


EECS 3481 APPLIED CRYPTOGRAPHY

YORK UNIVERSITY

CRYPTOGRAPHIC HASH FUNCTIONS

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WHAT

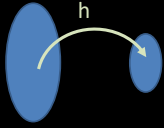
Some figures from Cryptography and Network Security, W. Stallings, Prentice-Hall

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WHAT IS A HASH FUNCTION?

Function $h: x \rightarrow y=h(x)$ maps a message of an arbitrary size to a fixed-size (n) bit sequence.

- Manifestly Many-to-One
 $|\text{domain}| = \infty, |\text{range}| = 2^n$
- Fast to compute (by S/W and H/W)
Often used on large inputs
- Digests the message
A single bit flip will likely not lead to a collision



Explore the `md5sum` and `sh1sum` Linux commands.

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HASH VIA JCE

```
MessageDigest md = MessageDigest.getInstance("<algo>");  
byte[] hash = md.digest( message expressed as byte[] );
```

ALGORITHM	GROUP	DIGEST	BLOCK
MD5	MD	128	512
SHA-1	SHA-1	160	512
SHA-224	SHA-2	224	512
SHA-256	SHA-2	256	512
SHA-512	SHA-2	512	1024
SHA3-256	SHA-3	256	1088
SHA3-512	SHA-3	512	576

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WHAT IS A CRYPTOGRAPHIC HASH FUNCTION?

It is a hash function with one or more of these properties:

1. Pre-image Resistance (one-way-ness)
Given y , infeasible to find any x : $h(x) = y$
2. 2nd Pre-image [Weak Collision] Resistance
Given any x_1 , infeasible to find $x_2 \neq x_1$: $h(x_1) = h(x_2)$
3. Strong Collision Resistance
Infeasible to find any x_1, x_2 pair ($x_2 \neq x_1$): $h(x_1) = h(x_2)$

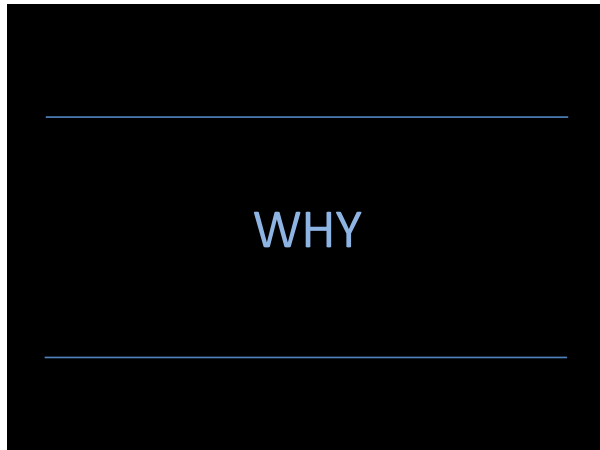
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ONE-WAY / WEAK / STRONG VENN DIAGRAM

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THE I IN CIA

Confidentiality ✓

How about the integrity of

- Content?
- Sender? (aka AUTH)
- Sending? (aka Non-Repudiation)
- Time of Sending? (aka Freshness)

Can symmetric or asymmetric Crypto (as-is / augmented) come to the rescue?

No, encryption does not automatically provide integrity. The third building block is needed.

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CONTENT INTEGRITY I

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CONTENT INTEGRITY II

- **Digest = the Hash**
AKA *tag, checksum, or PRF* (Pseudo-Random Function)
- **MAC** (Message Authenticated Code)
Encrypt digest with a secret key: $MAC = E[K, h(m)]$
- **HMAC**
Combine digest with a secret key
 $h[K1 || h(K2 || m)]$ where K1 and K2 are derived from K
- **Signature**
Encrypt digest with a private key.
For RSA = $[h(m)]^d \text{ mod } n$

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SENDER INTEGRITY (AKA AUTHENTICATION)

- **Symmetric Crypto**
Ensures auth but can repudiate and no freshness
- **Challenge-Response**
Alice sends nonce n to Bob; he returns $E(k, n)$ or $HMAC(k, n)$.
For mutual auth, she returns $E(k, f(n))$ or $HMAC(k, f(n))$. *Can still repudiate but ensures freshness.*
- **Asymmetric Crypto**
Alice signs a nonce (encrypts its hash with her private key and sends it. This yields auth. + freshness + non-repudiation.

