Recommender Systems: Latent Factor Models

Thanks to
Jure Leskovec, Anand Rajaraman, Jeff Ullman
http://www.mmds.org

The Netflix Prize

- **Training data**
  - 100 million ratings, 480,000 users, 17,770 movies
  - 6 years of data: 2000-2005
- **Test data**
  - Last few ratings of each user (2.8 million)
  - **Evaluation criterion:** Root Mean Square Error (RMSE) $= \frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$
  - **Netflix's system RMSE: 0.9514**
- **Competition**
  - 2,700+ teams
  - **$1 million** prize for 10% improvement on Netflix

The Netflix Utility Matrix *R*

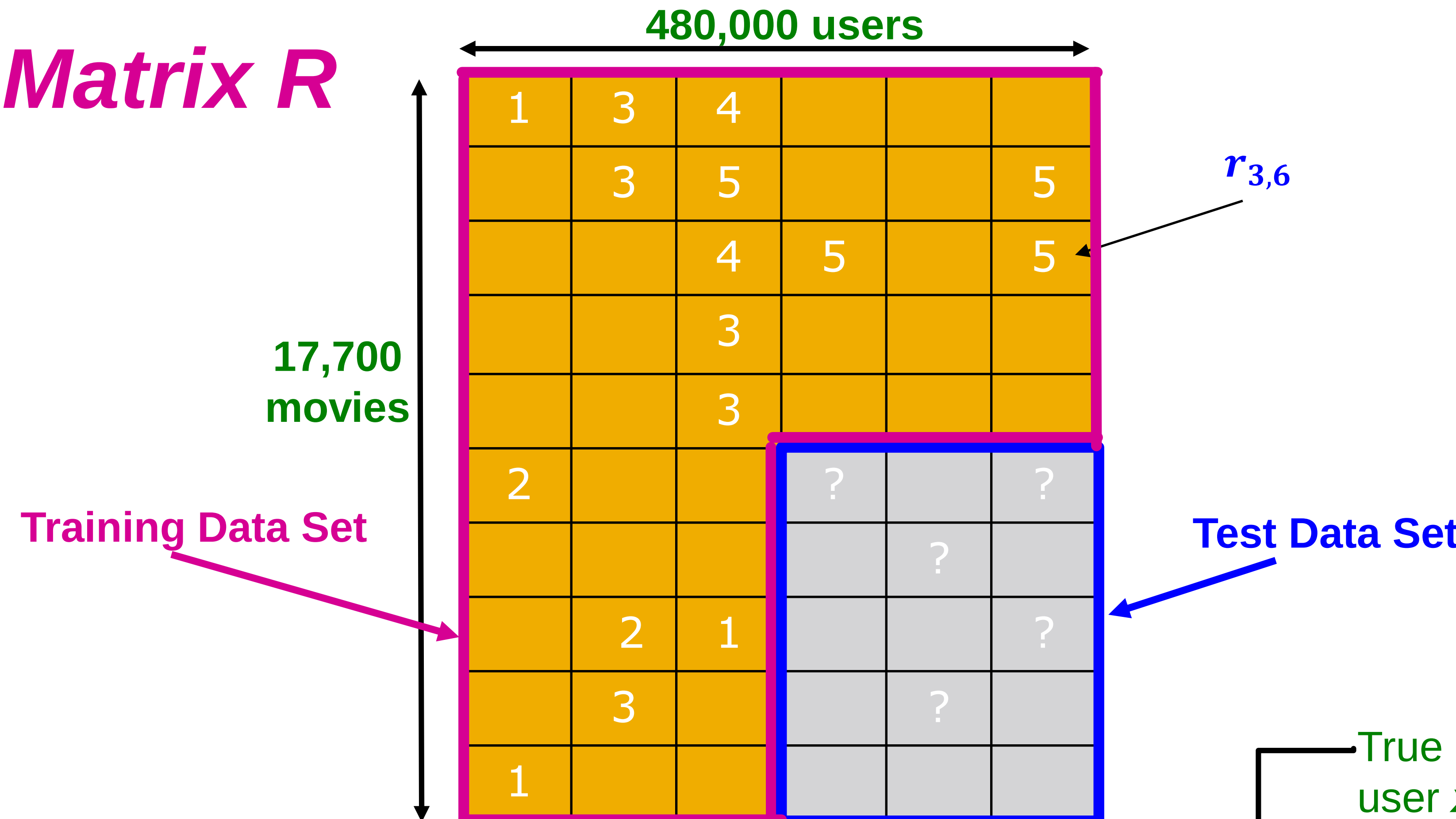
Matrix R

480,000 users

17,700 movies

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Utility Matrix *R*: Evaluation



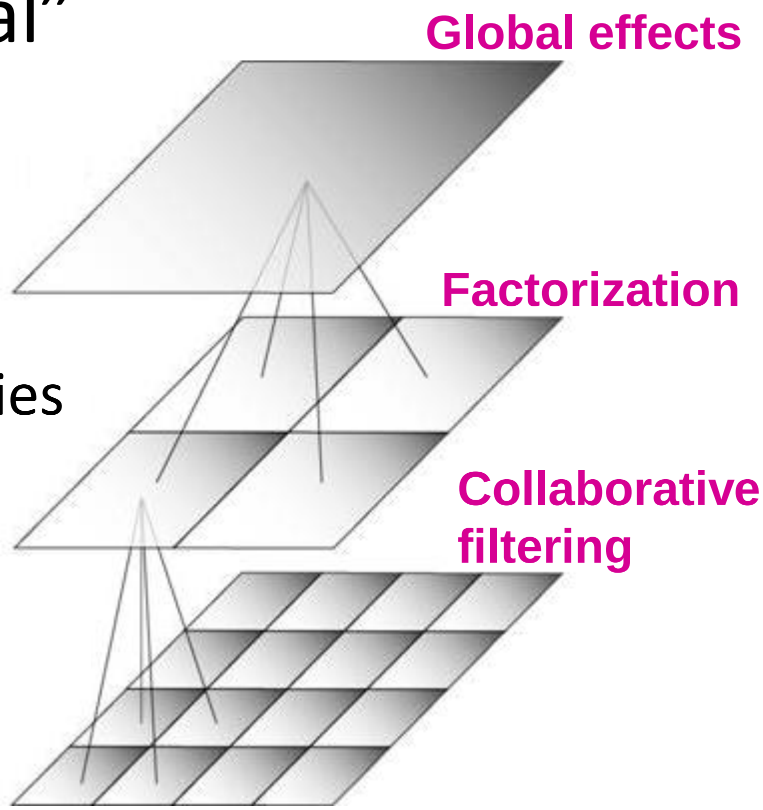
True rating of user ***x*** on item ***i***

$$\textbf{RMSE} = \frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$$

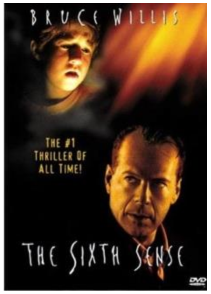
Predicted rating

BellKor Recommender System

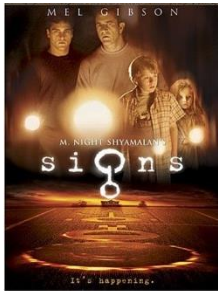
- **The winner of the Netflix Challenge!**
- **Multi-scale modeling of the data:** Combine top level, "regional" modeling of the data, with a refined, local view:
  - **Global:**
    - Overall deviations of users/movies
  - **Factorization:**
    - Addressing "regional" effects
  - **Collaborative filtering:**
    - Extract local patterns



Modeling Local & Global Effects



- **Global:**
  - Mean movie rating: **3.7 stars**
  - *The Sixth Sense* is **0.5** stars above avg.
  - Joe rates **0.2** stars below avg.
    ⇒ **Baseline estimation:**
    ***Joe* will rate *The Sixth Sense* 4 stars**
- **Local neighborhood (CF/NN):**
  - *Joe* didn't like related movie *Signs*
  - ⇒ **Final estimate:**
    ***Joe* will rate *The Sixth Sense* 3.8 stars**



Recap: Collaborative Filtering (CF)

- **Earliest and most popular collaborative filtering method**
- Derive unknown ratings from those of "**similar**" movies (item-item variant)
- Define **similarity measure** s_{ij} of items ***i*** and ***j***
- Select ***k***-nearest neighbors, compute the rating
  - ***N(i; x):*** items most similar to ***i*** that were rated by ***x***

$$\hat{r}_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***x*** on item ***j***
N(i;x)… set of items similar to item ***i*** that were rated by ***x***

Modeling Local & Global Effects

- **In practice we get better estimates if we model deviations:**

$$\hat{r}_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$\boldsymbol{b_{xi} = \mu + b_x + b_i}$$

μ = overall mean rating
b_x = rating deviation of user x
= (*avg. rating of* user x) – μ
b_i = (*avg. rating of* movie i) – μ

Problems/Issues:
1) Similarity measures are "arbitrary"
2) Pairwise similarities neglect interdependencies among users
3) Taking a weighted average can be restricting
Solution: **Instead of s_{ij} use w_{ij} that we estimate directly from data**

Idea: Interpolation Weights w_{ij}

- Use a **weighted sum** rather than **weighted avg.**:

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}(r_{xj} - b_{xj})$$

- **A few notes:**
  - $\boldsymbol{N(i;x)}$ … set of movies rated by user ***x*** that are similar to movie ***i***
  - $\boldsymbol{w_{ij}}$ is the interpolation weight (some real number)
    - We allow: $\sum_{j \in N(i,x)} \boldsymbol{w_{ij}} \neq \mathbf{1}$
  - $\boldsymbol{w_{ij}}$ models interaction between pairs of movies (it does not depend on user ***x***)

Idea: Interpolation Weights w_{ij}

- $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij}(r_{xj} - b_{xj})$
- **How to set w_{ij}?**
  - Remember, error metric is: $\frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$
    or equivalently **SSE:** $\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2$
  - Find w_{ij} that minimize **SSE** on **training data!**
    - Models relationships between item i and its neighbors j
  - w_{ij} can be **learned/estimated** based on x and all other users that rated i

Why is this a good idea?

