Network Effects and Cascading Behavior

Thanks to Jure Leskovec, Stanford and Panayiotis Tsaparas, Univ. of Ioannina for slides

Agenda

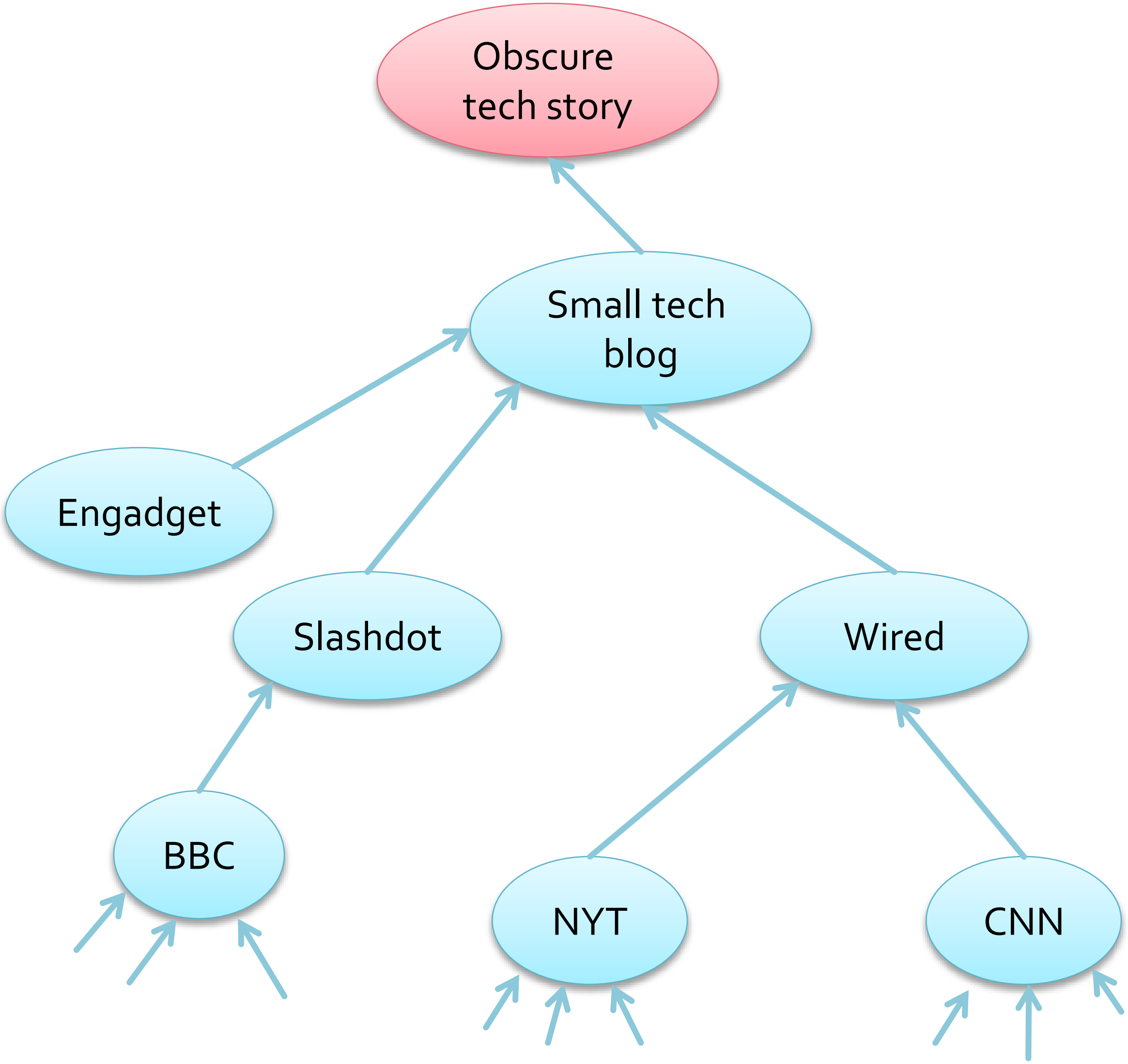
- Spreading Through Networks
- Granovetter's Model of Collective Action
- Decision Based Model of Diffusion
  - Game Theoretic Model of Cascades