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TIME INTEGRITY [DONE AT THE MESSAGE OR PROTOCOL LEVEL]

- **Sequence Numbers**
Adds session overhead: a counter per party.
- **Timestamps**
Requires frequent clock synchronization and tolerance to network delays (by providing time windows).
- **Request-Response Nonce**
Ensures "freshness" with an unpredictable, random nonce.
See Challenge-Response in previous slide.

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HASHING APPLICATIONS (BEYOND MESSAGING)

- **Software Download**
Provide a link to S/W and post its hash on a read-only site.
- **Password Storage Best-Practice**
Rather than storing the password, store only its hash.
- **Blob Indexing, Fingerprinting, and Caching**
Use the blob's hash as a key.
- **Online Bidding (Zero-Knowledge)**
Blind/Salt your bid then hash it.
- **Blockchain Immutability and Mining**
Each block has the hash of its predecessor. Proof of work thru hash constraints, e.g. $< 2^{254}$.

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EXERCISES

For each of the applications shown, what is the key property of the hashing function (Oneway/Weak/Strong)?

- *Software downloads*
- *Password storage*
- *Bidding*
- *Blob indexing*

For each of the three use cases in Slide #9, critique the security of the case in terms of:

- *Confidentiality*
- *Content Integrity*
- *Sender Integrity*
- *Source Repudiation*

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ATTACKS

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BIRTHDAY ATTACK

- x people in a room. What is the probability W of at least one sharing your birthday?
- x people in a room. What is the probability S of at least two sharing a birthday?
- $S \approx 1 - \exp(-x^2/2N)$ where $N=365$
- To achieve a probability of more than 50-50, we need $x \gg 1.177 * \text{sqrt}(N)$

>> Only $2^{n/2}$ evals to find collisions in an n-bit hash ! <<

Example: to fabricate a message, make $2^{n/2}$ variations in the real message and $2^{n/2}$ of the fraudulent. $\text{Prob}(\text{match}) > 50\%$

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MESSAGE FABRICATION EXAMPLE

Dear Anthony,

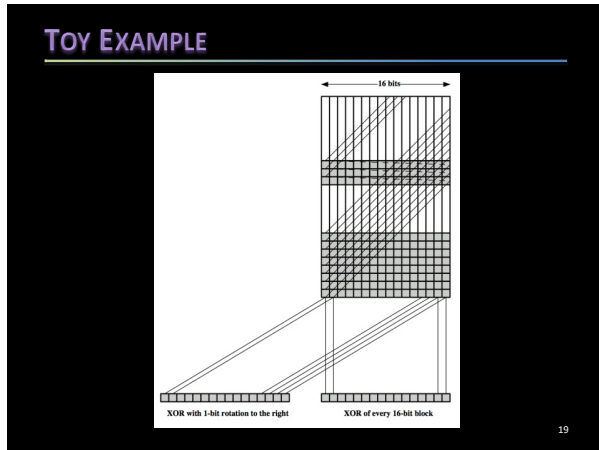
[This letter is] to introduce [you to] [Mr.] Alfred [P.] Barton, the [newly appointed] [senior] jewellery buyer for [our] [area] [division] - we [will take] over [the] [responsibility for] [all] [of] our interests in [watches and jewellery] [jewellery and watches] in the [area] - please [afford] his [every] help he [may need] to [seek out] the most [modern] [up to date] lines for the [top] end of the market. He is [empowered] to receive on our behalf [samples] of the [latest] [watches and jewellery] products, [up] to a [limit] of ten thousand dollars. He will [carry] a signed copy of this [letter] as proof of identity. An order with his signature, which is [attached] [authorizes] you to charge the cost to this company at the [above] [address. We [fully] expect that our [level] of orders will increase in the [following] year and [trust] that the new appointment will [be] [advantageous] to both our companies.

