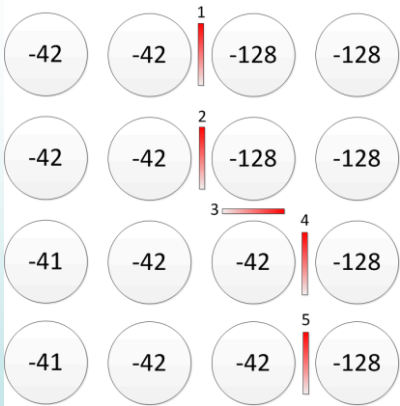
- *Signal-adaptive* transform:
 1. If two connected pixels are “similar”, then edge weight is large \rightarrow adjacency matrix A .
 2. Compute **graph Laplacian** $L = D - A$.
 3. Perform eigen-decomposition on L for GFT.

$$x = \sum_i a_i \varphi_i$$

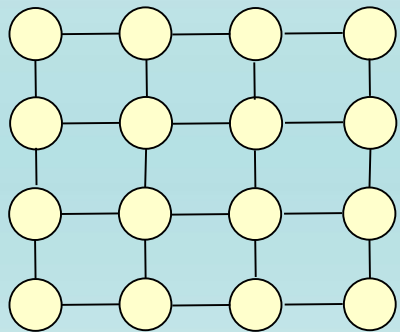
- **Intuition:** Embed geometric structure of signal as edge weights in graph.



Depth Map Compression



- DCT are **fixed** basis. Can we do better?
- **Idea**: use **adaptive** GFT to improve sparsity [1].
 1. Assign edge weight 1 to adjacent pixel pairs.
 2. Assign edge weight 0 to sharp depth discontinuity.
 3. Compute GFT for transform coding, transmit coeff.



$$\alpha = \Psi \mathbf{x}$$

← GFT

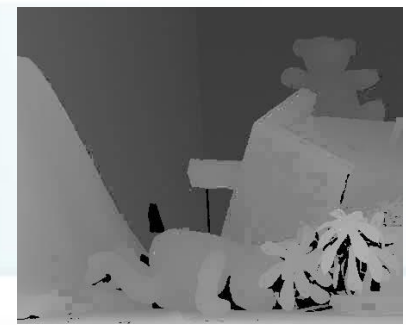
4. Transmit bits (**contour**) to identify chosen GFT to decoder (**overhead of GFT**).

Shape-adaptive wavelets can also be done.

[1] G. Shen et al., “Edge-adaptive Transforms for Efficient Depth Map Coding,” *IEEE Picture Coding Symposium*, Nagoya, Japan, December 2010.

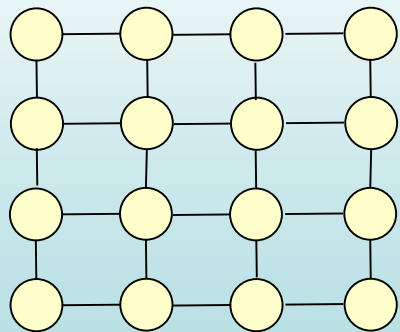
[2] M. Maitre et al., “Depth and depth-color Coding using Shape-adaptive Wavelets,” *Journal of Visual Communication and Image Representation*, vol.21, July 2010, pp.513-522.

Depth Map Compression



Q: Why GFT leads to sparseness?

Ans 1: Capture statistical structure of signal in edge weights of graph.



- Adjacent pixel correlation 0 or 1 for piecewise smooth (PWS) signal.
- Can be shown GFT approximates KLT given *Gaussian Random Markov Field (GRMF)* model.

Ans 2: Avoid filtering across sharp edges.



a 4x4 block



GFT $\alpha_1 = \begin{bmatrix} 237 & 0 & 0 & 0 \\ 163 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

DCT

$$\alpha_2 = \begin{bmatrix} 285 & -29 & -5 & -4 \\ 16 & 1 & -16 & -4 \\ -5 & 3 & 5 & -7 \\ -1 & -4 & 1 & 9 \end{bmatrix}$$

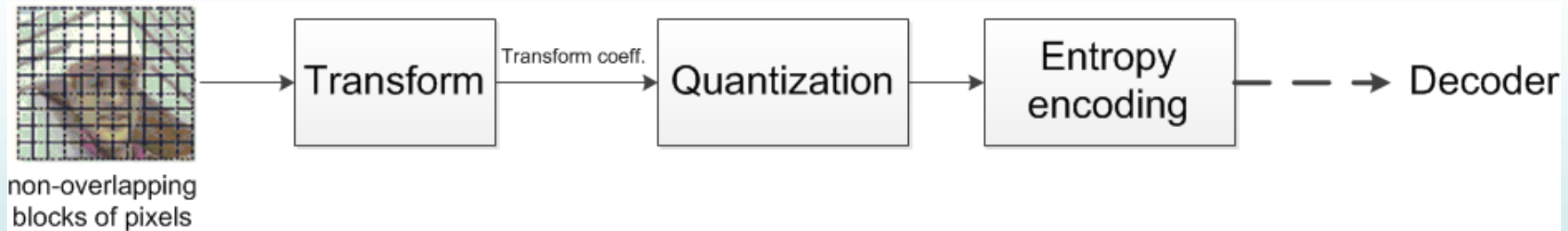
filtering operation



$$\alpha = \Psi X$$

Graph Fourier Transform (GFT) for Image Coding

- Block-based Transform coding of images*



Two things to transmit for **adaptive transforms**:

- transform coefficients → the cost of transform representation
- adaptive transform itself → the cost of transform description

- What's a *good* transform?

- minimize the cost of **transform representation** & the cost of **transform description**

Transform Comparison

	Transform Representation	Transform Description
Karhunen-Loeve Transform (KLT)	“Sparsest” signal representation given available data / statistical model	Can be expensive (if unstructured)
Discrete Cosine Transform (DCT)	<i>non-sparse signal representation</i> across sharp boundaries	little (fixed transform)
Graph Fourier Transform (GFT)	minimizes the total rate of signal’s transform representation & transform description	

GFT Comparison

HR-UGFT [1]	MR-GFT
unweighted graphs	unweighted & weighted graphs
no notion of optimality	define an optimality criterion
graphs are directly drawn from detected boundaries	propose efficient algorithms to search for the optimal GFT
Requires real-time eigen-decomposition	3 techniques to reduce computation complexity (multi-resolution, graph isomorphism, table lookup)

Search for Optimal GFT

- Rate-distortion performance: $D + \lambda R$

- **Assumption:** high bit rate, uniform quantization

Distortion does not change when considering different transforms! [5]

Consider Rate only!

- For a given image block $\mathbf{x} \in \mathbb{R}^N$ under fixed uniform quantization at high rate, the **optimal** GFT is the one that minimizes the total rate:

$$\min_{\mathbf{W}} R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W})$$

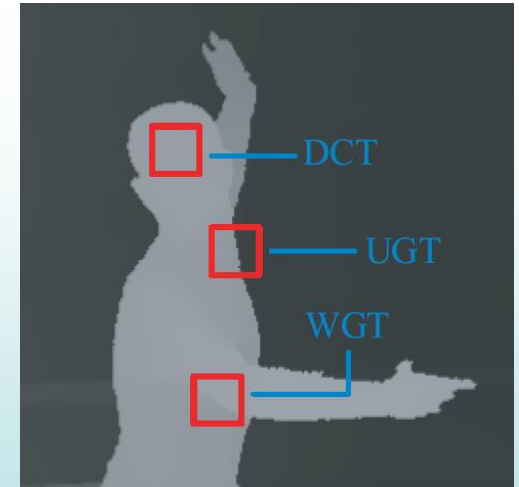
Rate of transform coefficient vector α

Rate of transform description T

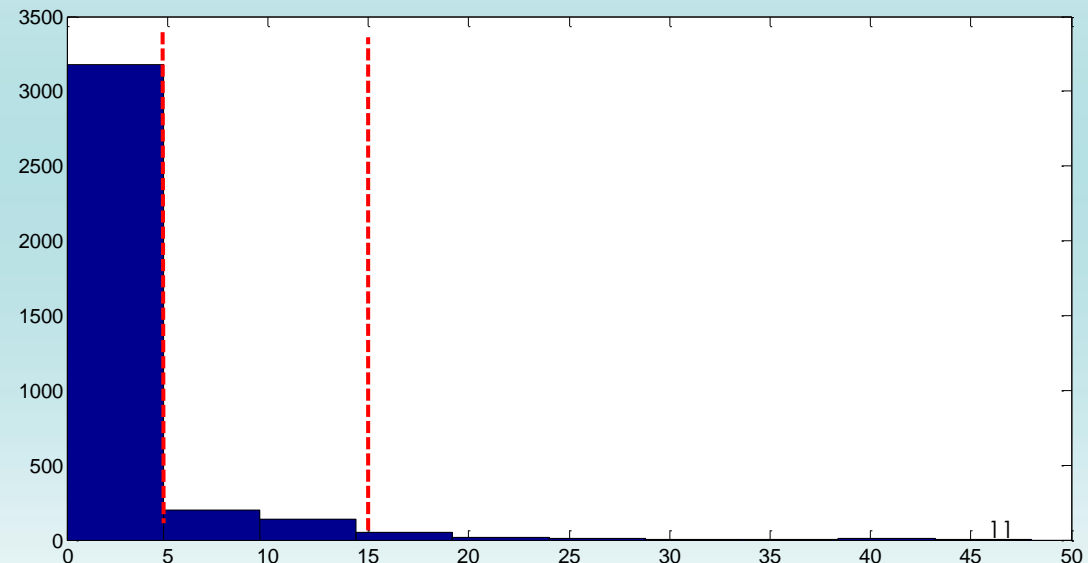
MR-GFT: Definition of the Search Space for Graph Fourier Transforms

$$\min_{\mathbf{W}} R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W})$$

- In general, weights could be any number in $[0,1]$
- **To limit the description cost** R_T
 - Restrict weights to a small discrete set $\mathcal{C} = \{1, 0, c\}$