Recommendations via Optimization

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

- **Goal:** Make good recommendations
  - Quantify goodness using **RMSE:**
    Lower RMSE $\Rightarrow$ better recommendations
  - Want to make good recommendations on items that user has not yet seen. Can't really do this!
  - **Let's set build a system such that it works well on known (user, item) ratings**
    And **hope** the system will also predict well the **unknown ratings**

Recommendations via Optimization

- **Idea: Let's set values *w* such that they work well on known (user, item) ratings**
- **How to find such values *w*?**
- **Idea:** Define an objective function and solve the optimization problem

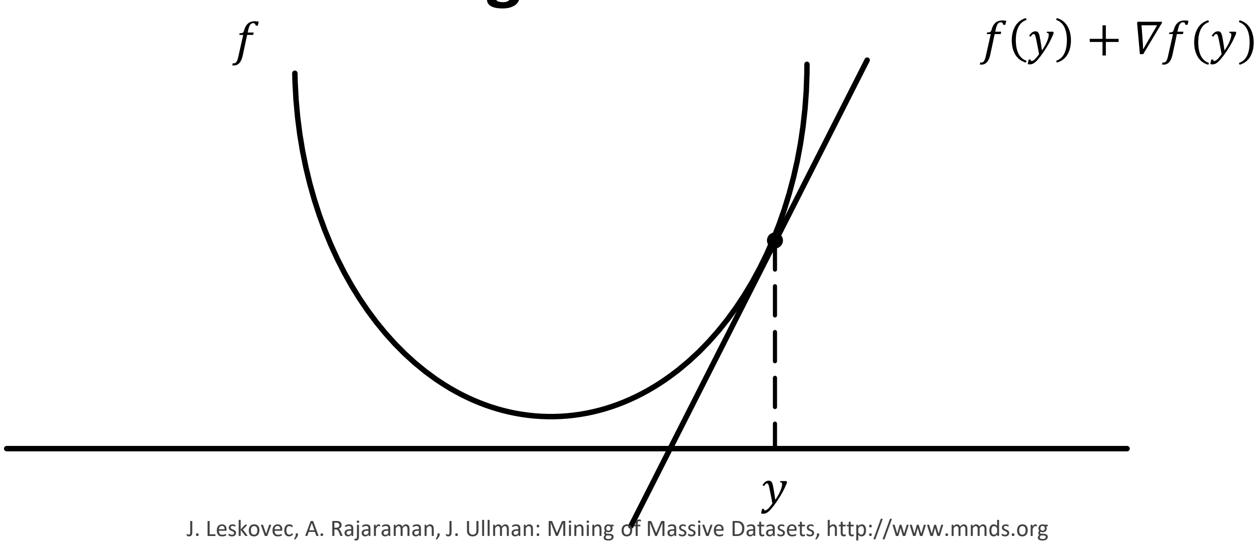
- Find $\boldsymbol{w_{ij}}$ that minimize **SSE** on **training data**!

$$J(w) = \sum_{x,i} \Bigg(\underbrace{\Big[b_{xi} + \sum_{j \in N(i;x)} w_{ij} \left(r_{xj} - b_{xj} \right) \Big]}_{\text{Predicted rating}} - \underbrace{r_{xi}}_{\text{True rating}} \Bigg)^2$$

- Think of ***w*** as a vector of numbers

Detour: Minimizing a function

- **A simple way to minimize a function $f(x)$:**
  - Compute the take a derivative ∇f
  - **Start at some point y and evaluate $\nabla f(y)$**
  - **Make a step in the reverse direction of the gradient: $y = y - \nabla f(y)$**
  - **Repeat until converged**



Interpolation Weights

- We have the optimization problem, now what?

$$J(w) = \sum_{x} \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} \left(r_{xj} - b_{xj} \right) \right] - r_{xi} \right)^2$$

- Gradient decent:
  - Iterate until convergence: $\boldsymbol{w} \leftarrow \boldsymbol{w} - \eta \nabla_{\boldsymbol{w}} \boldsymbol{J}$ η … learning rate
  - where $\nabla_{w} J$ is the gradient (derivative evaluated on data):

$$\nabla_w J = \left[\frac{\partial J(w)}{\partial w_{ij}} \right] = 2 \sum_{x,i} \left(\left[b_{xi} + \sum_{k \in N(i;x)} w_{ik} \left(r_{xk} - b_{xk} \right) \right] - r_{xi} \right) \left(r_{xj} - b_{xj} \right)$$

for $\boldsymbol{j} \in \{\boldsymbol{N}(\boldsymbol{i};\boldsymbol{x}), \forall \boldsymbol{i}, \forall \boldsymbol{x}\}$

else $\frac{\partial J(w)}{\partial w_{ij}} = \mathbf{0}$

  - **Note:** We fix movie ***i***, go over all $\boldsymbol{r_{xi}}$, for every movie ***j*** $\in \boldsymbol{N}(\boldsymbol{i};\boldsymbol{x})$, we compute $\frac{\partial \boldsymbol{J}(\boldsymbol{w})}{\partial \boldsymbol{w_{ij}}}$

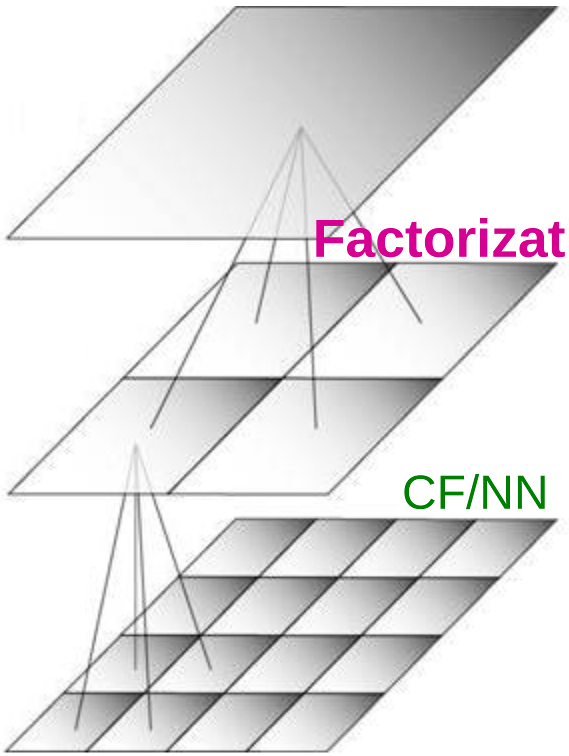
while $|w_{new} - w_{old}| > \varepsilon$:

$w_{old} = w_{new}$

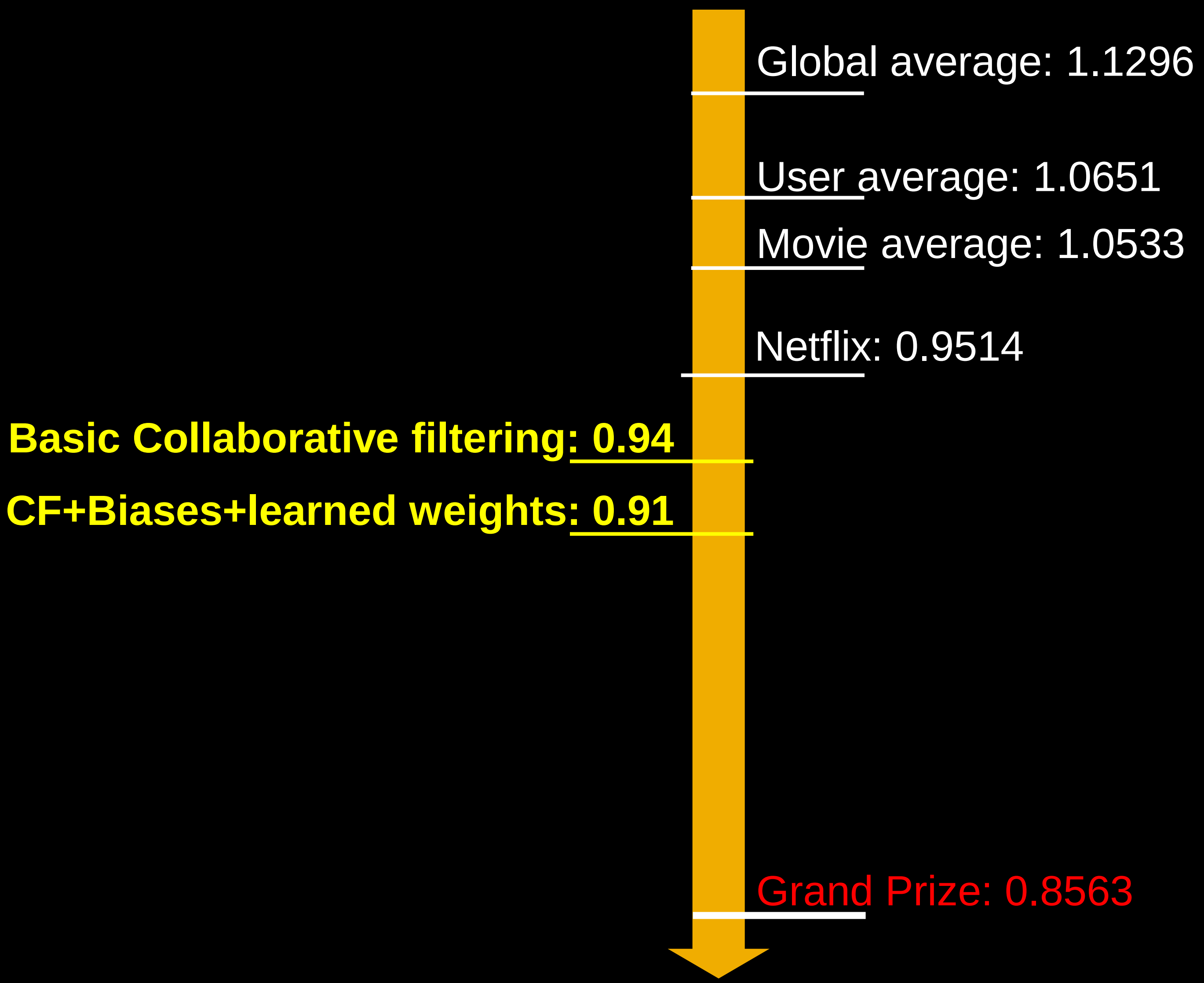
$w_{new} = w_{old} - \eta \cdot \nabla w_{old}$