Spreading Through Networks

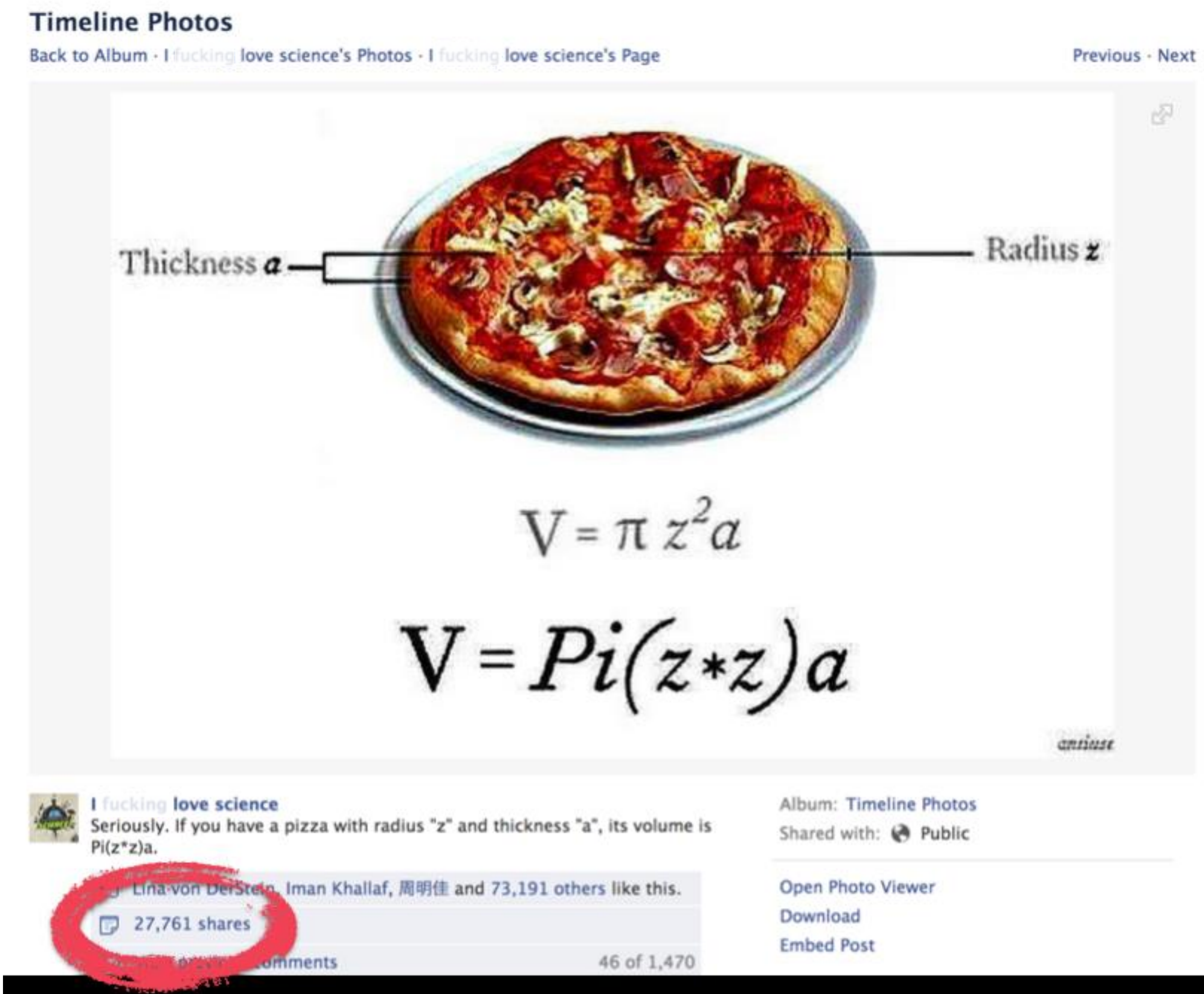
- **Spreading through networks:**
  - Cascading behavior
  - Diffusion of innovations
  - Network effects
  - Epidemics
- **Behaviors that cascade from node to node like an epidemic**

- **Examples:**
  - **Biological:**
    - Diseases via contagion
  - **Technological:**
    - Cascading failures
    - Spread of information
  - **Social:**
    - Rumors, news, new technology
    - Viral marketing