- "1": *strong correlation* in smooth regions
- "0": *zero correlation* in sharp boundaries
- "c": *weak correlation* in slowly-varying parts



MR-GFT: Definition of the Search Space for Graph Fourier Transforms

$$\begin{aligned} \min_{\mathbf{W}} \quad & R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W}) \\ \text{s.t.} \quad & W_{i,j} \in \{1, 0, c\} \quad \forall i, j \in \mathcal{V} \end{aligned}$$

- For ease of computation, divide the optimization into two sub-problems

1. **Weighted GFT** (WGFT): $\mathcal{C}_1 = \{1, c\}$

2. **Unweighted GFT** (UGFT): $\mathcal{C}_2 = \{1, 0\}$

Strong correlation only? Default to the DCT

◆ **What is the optimal c ?**

MR-GFT: Derivation of Optimal Edge Weights for Weak Correlation

- Assume a 1D first-order *autoregressive (AR) process* $\mathbf{x} = [x_1, \dots, x_N]^T$ where,

$$x_k = \begin{cases} \eta, & k = 1 \\ x_{k-1} + e_k, & 1 < k \leq N, [k-1, k] \in \mathcal{S} \leftarrow \text{smooth} \\ x_{k-1} + g + e_k, & 1 < k \leq N, [k-1, k] \in \mathcal{P} \leftarrow \text{jump} \end{cases}$$

non-zero mean random var.

- Assuming the only weak correlation exists between x_{k-1} and x_k

$$\mathbf{F}\mathbf{x} = \mathbf{b},$$

$$\begin{array}{l} x_1 = \eta \\ x_2 - x_1 = e_2 \\ \dots \\ x_k - x_{k-1} = g + e_k \\ \dots \\ x_N - x_{N-1} = e_N \end{array} \quad \Rightarrow \quad \mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ & \ddots & \ddots & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ e_2 \\ \vdots \\ e_k \\ \vdots \\ e_N \end{bmatrix} + \begin{bmatrix} \eta \\ 0 \\ \vdots \\ g \\ \vdots \\ 0 \end{bmatrix}$$

$$\mu = [0 \quad \dots \quad 0 \quad m_g \quad \dots \quad m_g]^T$$

|
k-th

$$\mathbf{x} = \mathbf{F}^{-1}\mathbf{b}$$

MR-GFT: Derivation of Optimal Edge Weights for Weak Correlation (cont'd)

- Covariance matrix

$$\begin{aligned}
 \mathbf{C} &= E[(\mathbf{x} - \mu)(\mathbf{x} - \mu)^T] \\
 &= E[\mathbf{xx}^T] - \mu\mu^T \\
 &= E[\mathbf{F}^{-1}\mathbf{bb}^T(\mathbf{F}^T)^{-1}] - \mu\mu^T \\
 &= \mathbf{F}^{-1}E[\mathbf{bb}^T](\mathbf{F}^T)^{-1} - \mu\mu^T
 \end{aligned}$$

$$\mathbf{x} = \mathbf{F}^{-1}\mathbf{b}$$

$$\mu = \left[0 \quad \cdots \quad 0 \quad \overset{\text{k-th}}{\underset{|}{m_g}} \quad \cdots \quad m_g \right]^T$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ e_2 \\ \vdots \\ e_k \\ \vdots \\ e_N \end{bmatrix} + \begin{bmatrix} \eta \\ 0 \\ \vdots \\ g \\ \vdots \\ 0 \end{bmatrix}$$

$$E[\mathbf{bb}^T] = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & \cdots & 0 \\ & & \ddots & & & \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \sigma_g^2 + m_g^2 + 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ & & & & & & \ddots & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \quad \text{--- k-th row}$$

MR-GFT: Derivation of Optimal Edge Weights for Weak Correlation (cont'd)

- Precision matrix (**tri-diagonal**)

$$\mathbf{Q} = \mathbf{C}^{-1} = \begin{bmatrix} 1 + \frac{1}{\sigma_g^2} & -1 & & & & \\ -1 & 2 & -1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & -1 & 2 & -1 & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 & -1 \\ & & & & & -1 & 2 & -1 \\ & & & & & & \ddots & \ddots & \ddots \\ & & & & & & & -1 & 2 & -1 \\ & & & & & & & & -1 & 1 \end{bmatrix}$$

The matrix is tri-diagonal. The central elements are circled in red, indicating the weights between nodes $k-1$ and k . An arrow labeled $W_{k-1,k}$ points to the top-right circled element, and an arrow labeled $W_{k,k-1}$ points to the bottom-left circled element. Blue lines on the right indicate the $(k-1)$ -th row and k -th row.

Graph Laplacian matrix! $\approx \mathcal{L}$

- the KLT basis = $\{\psi_l\}$ of the covariance matrix = $\{\psi_l\}$ of the precision matrix $\simeq \{\psi'_l\}$ of the Laplacian matrix

$$c = W_{k-1,k} = \frac{1}{\sigma_g^2 + 1}$$

MR-GFT: Adaptive Selection of Graph Fourier Transforms

$$\begin{aligned} \min_{\mathbf{W}} \quad & R_\alpha(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W}) \\ \text{s.t.} \quad & W_{i,j} \in \{1, 0, c\} \quad \forall i, j \in \mathcal{V} \end{aligned}$$

- Two sub-problems with two corresponding non-overlapping search spaces

1. **Weighted GFT** (WGFT): $\mathcal{C}_1 = \{1, c\}$ (weighted & connected graphs)

2. **Unweighted GFT** (UGFT): $\mathcal{C}_2 = \{1, 0\}$ (unweighted & disconnected graphs)

WGFT

$$\begin{aligned} \min_{\mathbf{W}} \quad & R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W}) \\ \text{s.t.} \quad & W_{i,j} \in \{1, c\} \quad \forall i, j \in \mathcal{V} \end{aligned}$$

- Cost function of transform coefficients

$$\hat{R}_{\alpha}(\mathbf{x}, \mathbf{W}) \approx \mathbf{x}^T \mathbf{L} \mathbf{x} = \sum_{(i,j) \in E} W_{i,j} (x_i - x_j)^2 = \sum_k \lambda_k \alpha_k^2$$

↙ GFT coeff
↖ graph freq.

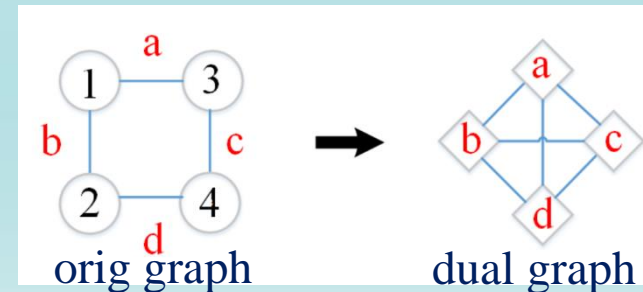
- Cost function of transform description

$$\hat{R}_T(\mathbf{W}) = \sum_{(e,s) \in \mathcal{E}^d} |W_e - W_s| + \sum_{e \in \mathcal{V}^d} \gamma \rho(1 - W_e)$$

↖ costly if many weight changes
↖ code only non-1's

- Problem formulation for WGFT

$$\begin{aligned} \min_{\mathbf{W}} \quad & \rho \sum_{e \in \mathcal{V}^d} [W_e (x_{v_1(e)} - x_{v_2(e)})^2 + \gamma(1 - W_e)] + \sum_{(e,s) \in \mathcal{E}^d} |W_e - W_s| \\ \text{s.t.} \quad & W_e \in \{1, c\} \quad \forall e \in \mathcal{V}^d. \end{aligned}$$

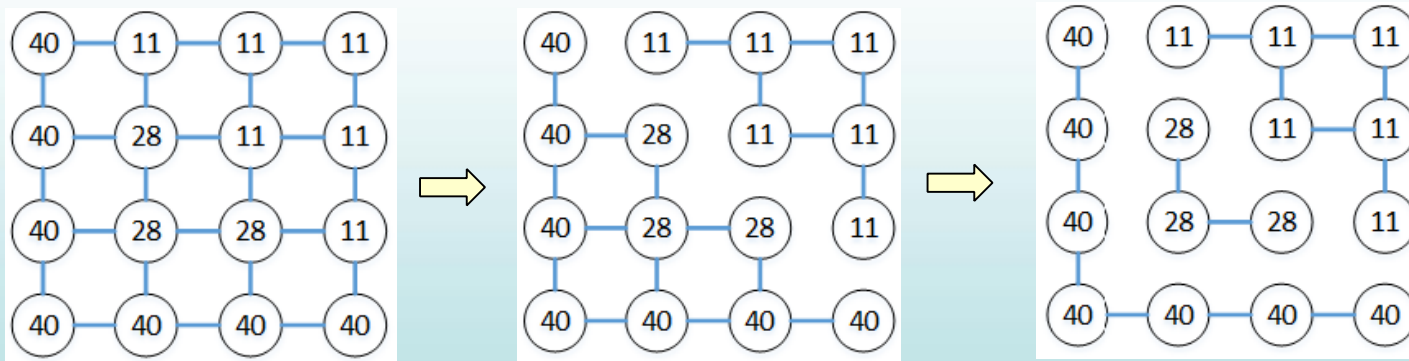


- Separation-Deviation** (SD) problem, solvable in polynomial time [6].