Interpolation Weights

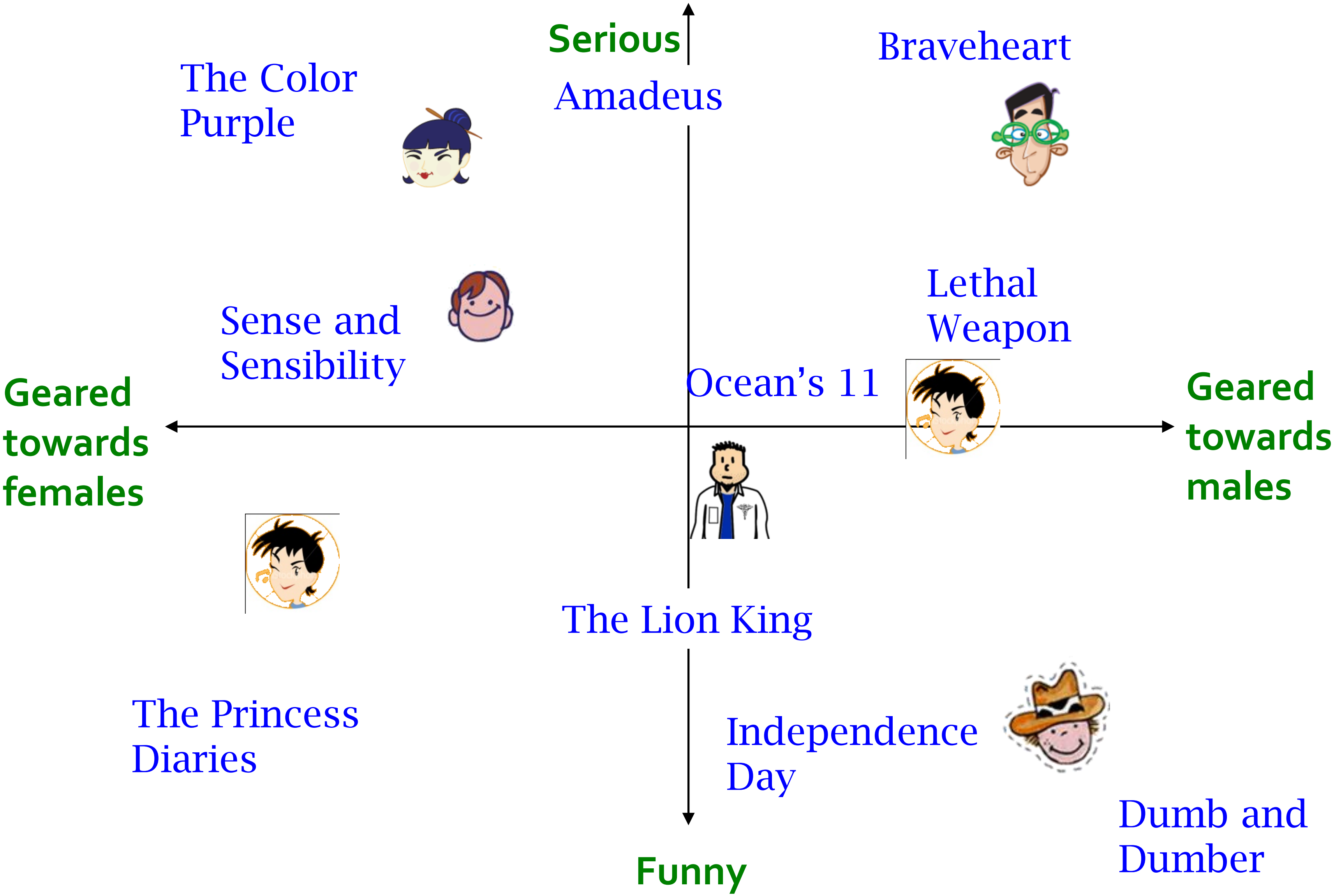
- **So far:** $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}(r_{xj} - b_{xj})$
  - Weights $\boldsymbol{w_{ij}}$ derived based on their role; **no use of an arbitrary similarity measure** ($\boldsymbol{w_{ij}} \neq \boldsymbol{s_{ij}}$)
  - Explicitly account for interrelationships among the neighboring movies
- **Next: Latent factor model**
  - Extract "regional" correlations



Performance of Various Methods



Latent Factor Models (e.g., SVD)



Latent Factor Models

SVD: $A = U \Sigma V^T$

- "SVD" on Netflix data: $\mathbf{R} \approx Q \cdot P^T$

users

1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

items

R

≈

factors

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

items

Q

users

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6

factors

P^T

- For now let's assume we can approximate the rating matrix ***R*** as a product of "thin" $Q \cdot P^T$
  - ***R*** has missing entries but let's ignore that for now!
    - Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x$$

$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q

p_x = column x of P^T

users (columns) / items (rows)

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

Q — items (rows) × factors (columns)

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

•

P^T — factors (rows) × users (columns)

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x$$

$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q

p_x = column x of P^T

users

items

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

$\approx$

items

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

factors

Q

$\bullet$

users

factors

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

P^T

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x = \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q
p_x = column x of P^T

users (columns) / items (rows)

1		3			5			5		4	
		5	4	**2.4**		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

Q (items × *f* factors)

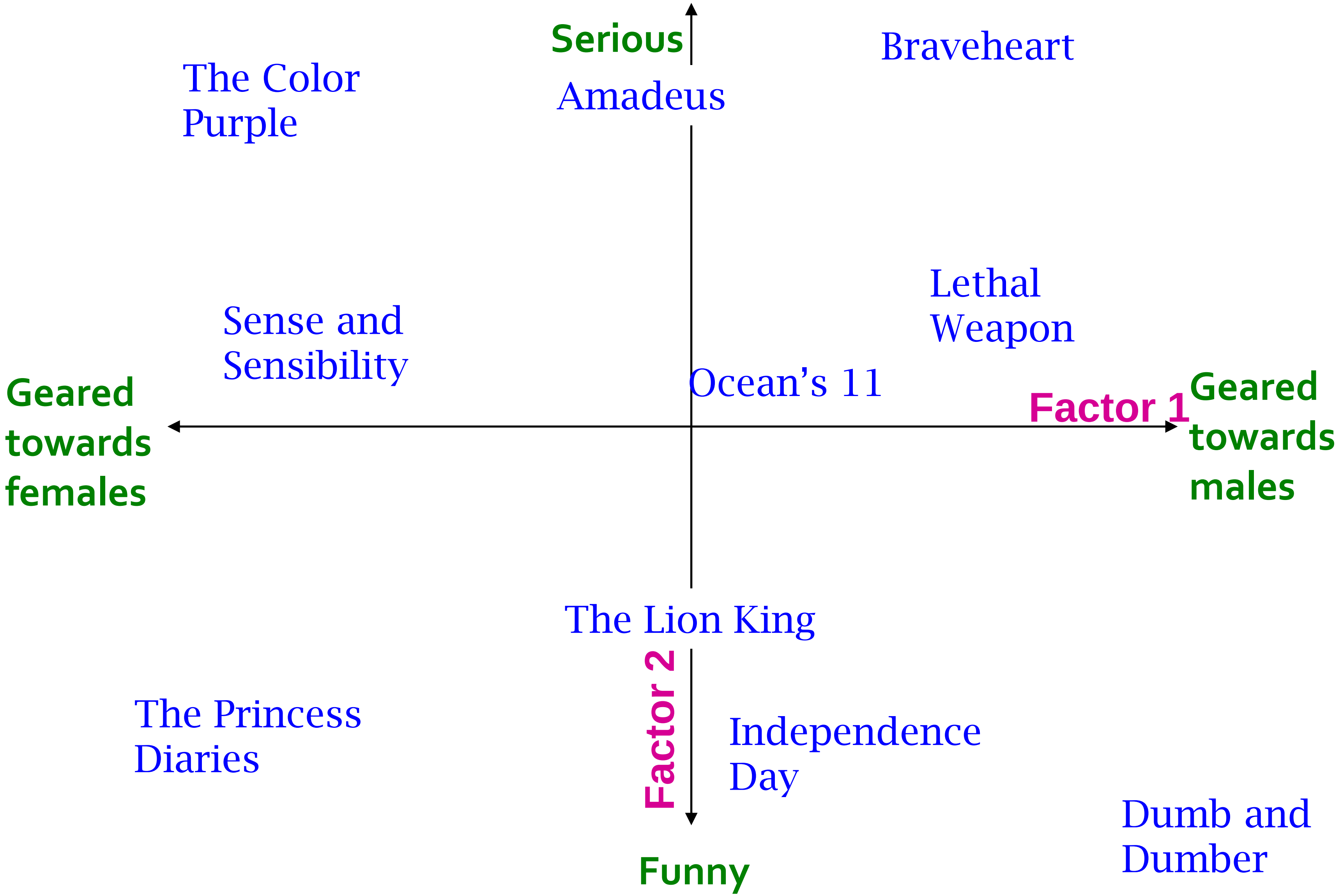
.1	-.4	.2
-.5	**.6**	**.5**
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

•

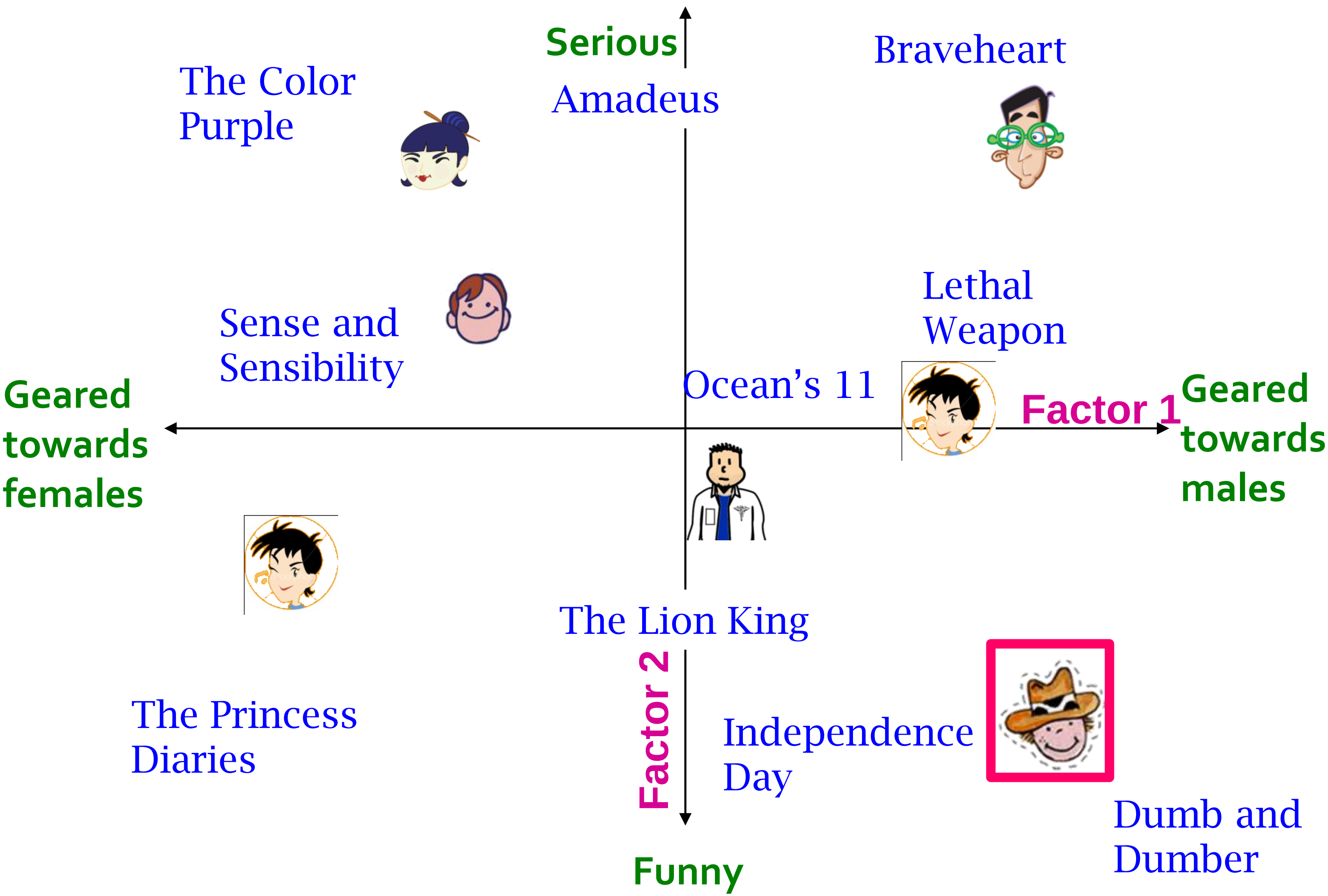
P^T (*f* factors × users)

1.1	-.2	.3	.5	**-2**	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	**.3**	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	**2.4**	.9	-.3	.4	.8	.7	-.6	.1