Information Diffusion: Media



Twitter & Facebook post sharing


Timeline Photos
Back to Album · I fucking love science's Photos · I fucking love science's Page
Previous · Next
Thickness a
Radius z
V = π z²a
V = Pi(z*z)a
I fucking love science
Seriously. If you have a pizza with radius "z" and thickness "a", its volume is Pi(z*z)a.
Lina von DerStein, Iman Khallaf, 周明佳 and 73,191 others like this.
27,761 shares
omments
46 of 1,470
Album: Timeline Photos
Shared with: Public
Open Photo Viewer
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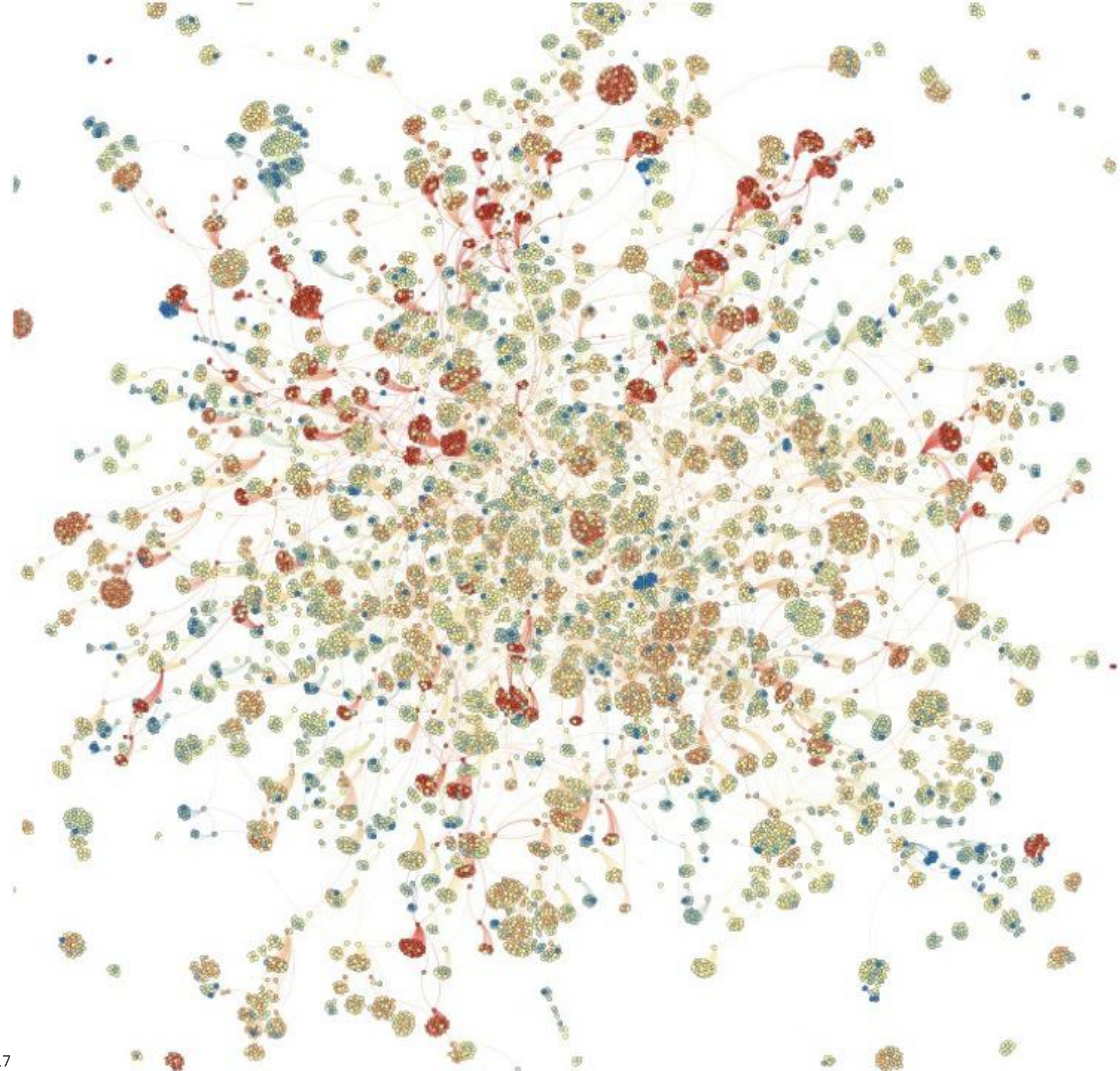
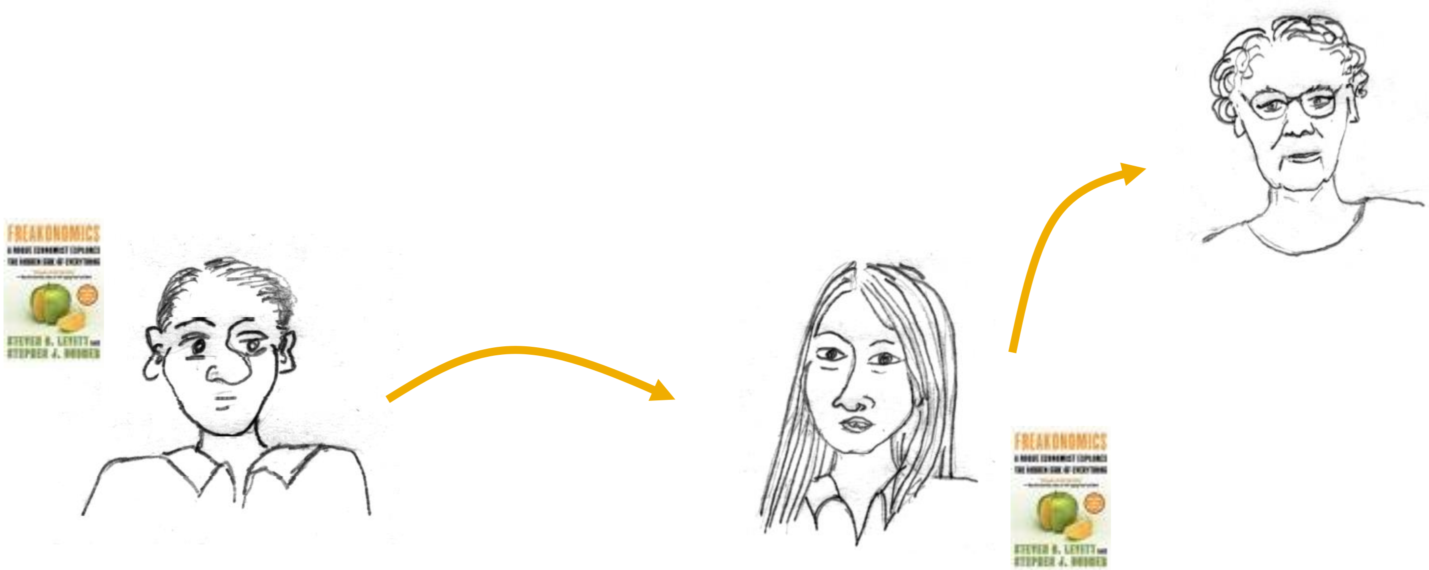
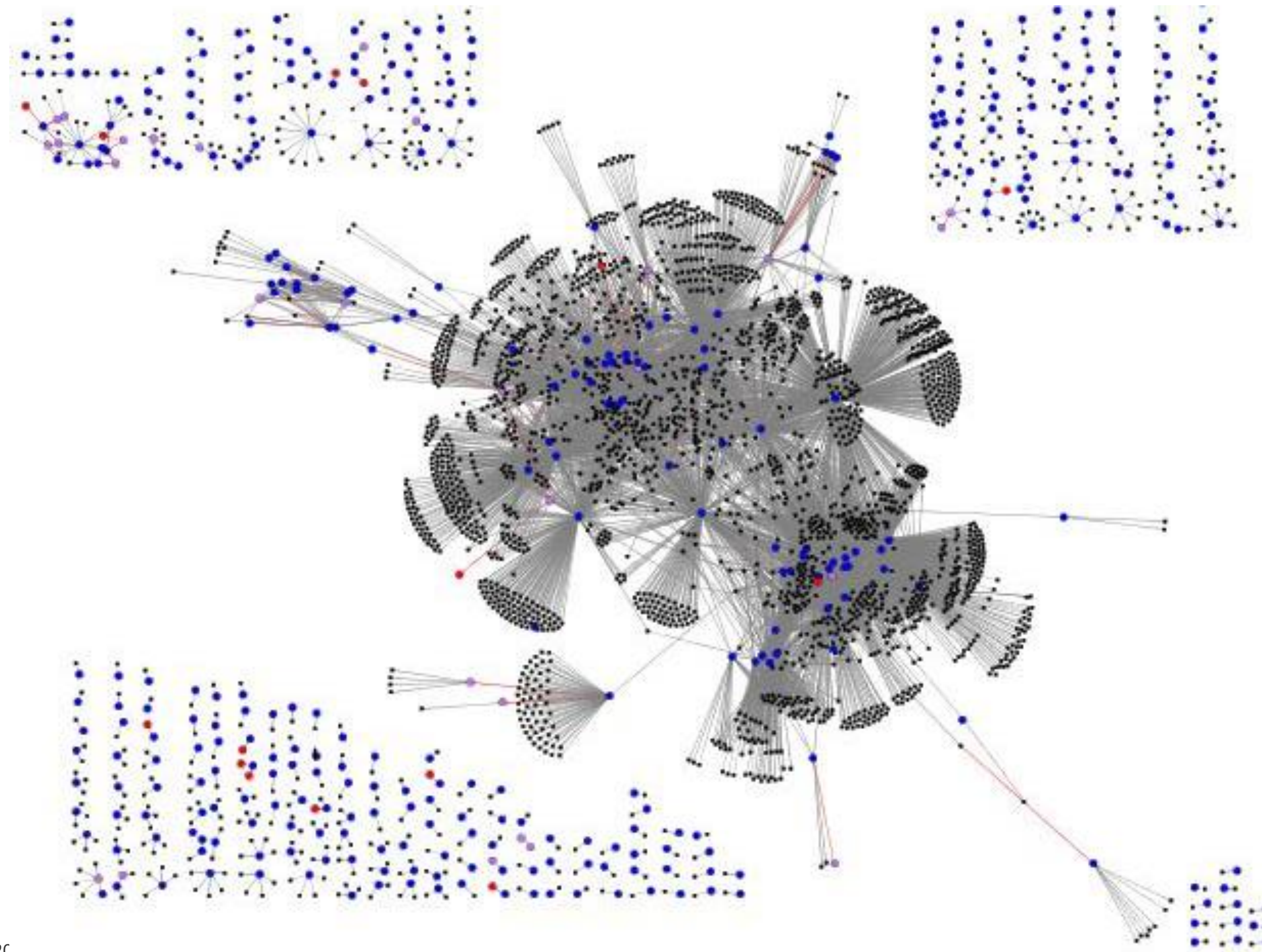