UGFT

$$\begin{aligned} \min_{\mathbf{W}} \quad & R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W}) \\ \text{s.t.} \quad & W_{i,j} \in \{1, 0\} \quad \forall i, j \in \mathcal{V} \end{aligned}$$

- A greedy algorithm



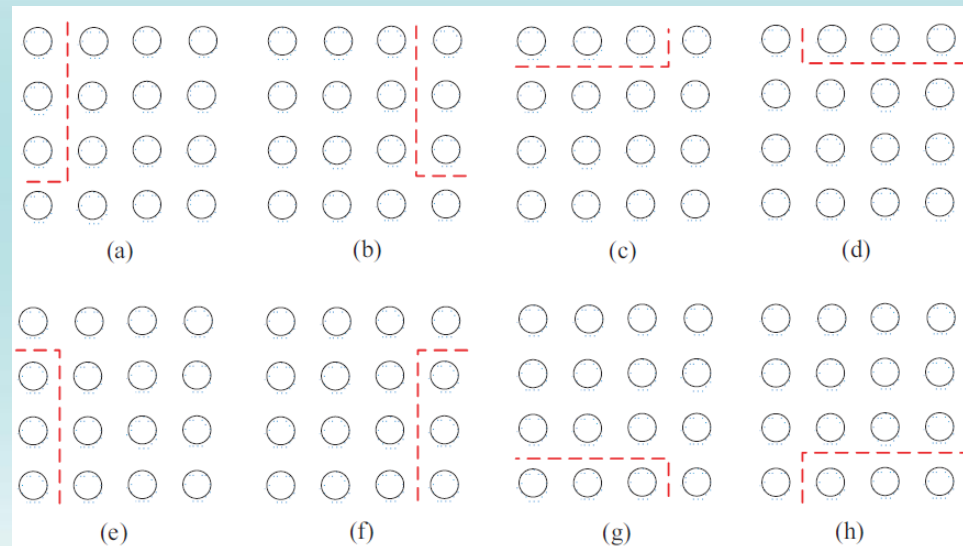
- Divide graph into disconnected sub-graphs via spectral clustering [7].
- Check objective function, further sub-divide if cost decreases.

MR-GFT: Adaptive Selection of Graph Fourier Transforms

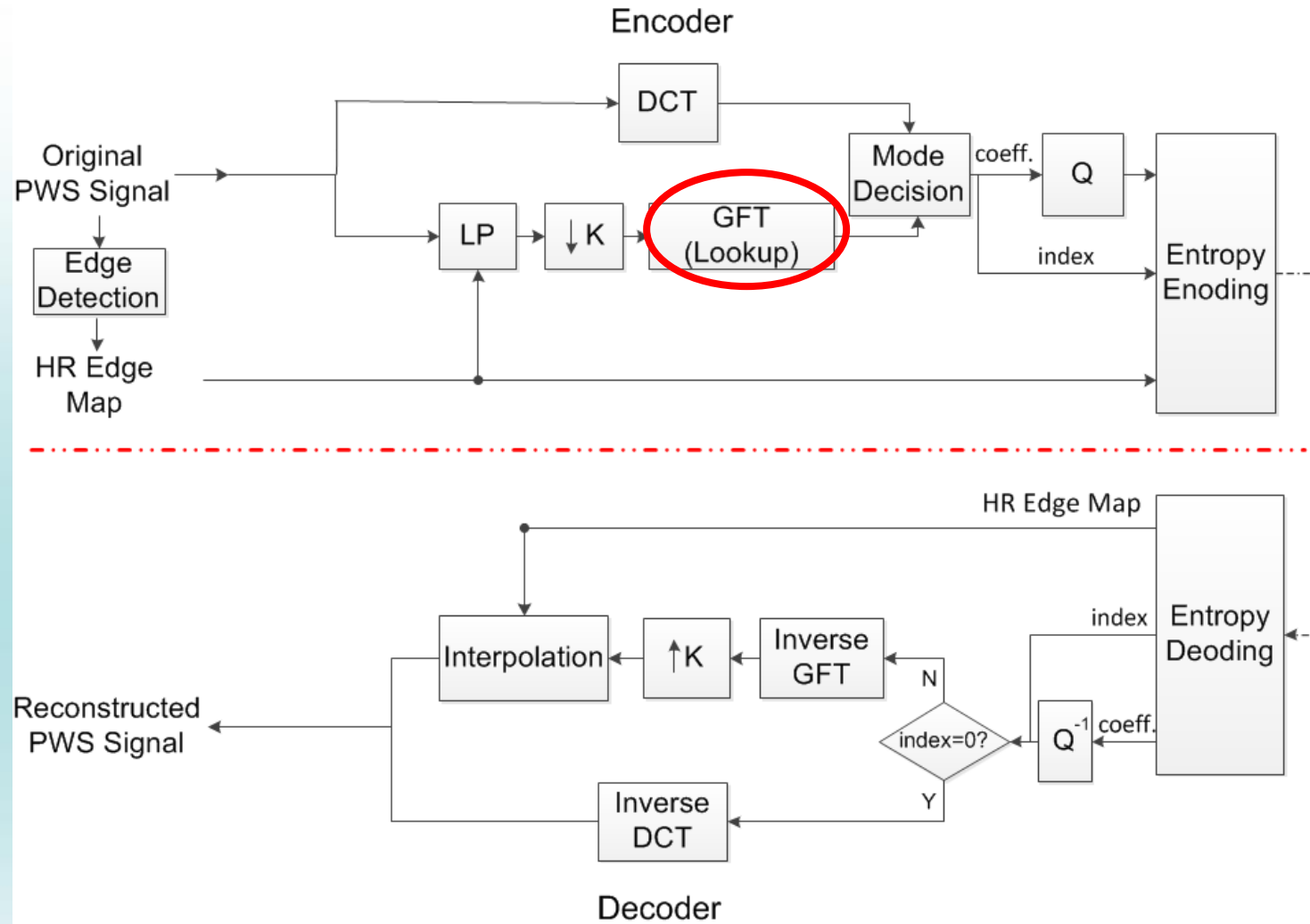
- Online eigen-decomposition: a hurdle to practical implementation
- Pre-compute and store GFTs in a table for simple lookups
 - Perform GFT on a **small block**
 - Store the **most frequently used** GFTs
 - Exploit **graph isomorphism**