Latent Factor Models

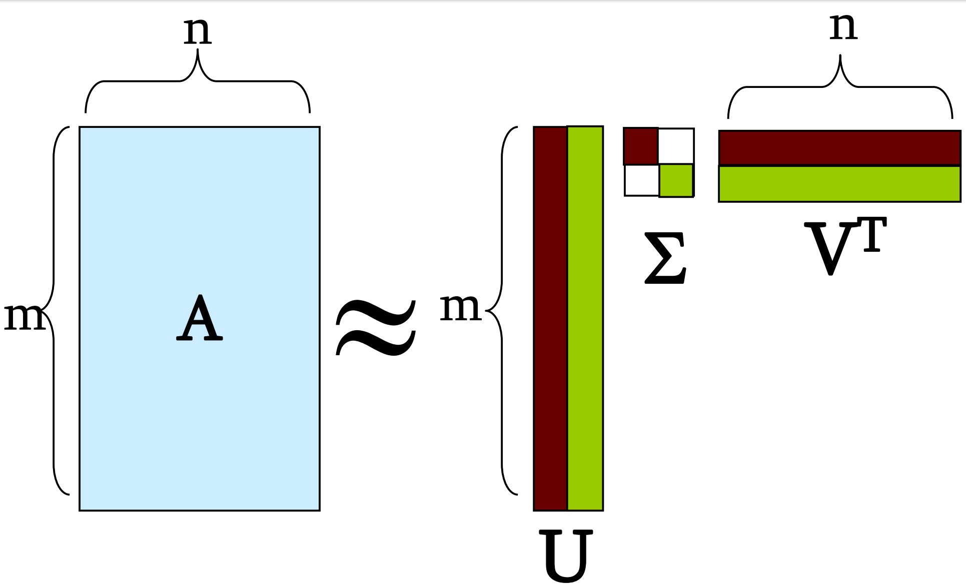


Latent Factor Models



Recap: SVD

- **Remember SVD:**
  - **A**: Input data matrix
  - **U**: Left singular vecs
  - **V**: Right singular vecs
  - **Σ**: Singular values



- **So in our case:**
  "SVD" on Netflix data: $R \approx Q \cdot P^T$
  A = R, Q = U, $P^T = \Sigma V^T$

$$\hat{r}_{xi} = q_i \cdot p_x$$

SVD: More good stuff

- **We already know that SVD gives minimum reconstruction error (Sum of Squared Errors):**

$$\min_{U,\mathrm{V},\Sigma} \sum_{ij \in A} \left(A_{ij} - [U\Sigma V^{\mathrm{T}}]_{ij} \right)^2$$

- **Note two things:**
  - **SSE** and **RMSE** are monotonically related:
    - $\boldsymbol{RMSE = \frac{1}{c}\sqrt{SSE}}$ **Great news: SVD is minimizing RMSE**
  - **Complication:** The sum in SVD error term is over all entries (no-rating is interpreted as zero-rating). But our ***R*** has missing entries!

Latent Factor Models

users (columns) × items (rows):

1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

Q: factors (columns) × items (rows)

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

P^T: users (columns) × factors (rows)

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

- **SVD isn't defined when entries are missing!**
- **Use specialized methods to find *P, Q***
  - $\min_{P,Q} \sum_{(i,x)\in \mathrm{R}} \left(r_{xi} - q_i \cdot p_x\right)^2$ $\qquad \hat{r}_{xi} = q_i \cdot p_x$
  - **Note:**
    - We don't require cols of ***P, Q*** to be orthogonal/unit length
    - ***P, Q*** map users/movies to a latent space
    - The most popular model among Netflix contestants

Finding the Latent Factors

Latent Factor Models

- **Our goal is to find P and Q such tat:**

$$\min_{P,Q} \sum_{(i,x)\in R} \left(r_{xi} - q_i \cdot p_x\right)^2$$

users

items

1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

factors

items

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

Q

users

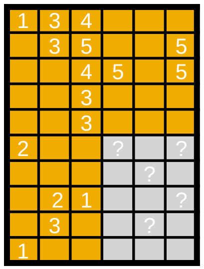
factors

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

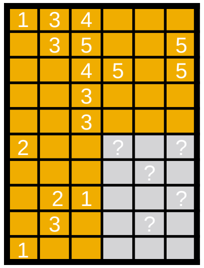
$\boldsymbol{P^T}$

Back to Our Problem

- **Want to minimize SSE for unseen test data**
- **Idea: Minimize SSE on training data**
  - Want large ***k*** (# of factors) to capture all the signals
  - But, **SSE** on **test data** begins to rise for ***k*** > 2
- This is a classical example of **overfitting:**
  - With too much freedom (too many free parameters) the model starts fitting noise
    - That is it fits too well the training data and thus **not generalizing** well to unseen test data



Dealing with Missing Entries



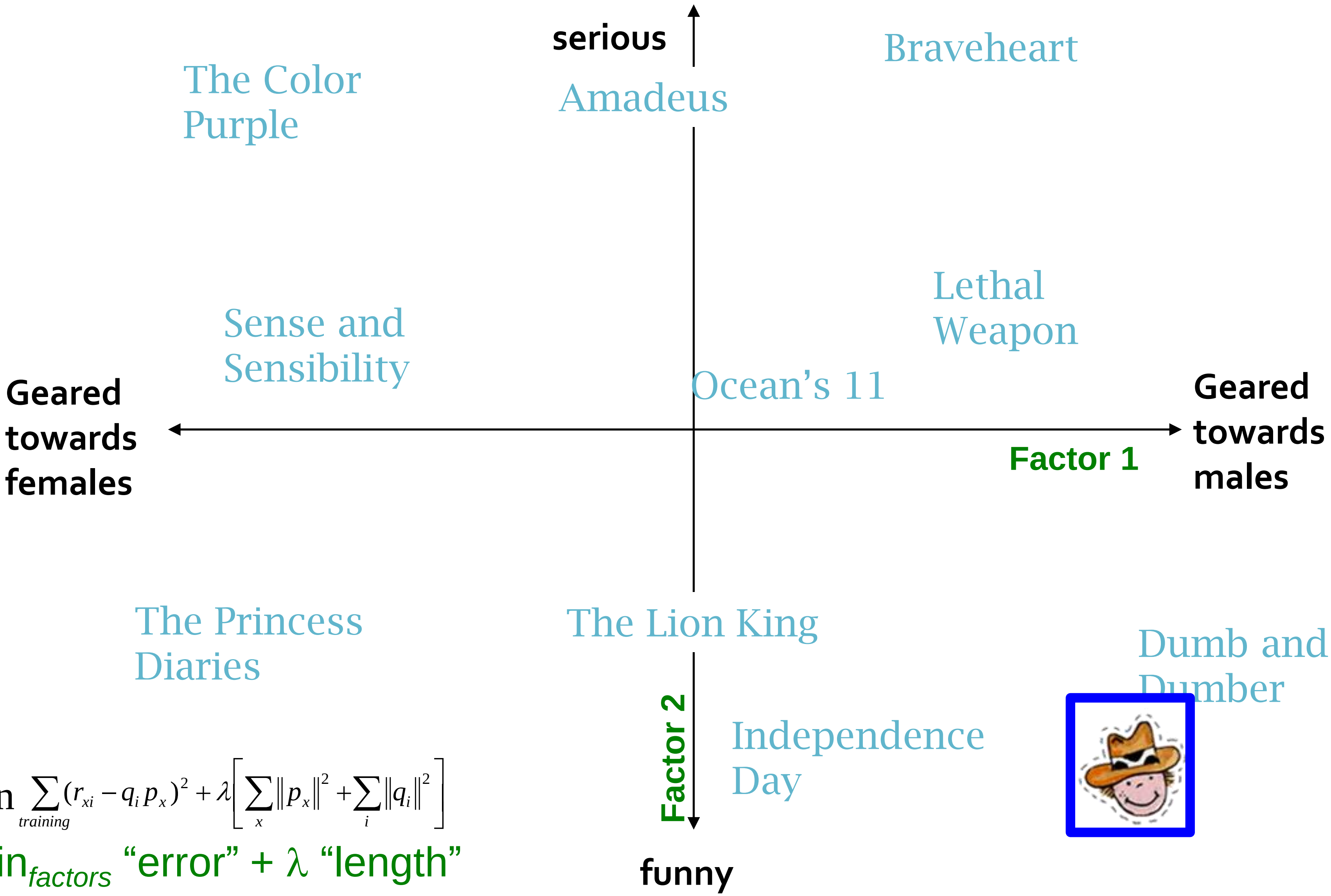
- **To solve overfitting we introduce regularization:**
  - Allow rich model where there are sufficient data
  - Shrink aggressively where data are scarce

$$\min_{P,Q} \underbrace{\sum_{training} (r_{xi} - q_i p_x)^2}_{\text{"error"}} + \underbrace{\left[\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2 \right]}_{\text{"length"}}$$

λ_1, λ_2 … user set regularization parameters

Note: We do not care about the "raw" value of the objective function, but we care in P,Q that achieve the minimum of the objective

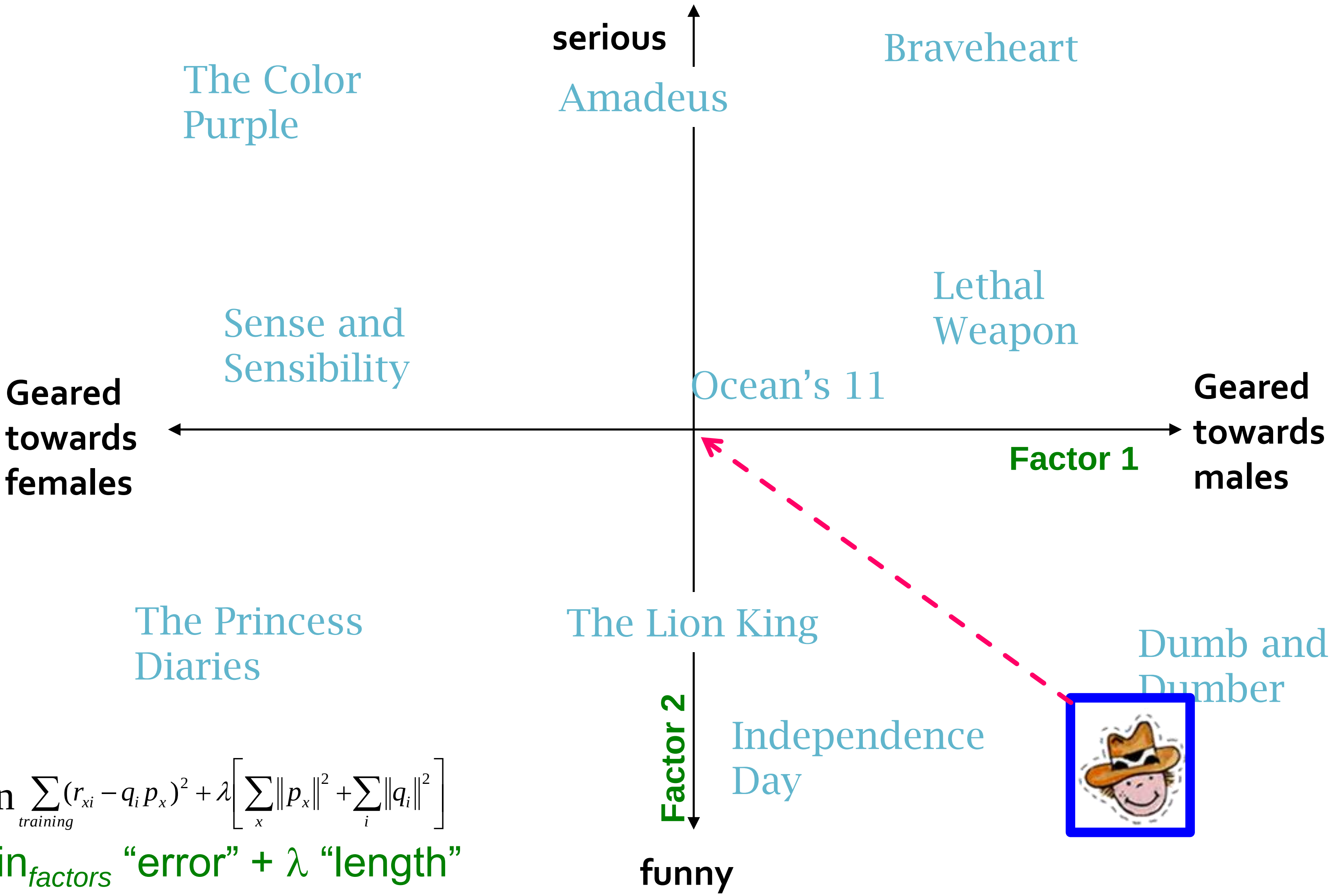
The Effect of Regularization



$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \lambda \left[\sum_x \|p_x\|^2 + \sum_i \|q_i\|^2 \right]$$