Diffusion in Viral Marketing

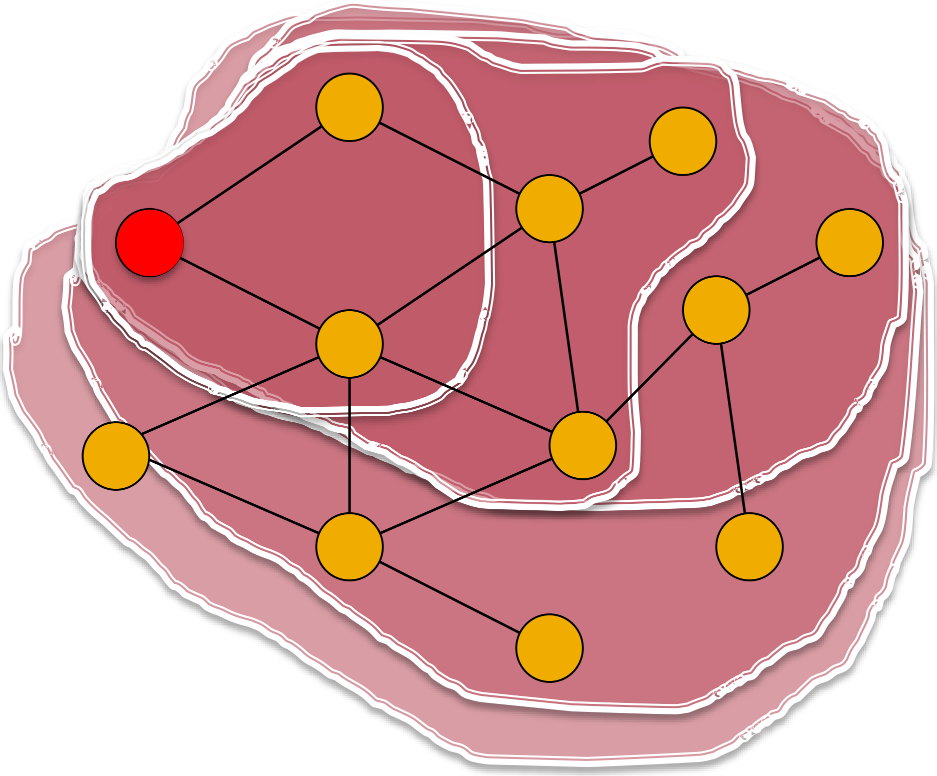
- **Product adoption:**
  - **Senders and followers of recommendations**