$$f : \mathcal{V}(\mathcal{G}) \rightarrow \mathcal{V}(\mathcal{H})$$

- Graph isomorphism



MR-GFT: Adaptive Selection of Graph Fourier Transforms

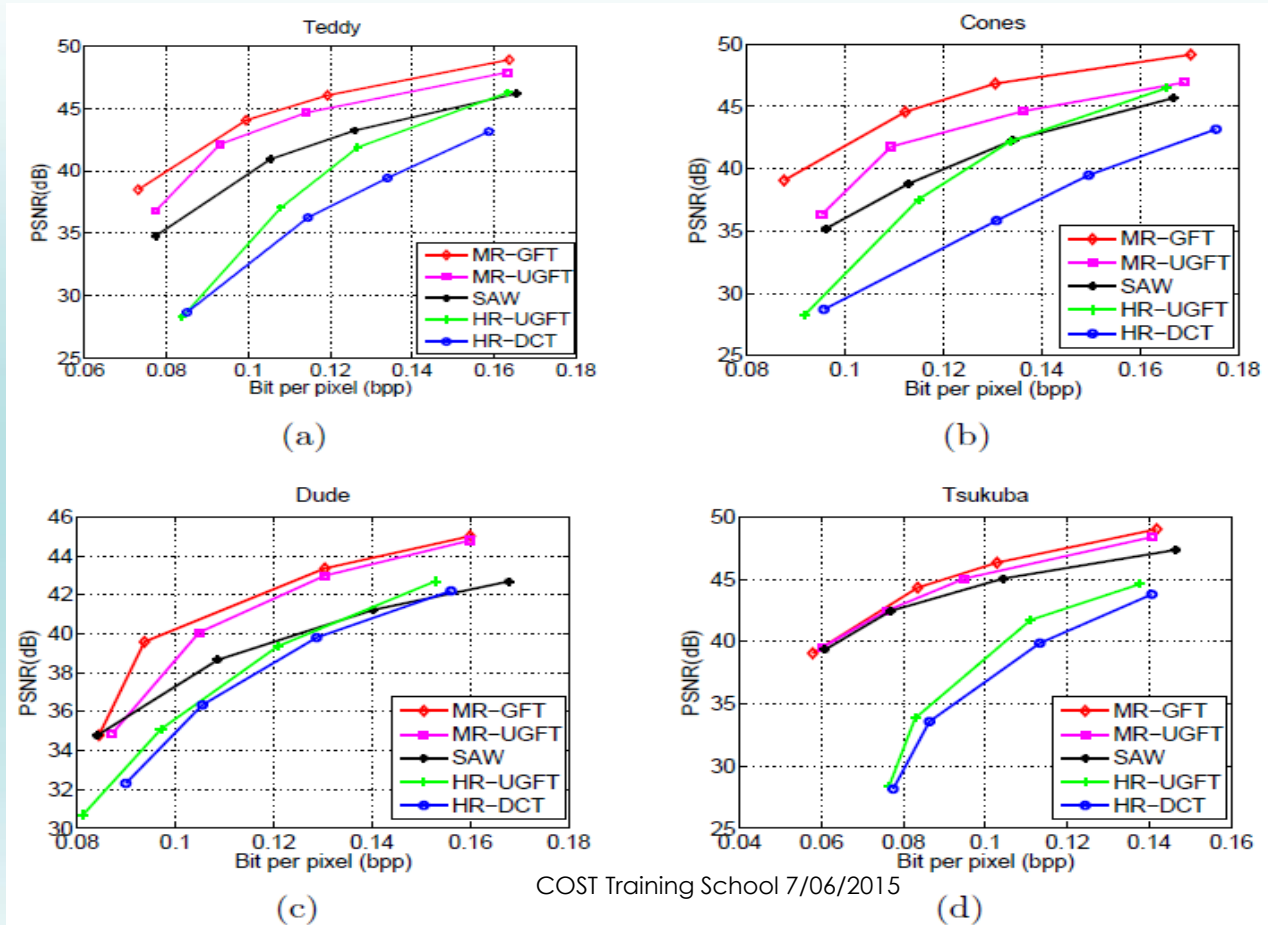


Experimentation

- Setup

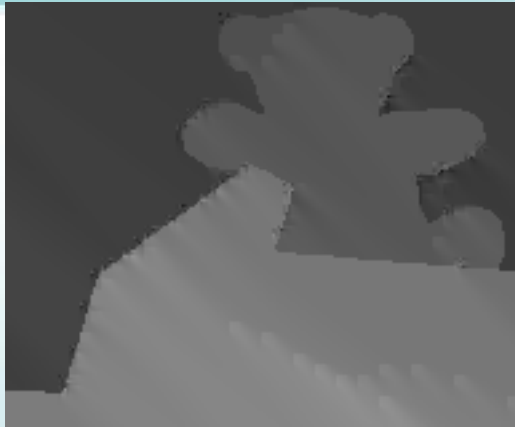
- Test images: depth maps of *Teddy* and *Cones*, and graphics images of *Dude* and *Tsukuba*.
- Compare against: HR-DCT, HR-SGFT, SAW, MR-SGFT

- Results



HR-DCT: 6.8dB
 HR-SGFT: 5.9dB
 SAW: 2.5dB
 MR-SGFT: 1.2dB

Subjective Results



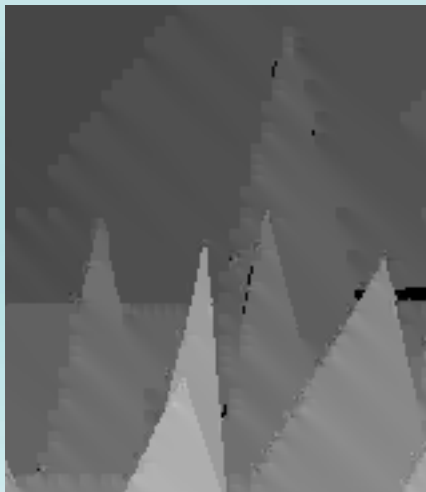
HR-DCT



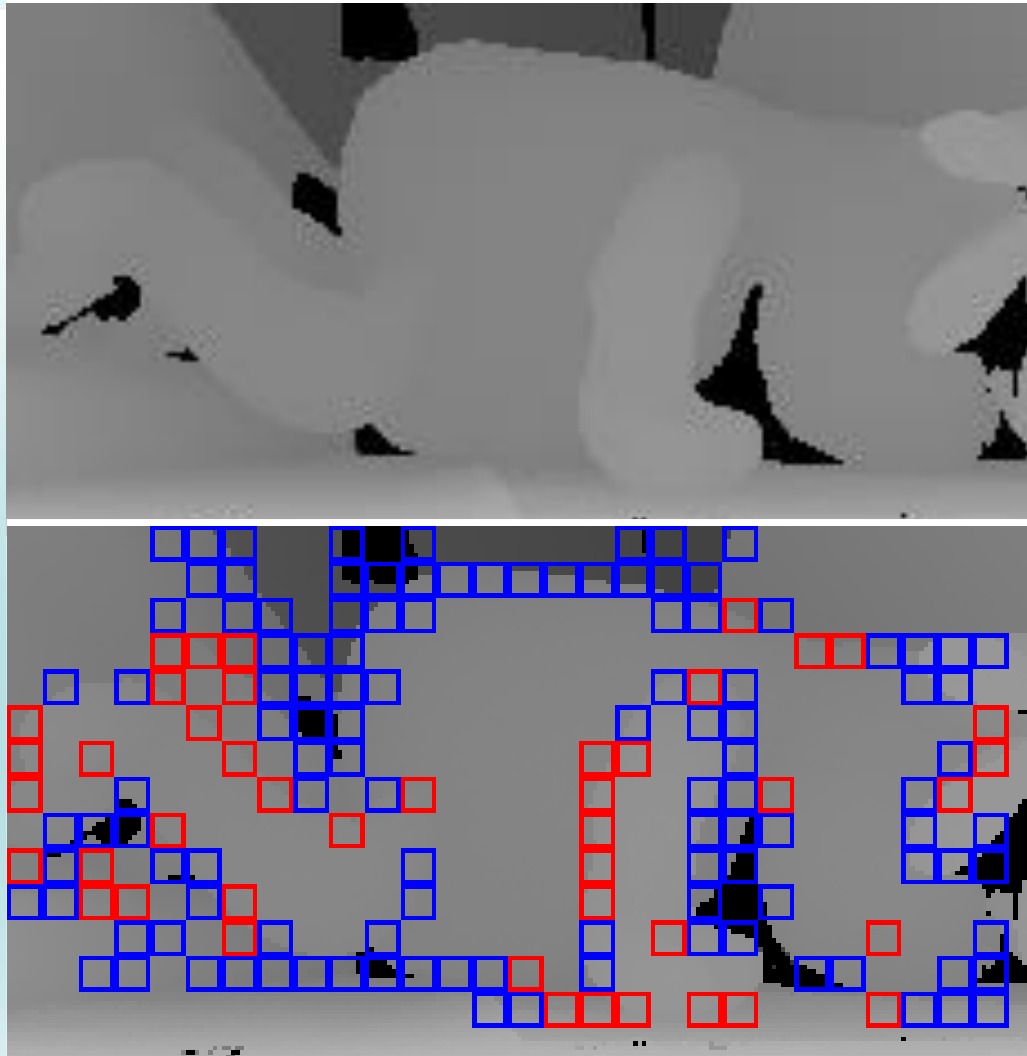
HR-SGFT



MR-GFT



Mode Selection



red: WGFT
blue: UGFT

Summary for Multi-resolution Graph Fourier Transform

- A *multi-resolution (MR) graph Fourier transform (GFT)* coding scheme for compression of piecewise smooth images.
- Minimize transform representation cost + transform description cost given weight $\{0, 1, c\}$.
- Solve for optimal c , show optimality of GFT.
- **WGFT** $\{1, c\}$: formulate a separation-deviation (SD) problem.
- **UGFT** $\{1, 0\}$: greedy algorithm via spectral clustering.
- Practical implementation via **multi-resolution**, **graph isomorphism** and **lookup tables**.
- Excellent experimental results!

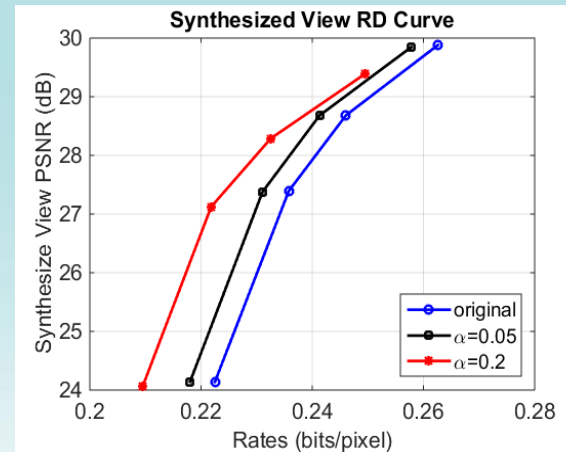
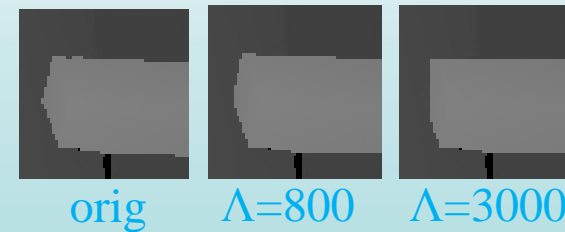
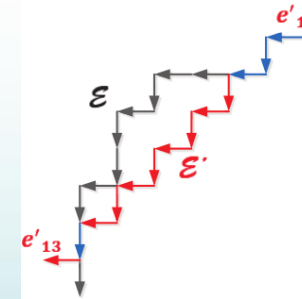
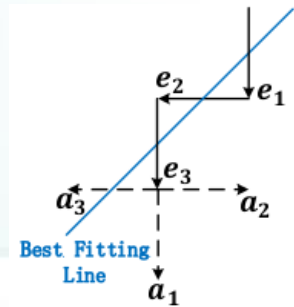
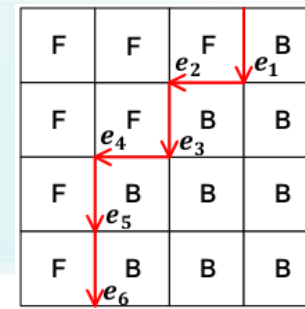
Edge Coding for PWS Image Compression

- **Arithmetic Edge Coding** [8]:

- Coding of sequence of *between-pixel edges*, or chain code with symbols {L, S, R}.
- Design a *context* to compute symbol probabilities for arithmetic coding.
- **Extension:** better context based purely on symbol statistics analysis.