$\min_{factors}$ "error" + λ "length"

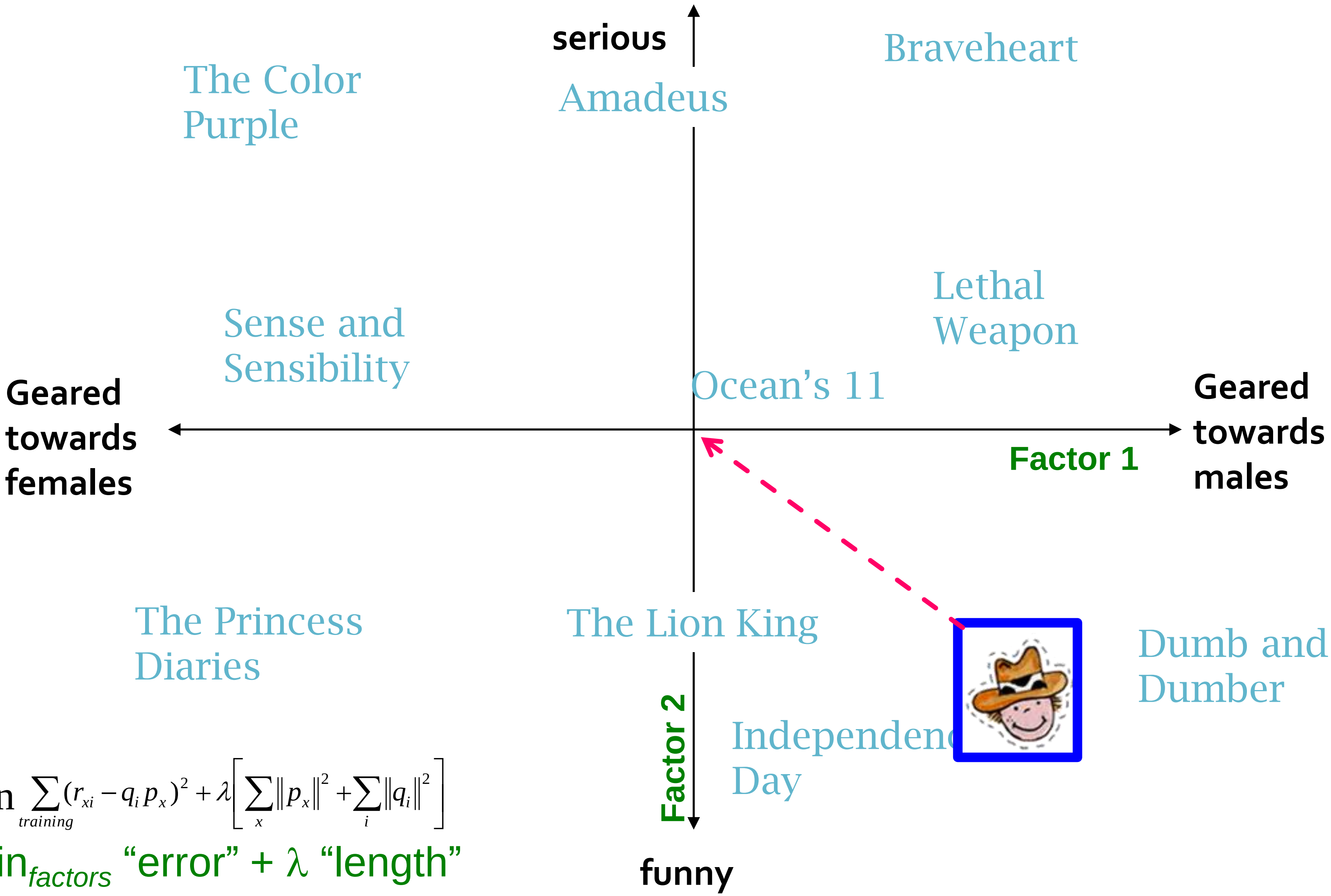
The Effect of Regularization



$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \lambda \left[\sum_x \|p_x\|^2 + \sum_i \|q_i\|^2 \right]$$

$\min_{factors}$ "error" + λ "length"

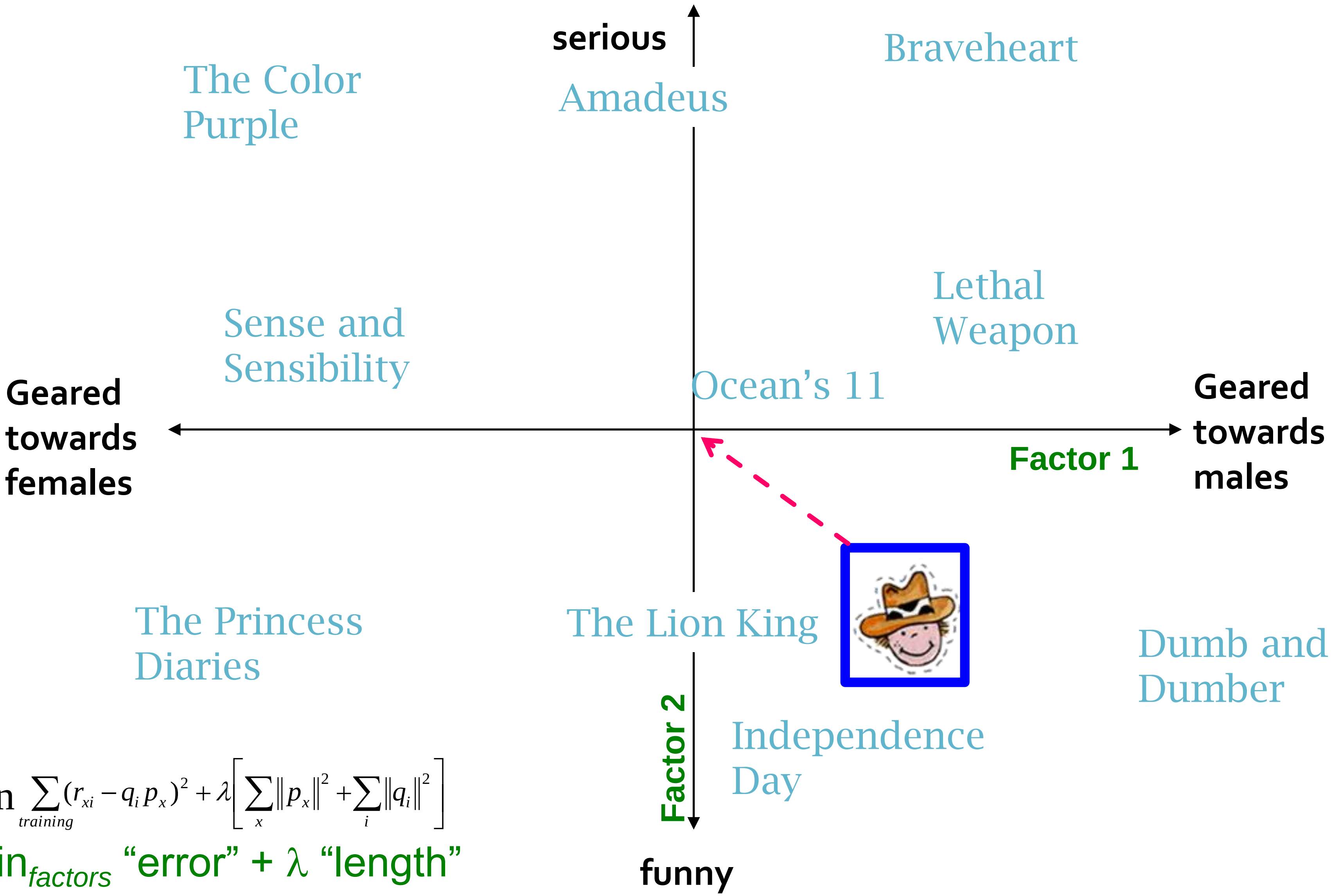
The Effect of Regularization



$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \lambda \left[\sum_x \|p_x\|^2 + \sum_i \|q_i\|^2 \right]$$

$\min_{factors}$ "error" + λ "length"

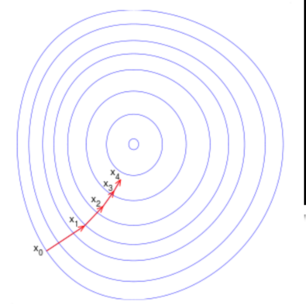
The Effect of Regularization



$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \lambda \left[\sum_x \|p_x\|^2 + \sum_i \|q_i\|^2 \right]$$

$\min_{factors}$ "error" + λ "length"

Stochastic Gradient Descent



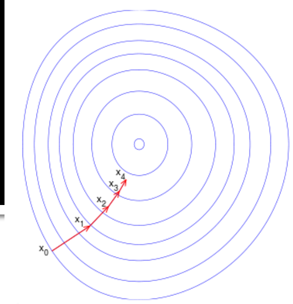
- **Want to find matrices *P* and *Q*:**

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2 \right]$$

- **Gradient decent:**
  - Initialize ***P*** and ***Q*** (using SVD, pretend missing ratings are 0)
  - Do gradient descent:
    - $\boldsymbol{P} \leftarrow \boldsymbol{P} - \eta \cdot \nabla \mathbf{P}$
    - $\boldsymbol{Q} \leftarrow \boldsymbol{Q} - \eta \cdot \nabla \mathbf{Q}$
    - where $\nabla\boldsymbol{Q}$ is gradient/derivative of matrix ***Q***:
    $\nabla Q = [\nabla q_{if}]$ and $\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x) p_{xf} + 2\lambda_2 q_{if}$
      - Here $\boldsymbol{q_{if}}$ is entry ***f*** of row $\boldsymbol{q_i}$ of matrix ***Q***
  - **Observation: Computing gradients is slow!**

How to compute gradient of a matrix?
Compute gradient of every element independently!