Diffusion in Viral Marketing

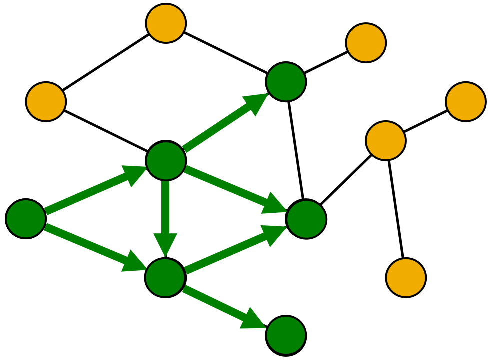


Spread of Diseases (e.g., Ebola)

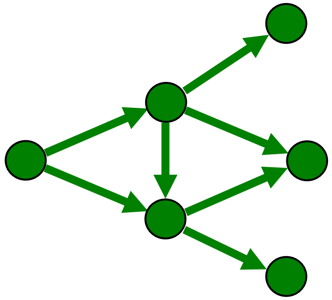


Network Cascades

- **Contagion that spreads over the edges of the network**
- It creates a propagation tree, i.e., **cascade**



Network

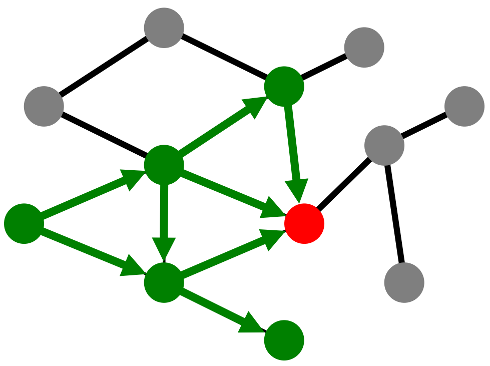


Cascade
(propagation graph)

Terminology:
- Stuff that spreads: Contagion
- "Infection" event: Adoption, infection, activation
- We have: Infected/active nodes, adoptors

How Do We Model Diffusion?

- **Decision based models (today!):**
  - Models of product adoption, decision making
    - A node observes decisions of its neighbors and makes its own decision
  - **Example:**
    - You join demonstrations if *k* of your friends do so too
- **Probabilistic models (later):**
  - **Models of influence or disease spreading**
    - An infected node tries to "push" the contagion to an uninfected node
  - **Example:**
    - You "catch" a disease with some prob. from each active neighbor in the network



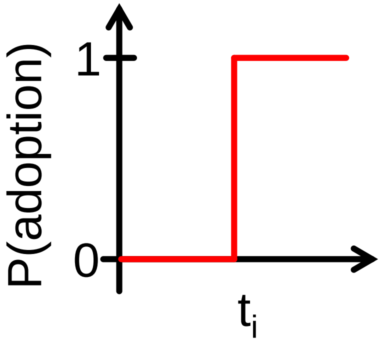
Granovetter's Model of Collective Action

Decision Based Models

- **Collective Action** [Granovetter, '78]
  - **Model where everyone sees everyone else's behavior (that is, we assume a complete graph)**
  - **Examples:**
    - Clapping or getting up and leaving in a theater
    - Keeping your money or not in a stock market
    - Neighborhoods in cities changing ethnic composition
    - Riots, protests, strikes
- **How does the number of people participating in a given activity grow or shrink over time?**