- **Contour Approximation & Depth Image Coding** [9]:

- Approximate contour while maintaining edge sharpness.
- Edge-adaptive blocked-based image coding (GFT).
- Average 1.68dB over GFT coding original contours at low-bitrate regions.



[8] I. Daribo, G. Cheung, D. Florencio, "Arbitrarily Shaped Sub-block Motion Prediction in Depth Video Compression using Arithmetic Edge Coding," *IEEE Trans on Image Processing*, Nov 2014.

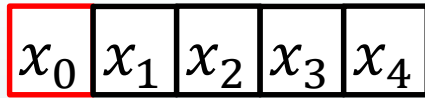
[9] Y. Yuan G Cheung, P. Frossard, P. Le Callet, V. Zhao, "Contour Approximation & Depth Image Coding for Virtual View Synthesis," submitted to *IEEE MMSP*, October, 2015.

Outline

- Depth Image Coding
 - Graph Fourier Transform
 - Multi-resolution Graph Fourier Transform
 - Generalized Graph Fourier Transform
- Graph based Representation (GBR)

Motivation

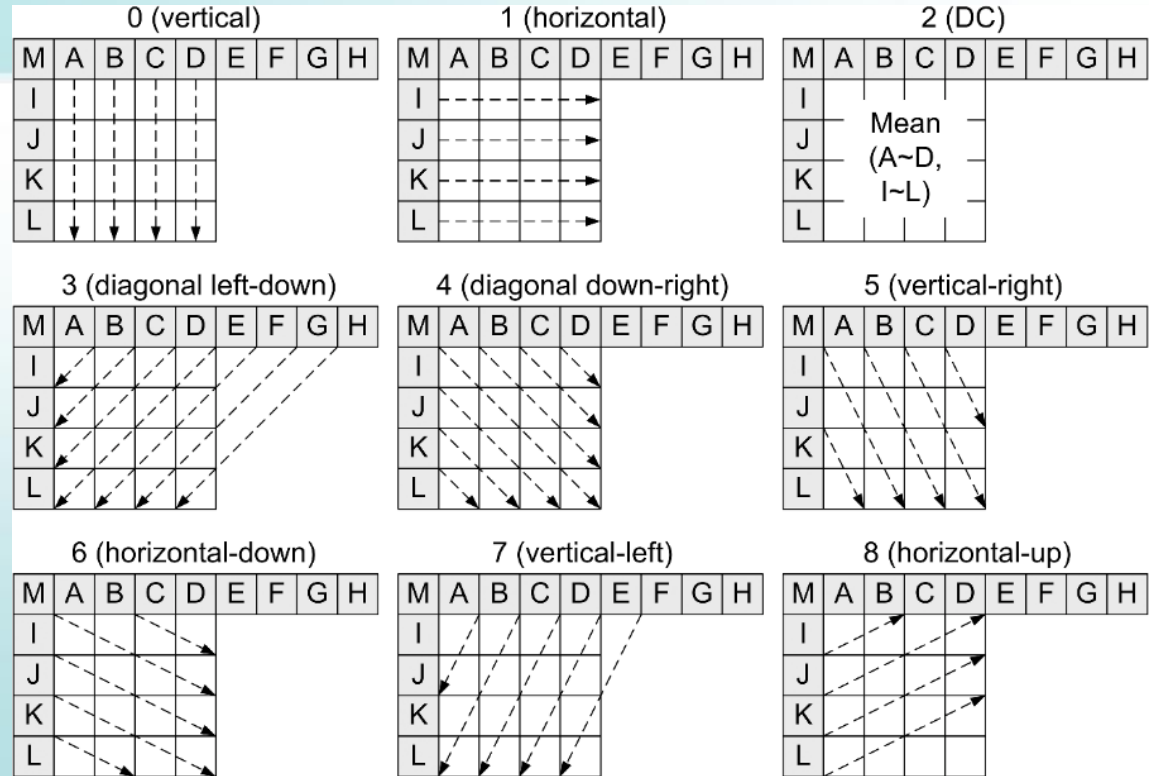
- Intra-prediction



Boundary pixel
(predictor)

Predicted pixels x_i

$x_i - x_0$: prediction residuals



Intra-prediction in H.264

- Discontinuities at block boundaries
 - intra-prediction will not be chosen or bad prediction

Contributions

- *Clustered-based* intra-prediction
 - **cluster** discontinuities at block boundaries
 - $\mu + x_0$: shift by cluster mean μ (side information)
- *Generalized Graph Fourier Transform (GGFT)*
 - **optimized** for intra-prediction residuals
 - generalized graph Laplacian: extra weight added at block boundaries
 - default to the DCT and ADST in some cases

Related Work

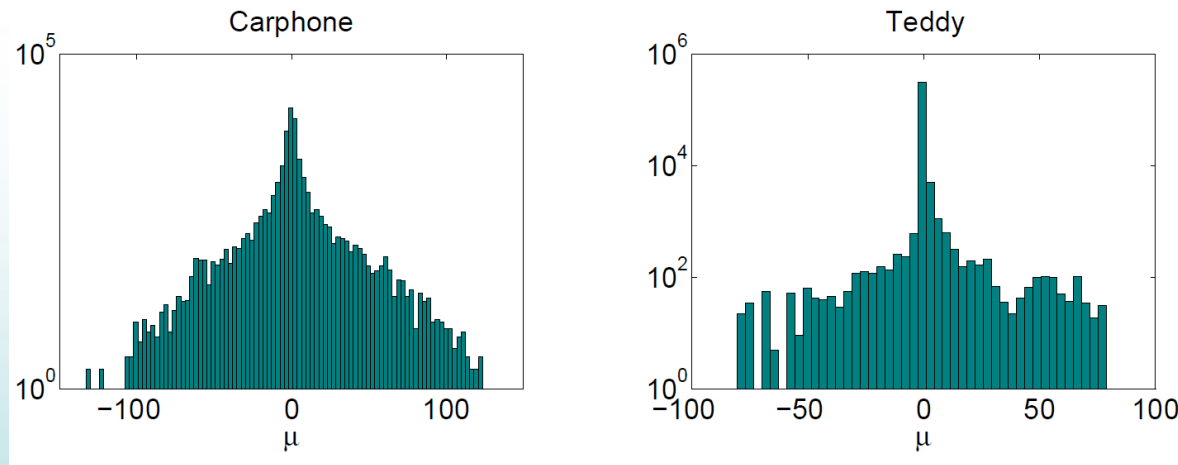
- Zhang et al, graph-based predictive transform coding, GMRF [3]
 - Assume given model. No discussion on how to derive model parameters.
- Wang et al, intra predictive graph transform coding [11]
 - Intra-prediction plus KLT, optional graph sparsification.
- Ye et al, MDDT [12]
 - Completely data-driven resulting in unstructured transform.

[3] C. Zhang and D. Florencio, "Analyzing the optimality of predictive transform coding using graph-based models," in *IEEE Signal Processing Letters*, vol. 20, NO. 1, January 2013, pp. 106–109.

[11] Y. Wang, A. Ortega, and G. Cheung, "Intra predictive transform coding based on predictive graph transform," *ICIP*, September 2013.

[12] Y. Ye and M. Karczewicz, "Improved H.264 intra coding based on bidirectional intra prediction, directional transform, and adaptive coefficient scanning," *ICIP*, October 2008.