Stochastic Gradient Descent



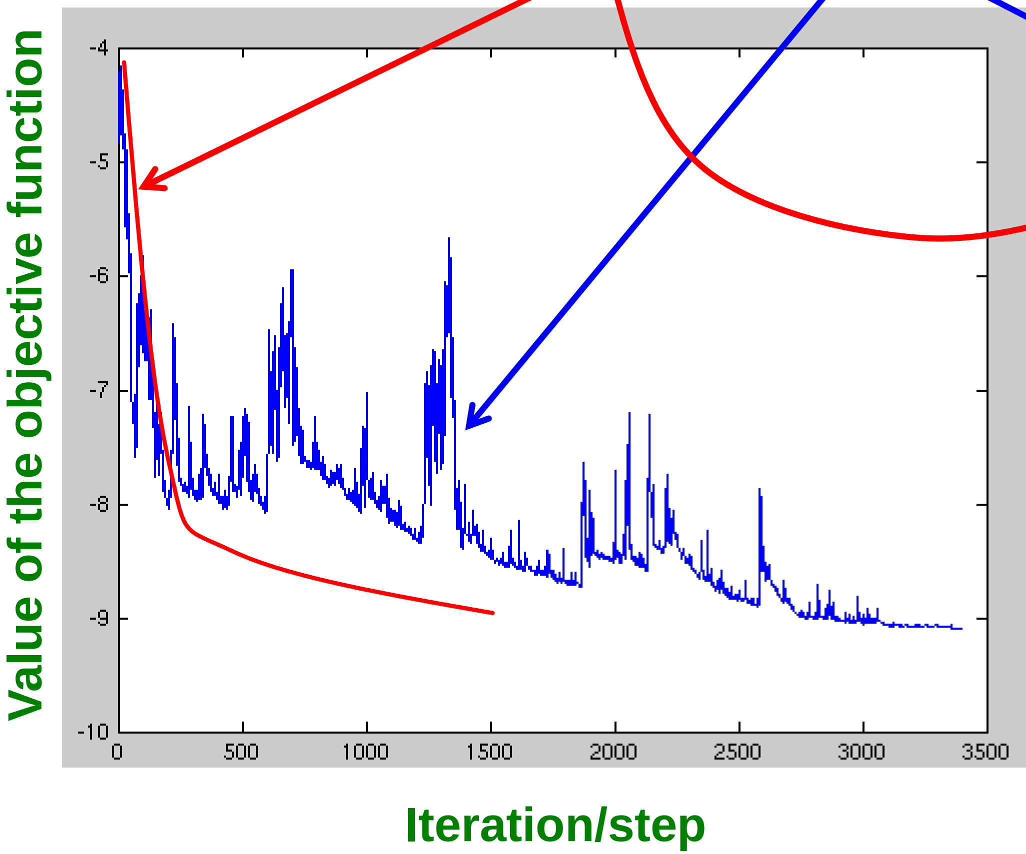
- **Gradient Descent (GD) vs. Stochastic GD**
  - **Observation:** $\nabla Q = [\nabla q_{if}]$ where

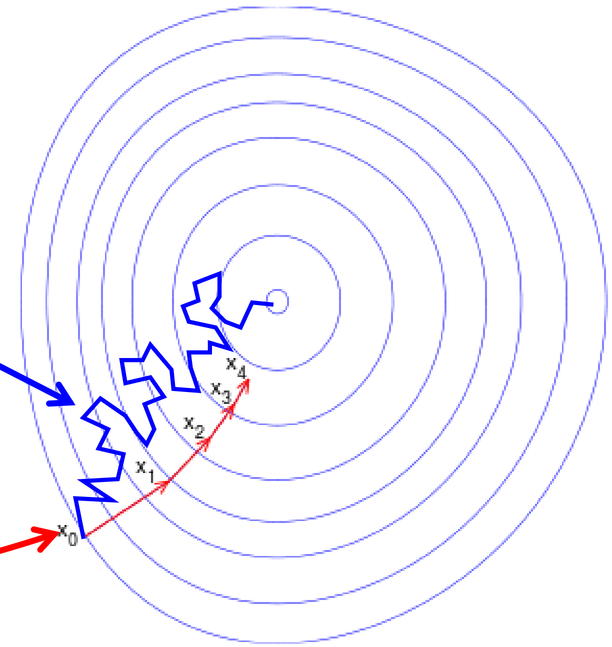
$$\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_{if} p_{xf}) p_{xf} + 2\lambda q_{if} = \sum_{x,i} \nabla \boldsymbol{Q}\,(\boldsymbol{r}_{xi})$$

    - Here $\boldsymbol{q}_{if}$ is entry $\boldsymbol{f}$ of row $\boldsymbol{q}_i$ of matrix $\boldsymbol{Q}$
  - $\boldsymbol{Q} = \boldsymbol{Q} - \eta \nabla \boldsymbol{Q} = \boldsymbol{Q} - \eta\left[\sum_{x,i} \nabla \boldsymbol{Q}\,(\boldsymbol{r}_{xi})\right]$
  - **Idea:** Instead of evaluating gradient over all ratings evaluate it for each individual rating and make a step
- **GD:** $\boldsymbol{Q} \leftarrow \boldsymbol{Q} - \eta\left[\sum_{\boldsymbol{r}_{xi}} \nabla \boldsymbol{Q}(\boldsymbol{r}_{xi})\right]$
- **SGD:** $\boldsymbol{Q} \leftarrow \boldsymbol{Q} - \mu \nabla \boldsymbol{Q}(\boldsymbol{r}_{xi})$
  - **Faster convergence!**
    - Need more steps but each step is computed much faster

SGD vs. GD

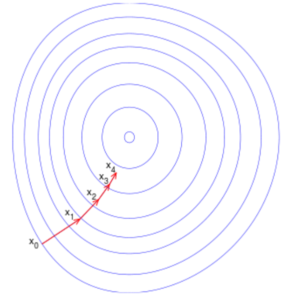
- **Convergence of GD vs. SGD**





GD improves the value of the objective function at every step.
SGD improves the value but in a “noisy” way.
GD takes fewer steps to converge but each step takes much longer to compute.
In practice, **SGD** is much faster!

Stochastic Gradient Descent



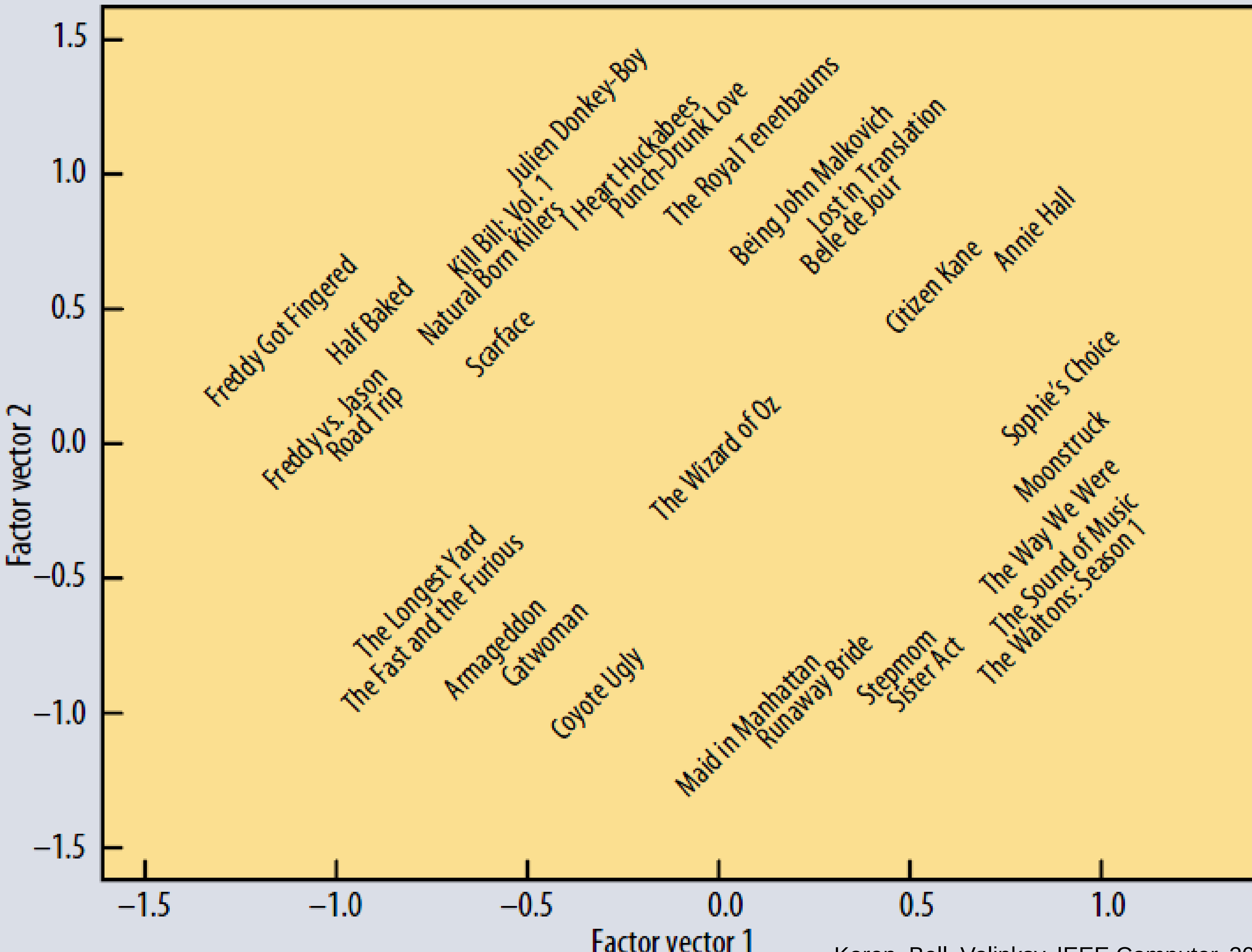
- **Stochastic gradient decent:**
  - Initialize ***P*** and ***Q*** (using SVD, pretend missing ratings are 0)
  - Then iterate over the ratings (multiple times if necessary) and update factors:

  For each *r*$_{xi}$:

    - $\varepsilon_{xi} = 2(r_{xi} - q_i \cdot p_x)$ (derivative of the "error")
    - $q_i \leftarrow q_i + \mu_1 (\varepsilon_{xi}\, p_x - \lambda_2\, q_i)$ (update equation)
    - $p_x \leftarrow p_x + \mu_2 (\varepsilon_{xi}\, q_i - \lambda_1\, p_x)$ (update equation)

  μ … learning rate

- **2 for loops:**
  - For until convergence:
    - For each $\mathbf{r}_{xi}$
      - Compute gradient, do a "step"