Collective Action: The Model

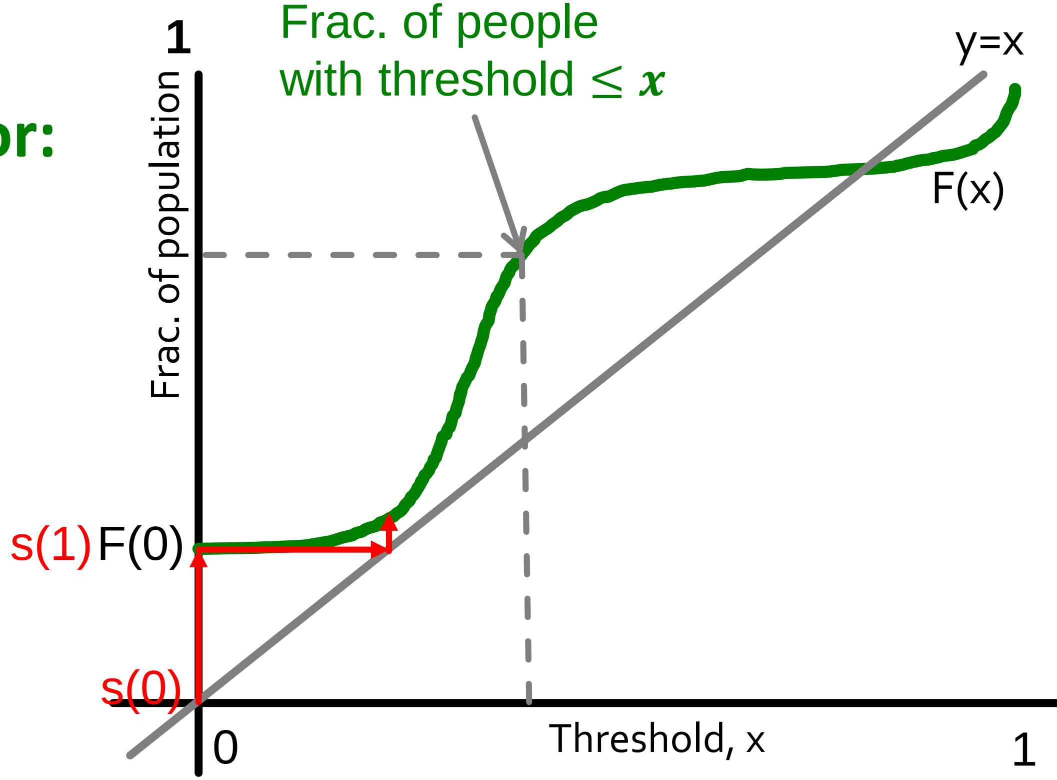
- ***n* people – everyone observes all actions**
- Each person ***i*** has a threshold t_i $(0 \leq t_i \leq 1)$
  - Node ***i*** will adopt the behavior iff at least t_i fraction of people have already adopted:
    - **Small t_i**: early adopter
    - **Large t_i**: late adopter
  - Time moves in discrete steps



- **The population is described by $\{t_1,\ldots,t_n\}$**
  - **F(x)** … fraction of people with threshold $t_i \leq x$
    - F(x) is a property of the contagion given to us. F(x) is the **c.d.f.** of *x*