1D Signal Modeling



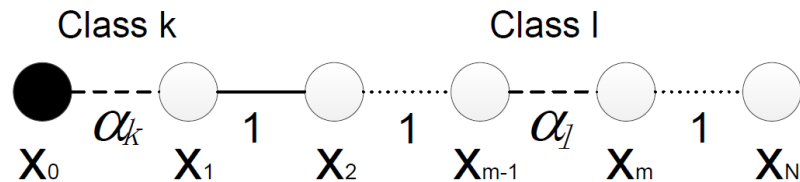
- Inter-pixel differences are concentrated around 0, occasionally large.
- Quantize inter-pixel differences into K **bins**, leveraging on *Lloyd algorithm*

$$x_n = x_{n-1} + \hat{\mu}_{i(\mu_n)} + g_{i(\mu_n)}$$

bin average

approximation error

Optimal 1D Intra prediction



$$\begin{bmatrix} x_0 + \hat{\mu}_a \\ \vdots \\ x_0 + \hat{\mu}_a \\ x_0 + \hat{\mu}_a + \hat{\mu}_b \\ \vdots \\ x_0 + \hat{\mu}_a + \hat{\mu}_b \end{bmatrix}$$

→ Class k

→ Class l

- **Optimal** in terms of resulting in a *zero-mean* prediction residual
- Default to conventional intra-prediction when $\hat{\mu}_a = \hat{\mu}_b = 0$, i.e.,

$$[x_0, \dots, x_0]^T$$

Generalized Graph Fourier Transform

- The precision matrix of the prediction residual

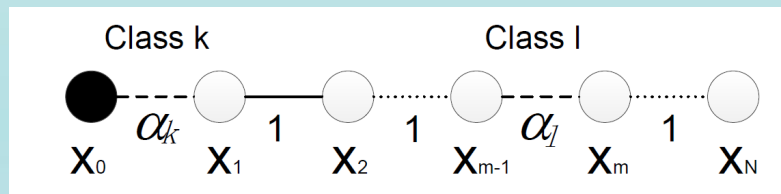
$$\begin{bmatrix} \alpha_a + 1 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & -1 & 1 + \alpha_b & -\alpha_b & \\ & & & -\alpha_b & \alpha_b + 1 & -1 \\ & & & & \ddots & \ddots & \ddots \\ & & & & & -1 & 2 & -1 \\ & & & & & & -1 & 1 \end{bmatrix}$$

$$\alpha_a = \sigma_{g_0}^2 / \sigma_{g_a}^2 \quad (\text{inaccuracy of intra-prediction})$$

$$\alpha_b = \sigma_{g_0}^2 / \sigma_{g_b}^2 \quad (\text{discontinuities within signal})$$

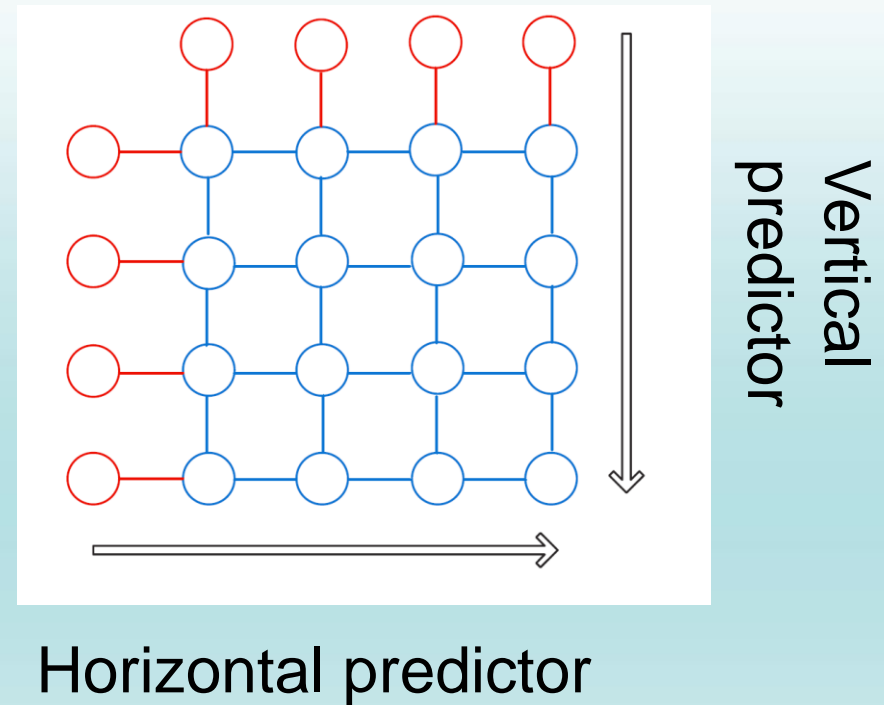


 Variance of approximation error



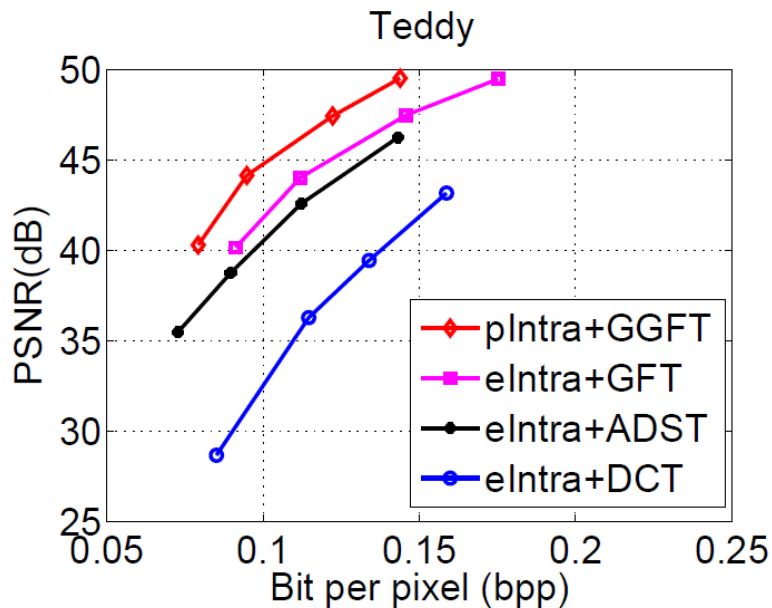
Proposed Coding System

- Four clusters:
 - Strong correlation: $\hat{\mu}_0 = 0$
 - Weak correlations: $\hat{\mu}_{-1} < 0 < \hat{\mu}_1$
 - Zero correlation
- Side information:
 - contours: arithmetic edge coding
 - cluster indicator: arithmetic coding
- 2D prediction and transform

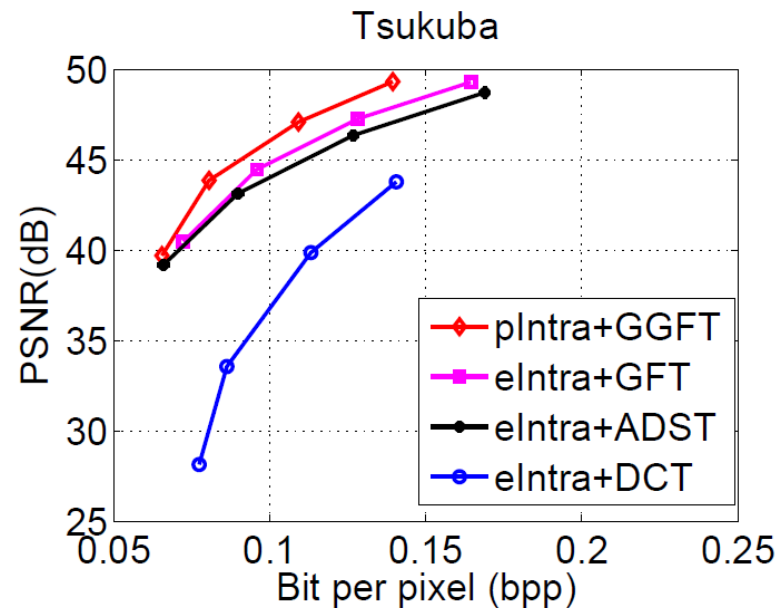


Experimental Results

- Test images: PWS images and natural images
- Compare *proposed intra-prediction (pIntra) + GGFT* against:
 - edge-aware intra-prediction (eIntra) + DCT
 - eIntra + ADST
 - eIntra + GFT



(a) Teddy



(b) Tsukuba

Experimental Results

TABLE I

AVERAGE GAIN IN PSNR MEASURED WITH THE BJONTEGAARD METRIC

Image	eIntra+GFT	eIntra+ADST	eIntra+DCT
Teddy	1.40	3.48	10.76
Cones	0.63	7.25	12.88
Tsukuba	1.97	2.36	13.28
Dude	3.46	4.59	5.26
Ballet	0.79	3.94	9.16
Carphone	0.59	1.13	1.96
Girl	0.42	0.31	1.74
Peppers	0.22	0.19	1.24
Cameraman	0.16	0.75	1.35
BasketballDrill	0.39	1.02	1.80

Subjective Quality



eIntra + DCT



eIntra + GFT



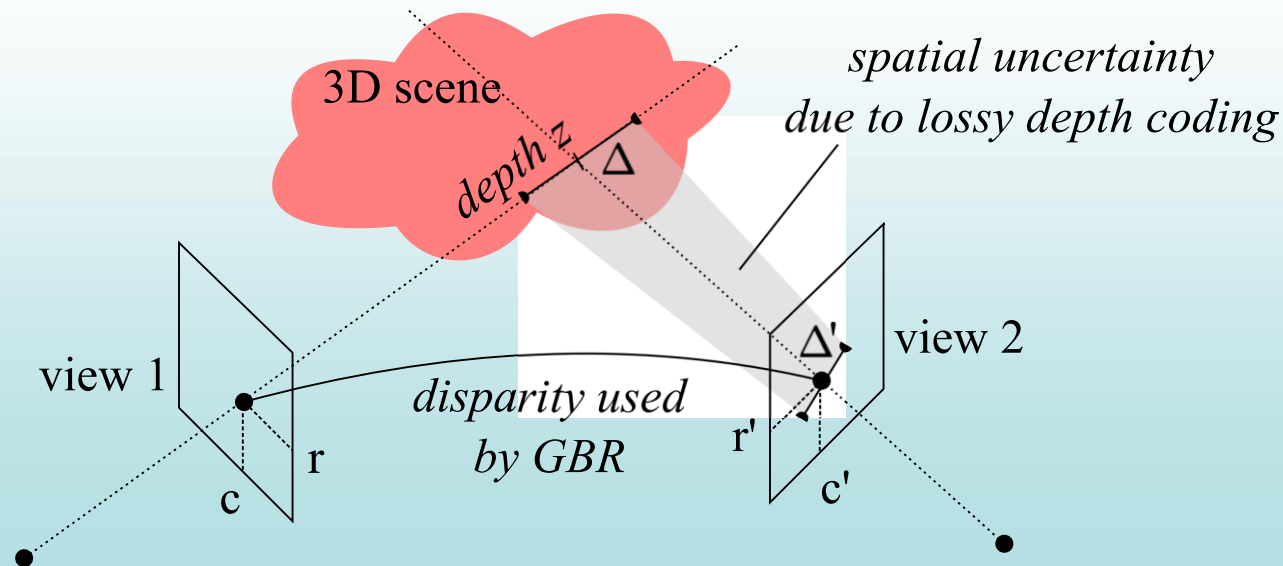
pIntra + GGFT

Outline

- Depth Image Coding
 - Graph Fourier Transform
 - Multi-resolution Graph Fourier Transform
 - Generalized Graph Fourier Transform
- Graph based Representation (GBR)