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HOW

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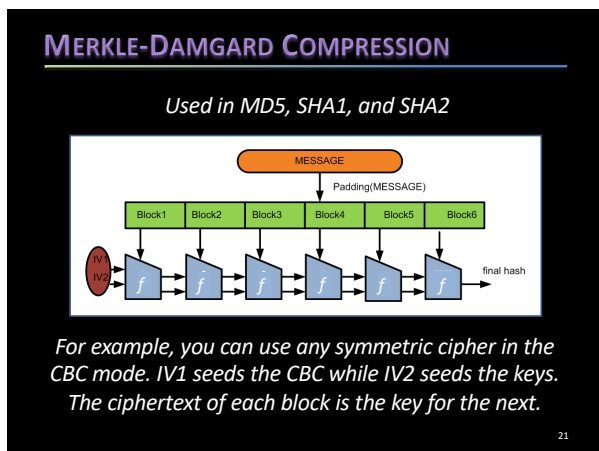
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APPROACHES

- **FORMULA-BASED**
Examples: $y = \sum x \text{ mod } n$ or $\sum x^2 \text{ mod } n$
- **ITERATIVE BLOCK COMPRESSION**
Examples: Merkle–Damgard (SHA1/2)
- **SPONGE**
Examples: Keccak (SHA3)

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KECCAK SPONGE

The diagram illustrates the Keccak sponge construction. It shows a sequence of blocks P_0, P_1, \dots, P_{n-1} being absorbed into a sponge. Each block P_i is padded with zeros to fit into a fixed-size block. The blocks are processed by a function f . The output of the sponge is a sequence of blocks Z_0, Z_1, \dots being squeezed out. The diagram also shows the internal state of the sponge, which is a 512-bit block divided into lanes a, b, c, d, e, f, g, h .

Used in SHA3 (224-512)

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TYPICAL KECCAK F FUNCTION

The diagram shows a typical Keccak f function. It takes an input block a, b, c, d, e, f, g, h and produces an output block a, b, c, d, e, f, g, h . The function consists of several operations: a majority function $Maj(a, b, c)$, a choice function $Ch(e, f, g)$, and a summation function Σ . The operations are performed on the input lanes and the results are added to the input lanes. The function also uses a 64-bit additive constant K_t and a 64-bit word W_t from the current 512-bit input block.

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HOW: TYPICAL f OPERATIONS

$Ch(e, f, g) = (e \text{ AND } f) \text{ XOR } (\text{NOT } e \text{ AND } g)$
 $Maj(a, b, c) = (a \text{ AND } b) \text{ XOR } (a \text{ AND } c) \text{ XOR } (b \text{ AND } c)$
 $\Sigma(a) = \text{ROTR}(a, 28) \text{ XOR } \text{ROTR}(a, 34) \text{ XOR } \text{ROTR}(a, 39)$
 $\Sigma(e) = \text{ROTR}(e, 14) \text{ XOR } \text{ROTR}(e, 18) \text{ XOR } \text{ROTR}(e, 41)$
 $+$ = addition modulo 2^{64}
 K_t = a 64-bit additive constant
 W_t = a 64-bit word from the current 512-bit input block
 $W_t = \sigma_0(W_{t-2}) + W_{t-7} + \sigma_1(W_{t-15}) + W_{t-16}$ ($t = 16 \dots 79$)
 $\sigma_0 = \text{ROTR}(1) \text{ XOR } \text{ROTR}(8) \text{ XOR } \text{SHR}(7)$, $\sigma_1 = \text{ROTR}(19) \text{ XOR } \text{ROTR}(60) \text{ XOR } \text{SHR}(6)$

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