Koren, Bell, Volinksy, IEEE Computer, 2009

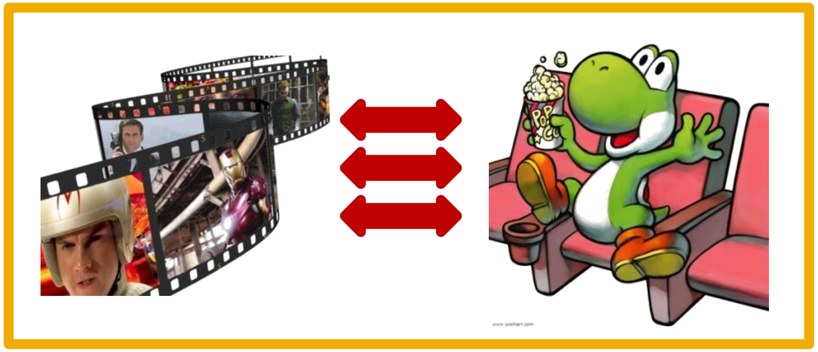
Extending Latent Factor Model to Include Biases

Modeling Biases and Interactions

user bias

movie bias

user-movie interaction



Baseline predictor

- Separates users and movies
- Benefits from insights into user's behavior
- Among the main practical contributions of the competition

User-Movie interaction

- Characterizes the matching between users and movies
- Attracts most research in the field
- Benefits from algorithmic and mathematical innovations

- $\boldsymbol{\mu}$ = overall mean rating
- $\boldsymbol{b_x}$ = bias of user $\boldsymbol{x}$
- $\boldsymbol{b_i}$ = bias of movie $\boldsymbol{i}$

Baseline Predictor

- We have expectations on the rating by user ***x*** of movie ***i***, even without estimating ***x***'s attitude towards movies like ***i***

- Rating scale of user ***x***
- Values of other ratings user gave recently (day-specific mood, anchoring, multi-user accounts)

- (Recent) popularity of movie ***i***
- Selection bias; related to number of ratings user gave on the same day ("frequency")

Putting It All Together

$$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

Overall mean rating — Bias for user ***x*** — Bias for movie ***i*** — User-Movie interaction

- **Example:**
  - Mean rating: ***μ = 3.7***
  - You are a critical reviewer: your ratings are 1 star lower than the mean: ***b_x = -1***
  - Star Wars gets a mean rating of *0.5* higher than average movie: ***b_i = + 0.5***
  - Predicted rating for you on Star Wars:
    = 3.7 - 1 + 0.5 = 3.2

Fitting the New Model

- **Solve:**

$$\min_{Q,P} \sum_{(x,i)\in R} \left(r_{xi} - (\mu + b_x + b_i + q_i\, p_x) \right)^2$$

goodness of fit

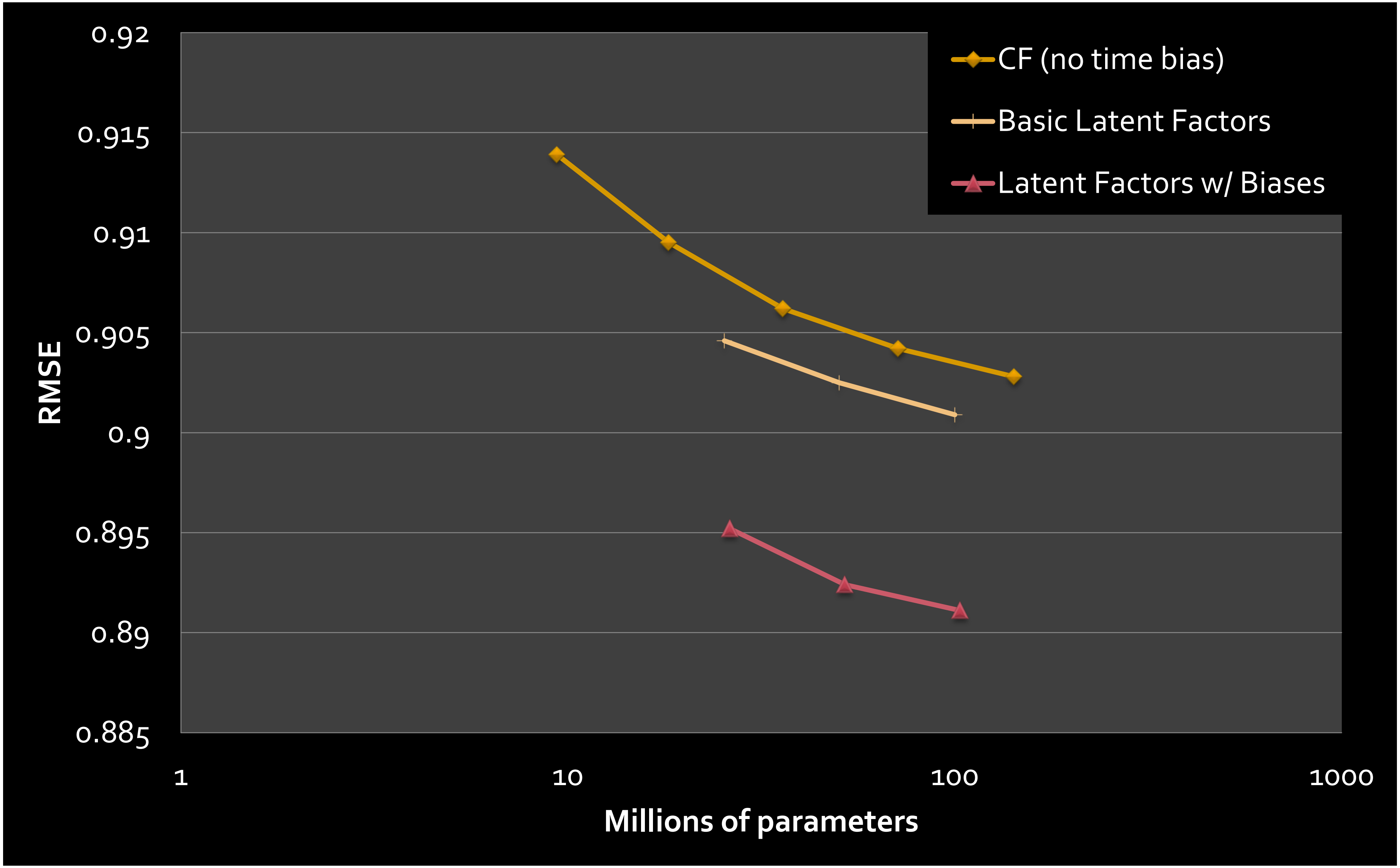
$$+ \left(\lambda_1 \sum_i \| q_i \|^2 + \lambda_2 \sum_x \| p_x \|^2 + \lambda_3 \sum_x \| b_x \|^2 + \lambda_4 \sum_i \| b_i \|^2 \right)$$

regularization

λ is selected via grid-search on a validation set

- **Stochastic gradient decent to find parameters**
  - **Note:** Both biases $\boldsymbol{b_x}$, $\boldsymbol{b_i}$ as well as interactions $\boldsymbol{q_i}$, $\boldsymbol{p_x}$ are treated as parameters (we estimate them)

Performance of Various Methods



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

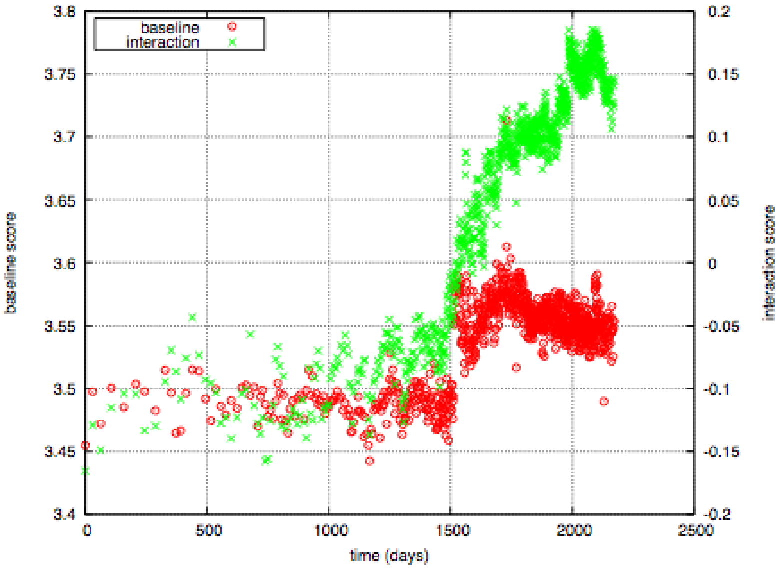
Latent factors+Biases: 0.89

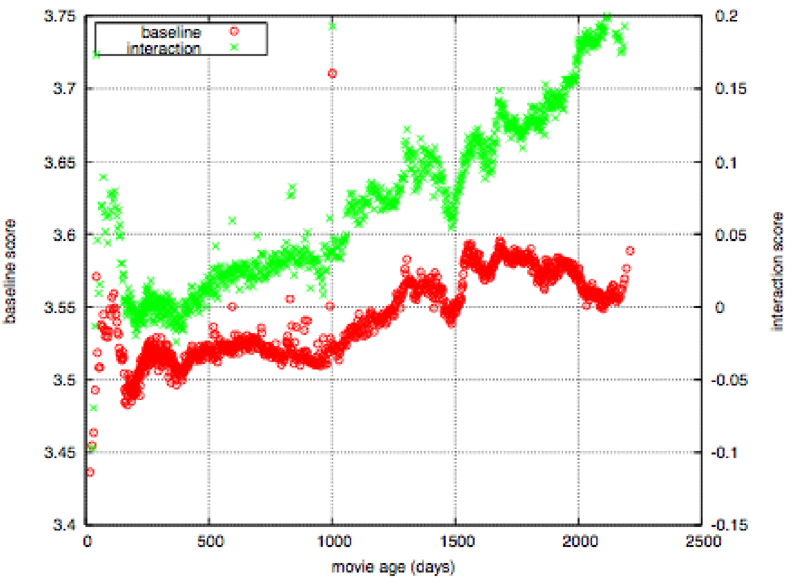
Grand Prize: 0.8563

The Netflix Challenge: 2006-09

Temporal Biases Of Users

- **Sudden rise in the average movie rating** (early 2004)
  - Improvements in Netflix
  - GUI improvements
  - Meaning of rating changed
- **Movie age**
  - Users prefer new movies without any reasons
  - Older movies are just inherently better than newer ones