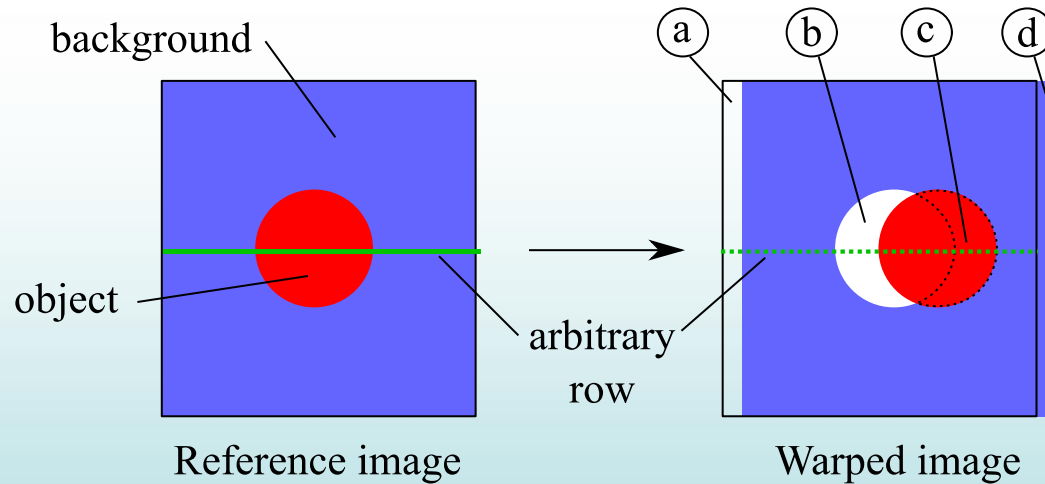
Motivations

- All depth-based representation methods for multiview images share the following drawback:



- Observation:** consequences of lossy depth compression are not well controlled.
- Analogy:** Motion vectors (MV) can be coarse (e.g., 16x16), but is always *losslessly* coded.

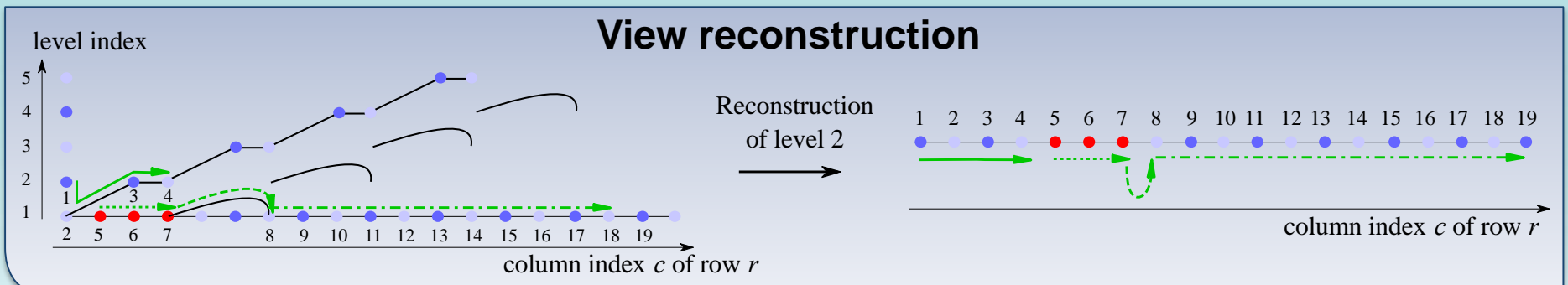
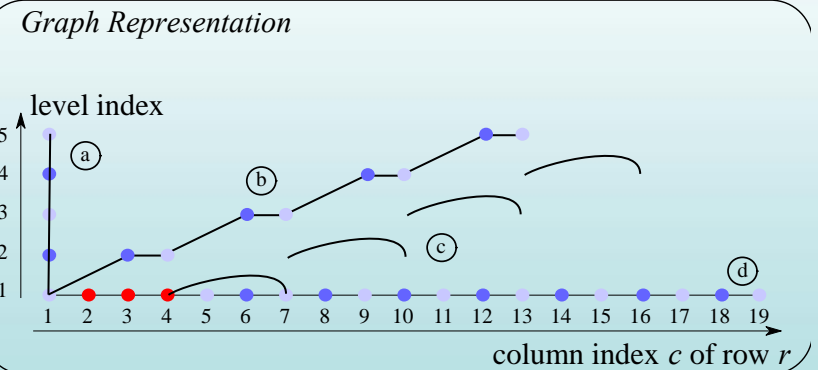
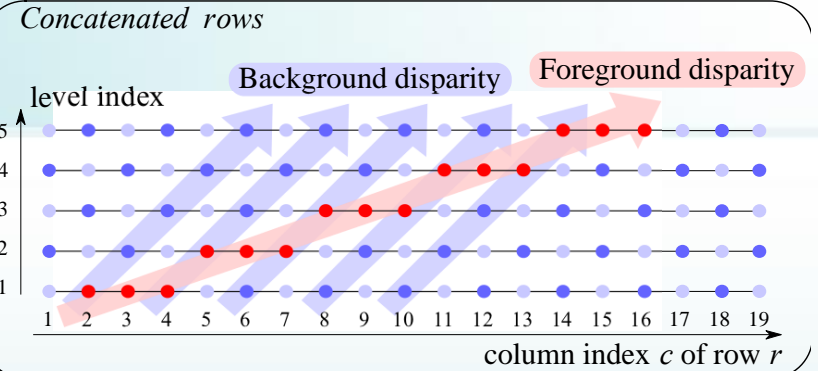
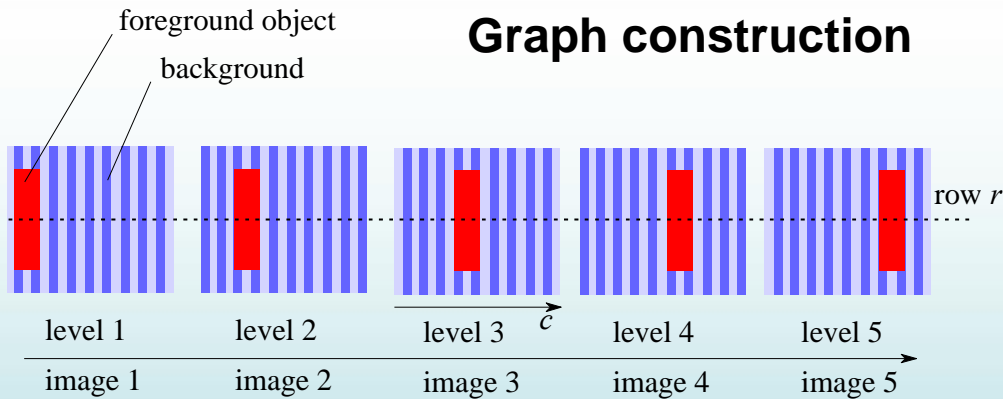
Intuitions