Y. Koren, Collaborative filtering with temporal dynamics, KDD ’09

Temporal Biases & Factors

- **Original model:**

  $$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

- **Add time dependence to biases:**

  $$r_{xi} = \mu + b_x(t) + b_i(t) + q_i \cdot p_x$$

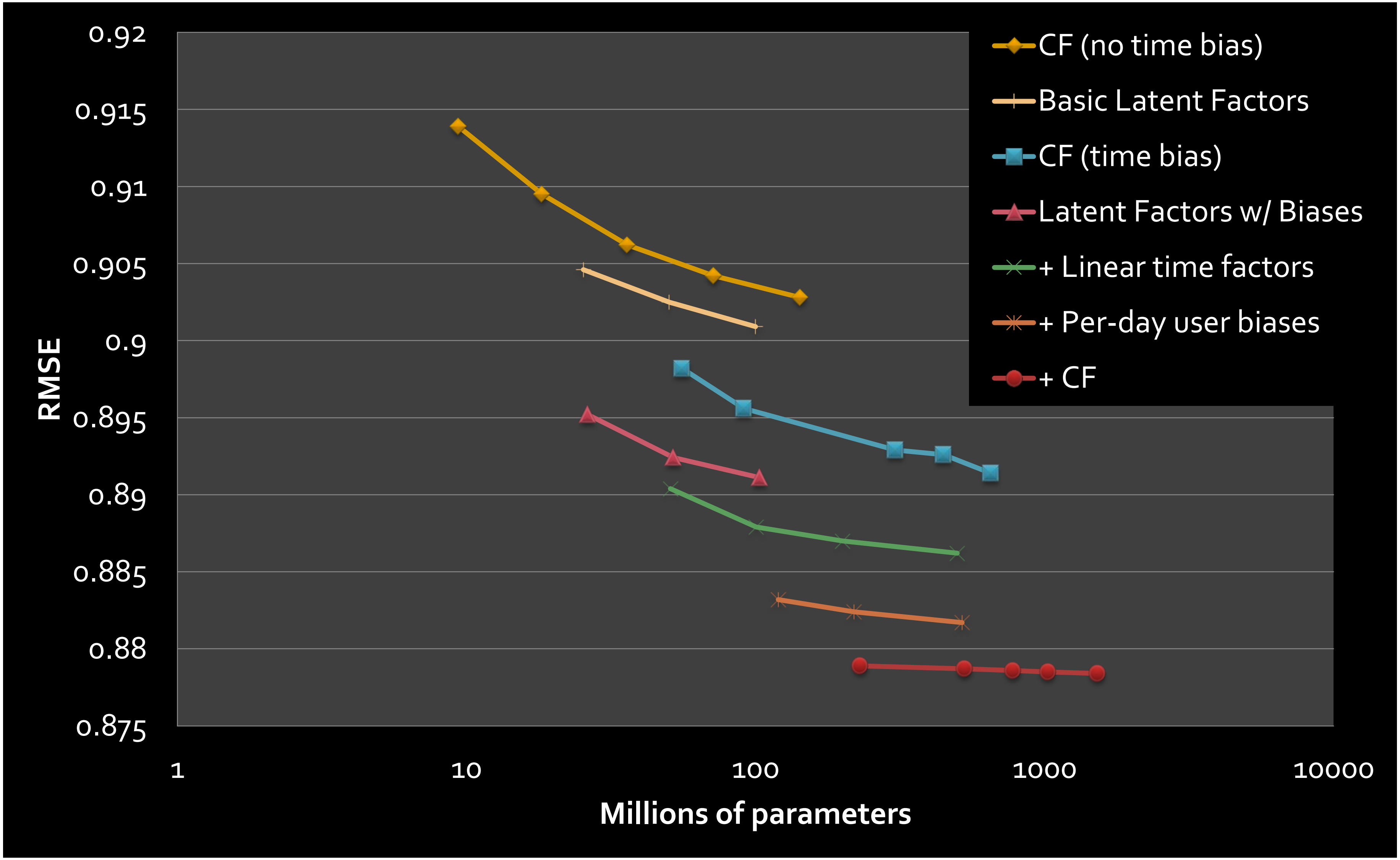
  - Make parameters b_x and b_i to depend on time
  - **(1)** Parameterize time-dependence by linear trends
    (2) Each bin corresponds to 10 consecutive weeks

    $$b_i(t) = b_i + b_{i,\mathrm{Bin}(t)}$$

- **Add temporal dependence to factors**
  - $p_x(t)$… user preference vector on day t

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09

Adding Temporal Effects



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Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

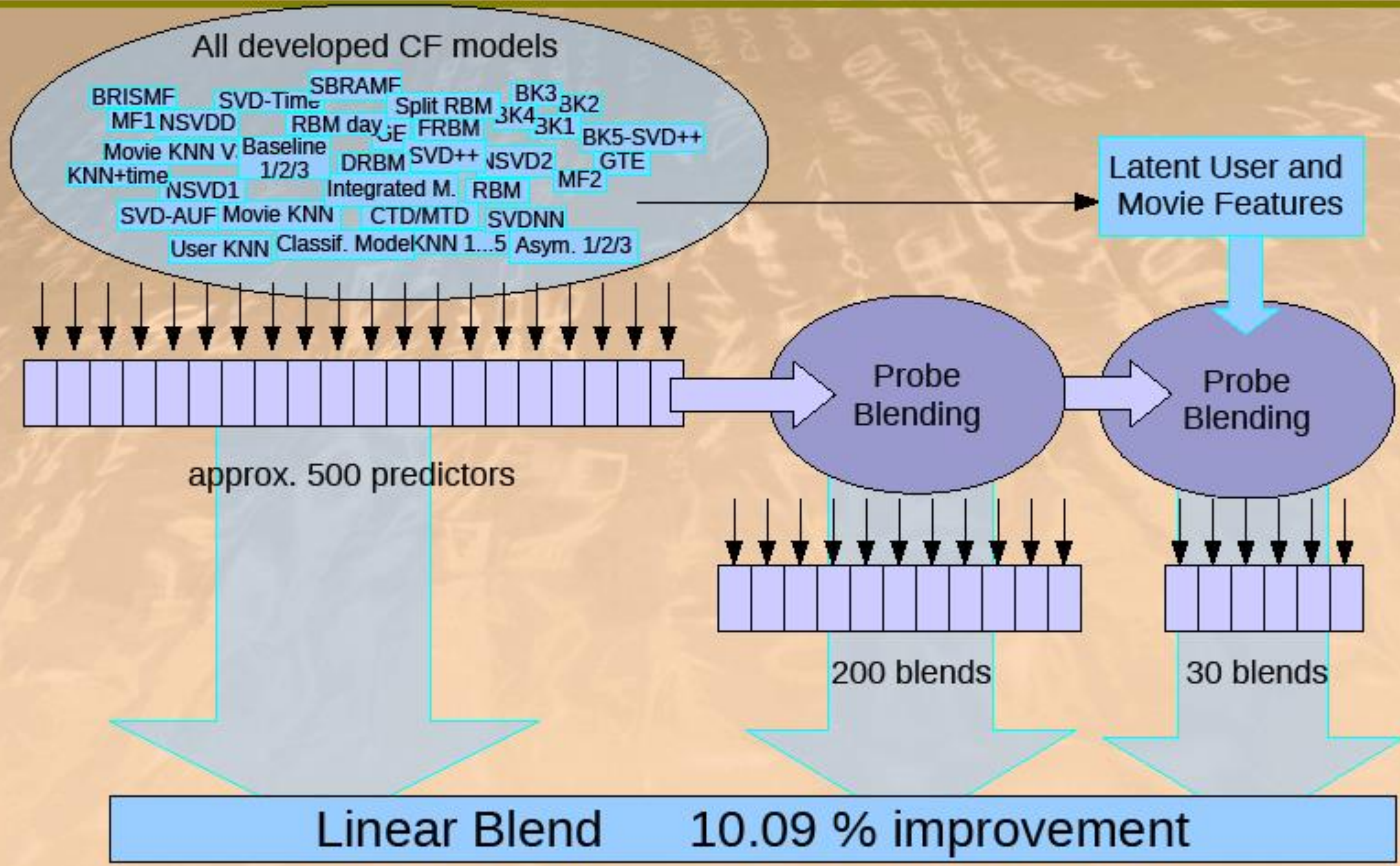
Latent factors+Biases: 0.89

Latent factors+Biases+Time: 0.876

Grand Prize: 0.8563

Still no prize! ☹
Getting desperate.
Try a "kitchen sink" approach!

The big picture
Solution of BellKor's Pragmatic Chaos



Michael Jahrer / Andreas Töscher – Team BigChaos – September 21, 2009

Standing on June 26th 2009

NETFLIX

Netflix Prize

Home | Rules | Leaderboard | Register | Update | Submit | Download

Leaderboard

Display top 20 leaders.

Rank	Team Name	Best Score	% Improvement	Last Submit Time
1	BellKor's Pragmatic Chaos	0.8558	10.05	2009-06-26 18:42:37
Grand Prize - RMSE <= 0.8563				
2	PragmaticTheory	0.8582	9.80	2009-06-25 22:15:51
3	BellKor in BigChaos	0.8590	9.71	2009-05-13 08:14:09
4	Grand Prize Team	0.8593	9.68	2009-06-12 08:20:24
5	Dace	0.8604	9.56	2009-04-22 05:57:03
6	BigChaos	0.8613	9.47	2009-06-23 23:06:52
Progress Prize 2008 - RMSE = 0.8616 - Winning Team: BellKor in BigChaos				
7	BellKor	0.8620	9.40	2009-06-24 07:16:02
8	Gravity	0.8634	9.25	2009-04-22 18:31:32
9	Opera Solutions	0.8638	9.21	2009-06-26 23:18:13
10	BruceDengDaoCiYiYou	0.8638	9.21	2009-06-27 00:55:55
11	pengpengzhou	0.8638	9.21	2009-06-27 01:06:43
12	xlvector	0.8639	9.20	2009-06-26 13:49:04
13	xiangliang	0.8639	9.20	2009-06-26 07:47:34

June 26th submission triggers 30-day "last call"

The Last 30 Days

- **Ensemble team formed**
  - Group of other teams on leaderboard forms a new team
  - Relies on combining their models
  - Quickly also get a qualifying score over 10%
- **BellKor**
  - Continue to get small improvements in their scores
  - Realize that they are in direct competition with Ensemble
- **Strategy**
  - Both teams carefully monitoring the leaderboard
  - Only sure way to check for improvement is to submit a set of predictions
    - This alerts the other team of your latest score

24 Hours from the Deadline

- **Submissions limited to 1 a day**
  - Only 1 final submission could be made in the last 24h
- **24 hours before deadline…**
  - **BellKor** team member in Austria notices (by chance) that **Ensemble** posts a score that is slightly better than BellKor's
- **Frantic last 24 hours for both teams**
  - Much computer time on final optimization
  - Carefully calibrated to end about an hour before deadline
- **Final submissions**
  - **BellKor** submits a little early (on purpose), 40 mins before deadline
  - **Ensemble** submits their final entry 20 mins later
  - **….and everyone waits….**

NETFLIX

Netflix Prize

COMPLETED

Home | Rules | Leaderboard | Update | Download

Leaderboard

Showing Test Score. Click here to show quiz score

Display top 20 leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.8582	9.90	2009-07-10 21:24:40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace_	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11
Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos				
13	xiangliang	0.8642	9.27	2009-07-15 14:53:22
14	Gravity	0.8643	9.26	2009-04-22 18:31:32
15	Ces	0.8651	9.18	2009-06-21 19:24:53
16	Invisible Ideas	0.8653	9.15	2009-07-15 15:53:04
17	Just a guy in a garage	0.8662	9.06	2009-05-24 10:02:54
18	J Dennis Su	0.8666	9.02	2009-03-07 17:16:17
19	Craig Carmichael	0.8666	9.02	2009-07-25 16:00:54
20	acmehill	0.8668	9.00	2009-03-21 16:20:50

Progress Prize 2007 - RMSE = 0.8723 [illegible]

Million $ Awarded Sept 21st 2009

Acknowledgments

- Some slides and plots borrowed from Yehuda Koren, Robert Bell and Padhraic Smyth
- **Further reading:**
  - Y. Koren, Collaborative filtering with temporal dynamics, KDD '09
  - http://www2.research.att.com/~volinsky/netflix/bpc.html
  - http://www.the-ensemble.com/