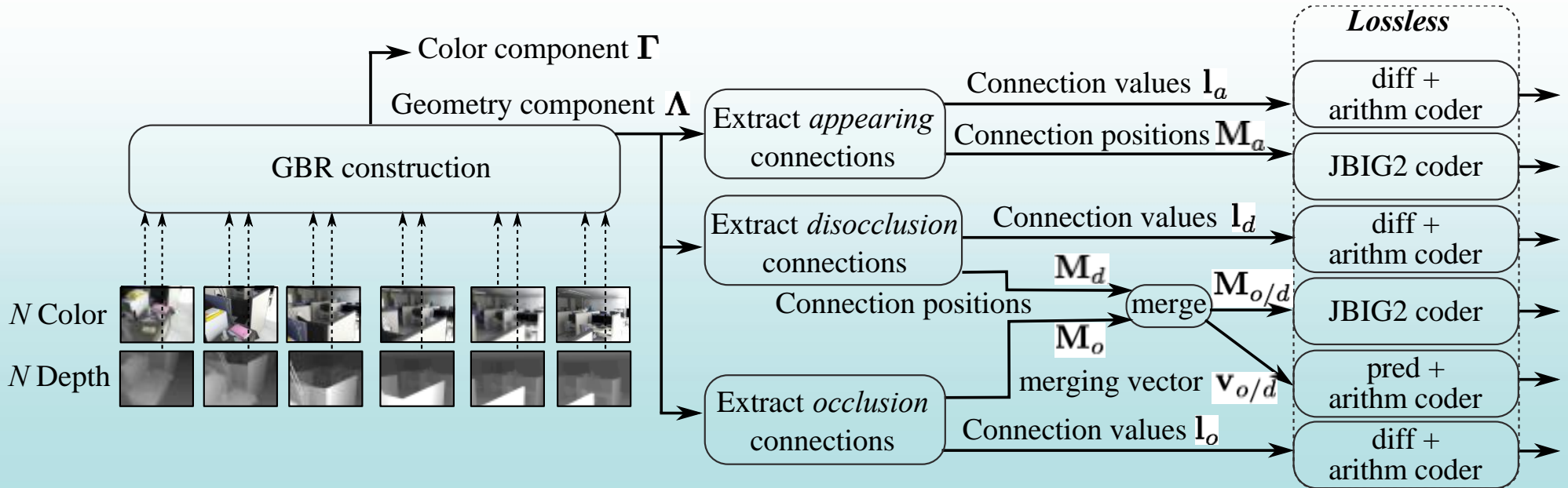
- (a) – appearing pixels
- (b) – disoccluded pixels
- (c) – occluded pixels
- (d) – disappearing pixels

- Replace depth by connections between pixels, for geometry representation
- Build a lossy representation followed by a lossless coding instead of directly lossy coding the depth

Graph construction rules



Graph coding



Compression results

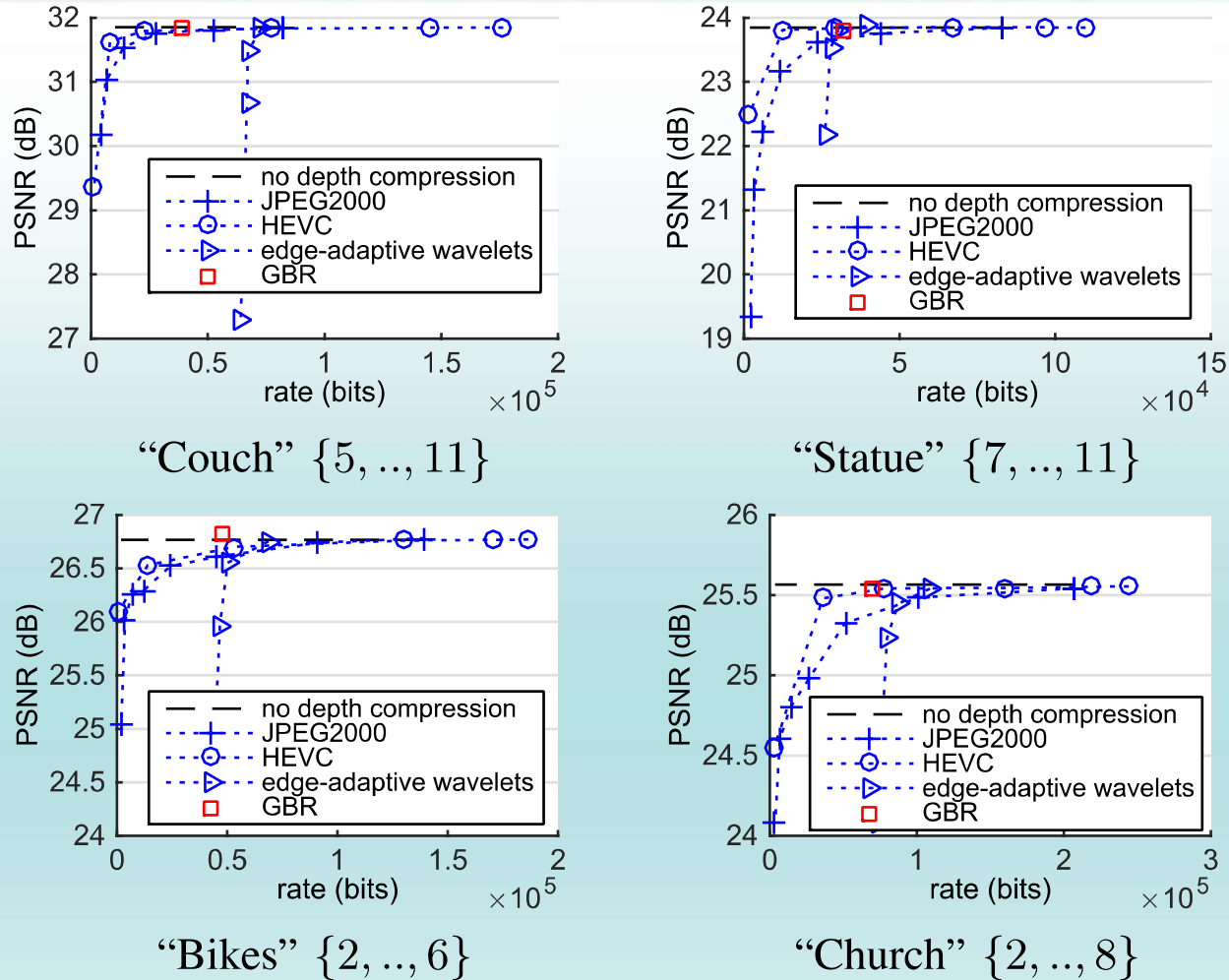


Fig. 10. Reference View 1 is used for the synthesis of multiple views, after the coding of geometry information with multiple techniques.

Naturally keeping the scene structure

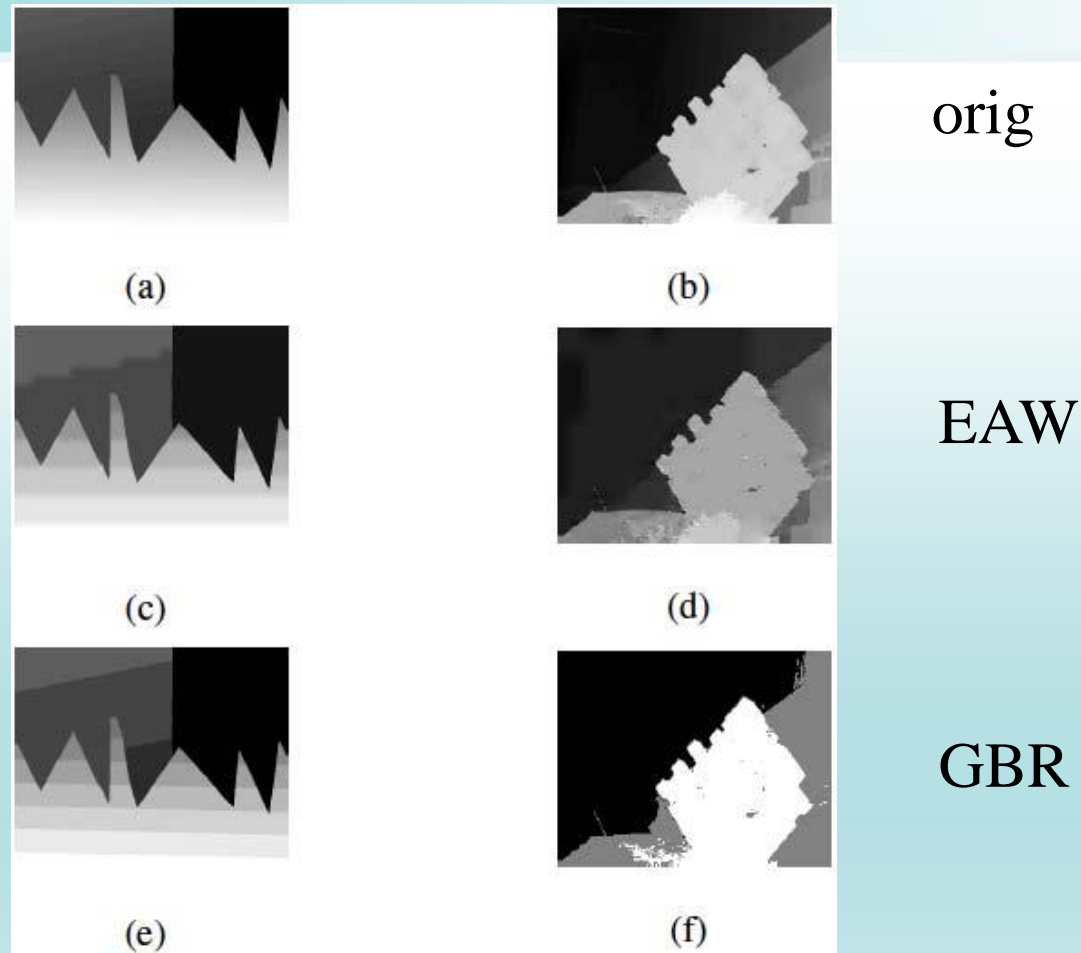


Fig. 12. Geometry images for “Sawtooth” (left) and “Statue” (right) sequences. Subfigures (a) and (b) are the original depth maps. Subfigures (c) and (d) are the depth maps coded with edge-adaptive wavelet (EAW) based coder [24], while (e) and (f) are geometry images extracted from our GBR. In these visual examples, the geometry coding rate of EAW is equal to the rate of our GBR (30 kb for “Sawtooth” and 10 kb for “Statue”).

Summary

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