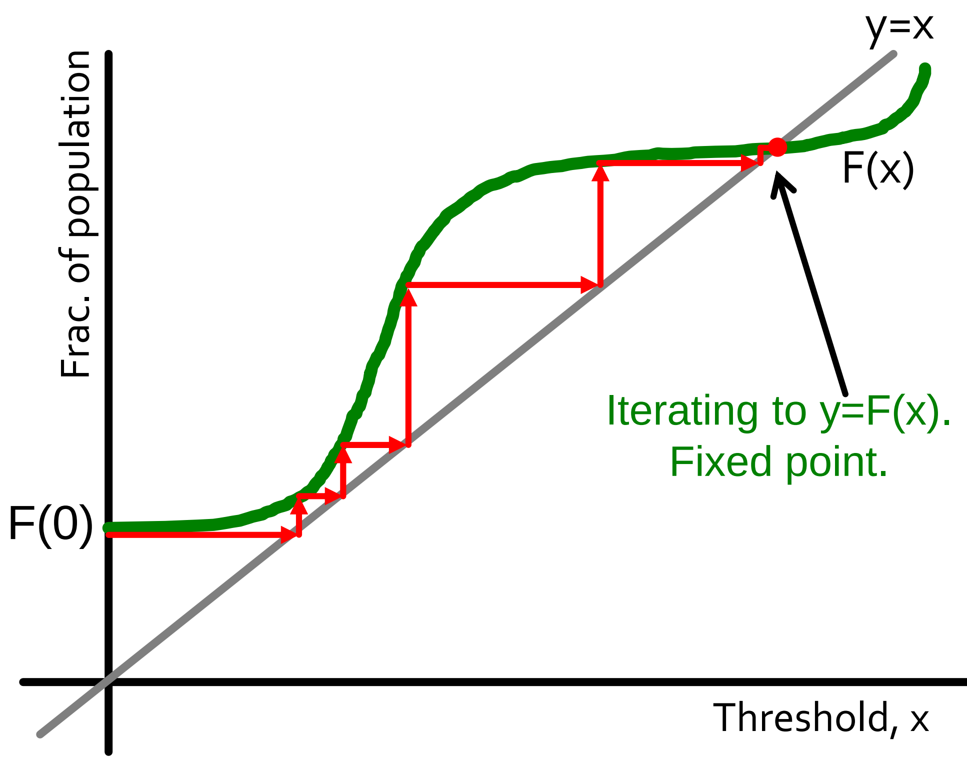
Collective Action: Dynamics

- **F(x)** … fraction of people with threshold $t_i \leq x$
  - **F(x) is non-decreasing:** $F(x+\varepsilon) \geq F(x)$
- **The model is dynamic:**
  - **Step-by-step change in number of people adopting the behavior:**
    - **F(x)** … frac. of people with threshold ≤ **x**
    - **s(t)** … frac. of people participating at time **t**
  - **Simulate:**
    - s(0) = 0
    - s(1) = F(0)
    - s(2) = F(s(1)) = F(F(0))



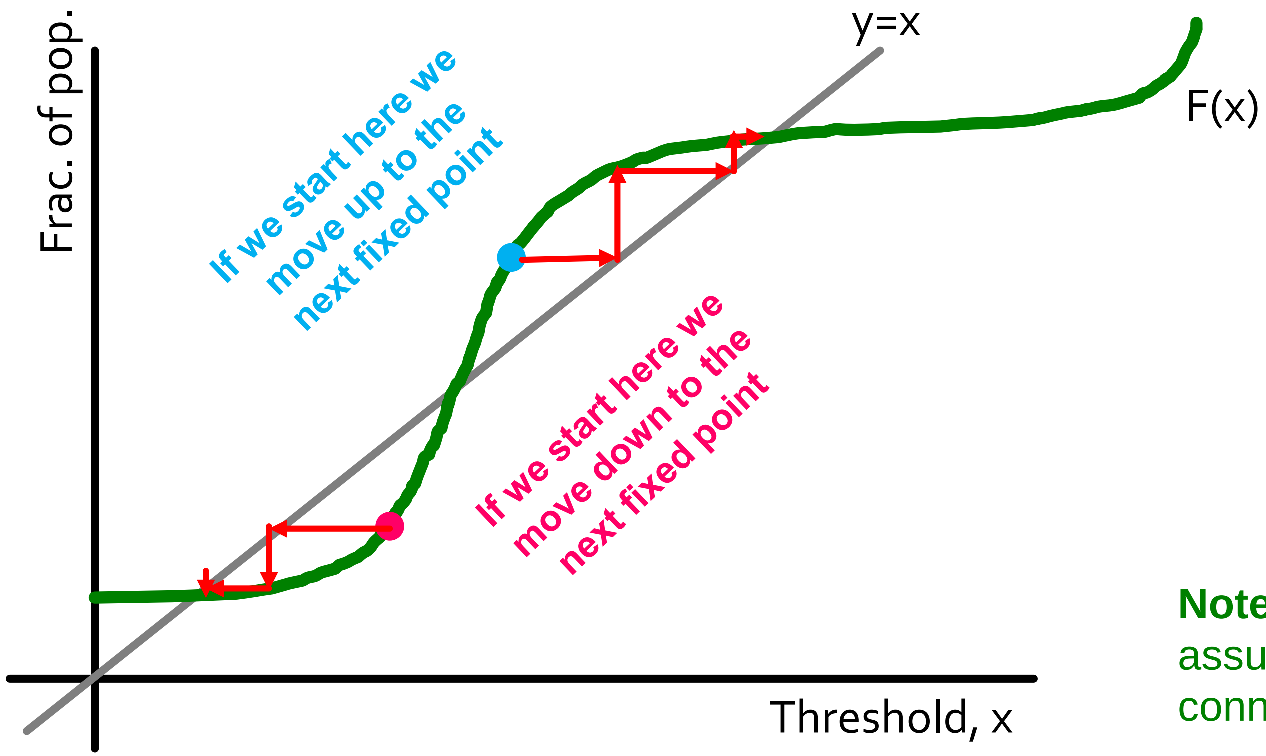
Collective Action: Dynamics

- **Step-by-step change in number of people :**
  - **F(x)** … fraction of people with threshold ≤ **x**
  - **s(t)** … number of participants at time **t**
- **Easy to simulate:**
  - $s(0) = 0$
  - $s(1) = F(0)$
  - $s(2) = F(s(1)) = F(F(0))$
  - $s(t+1) = F(s(t)) = F^{t+1}(0)$
- **Fixed point: F(x)=x**
  - Updates to **s(t)** to converge to a stable fixed point
  - There could be other fixed points but starting from **0** we only reach the first one



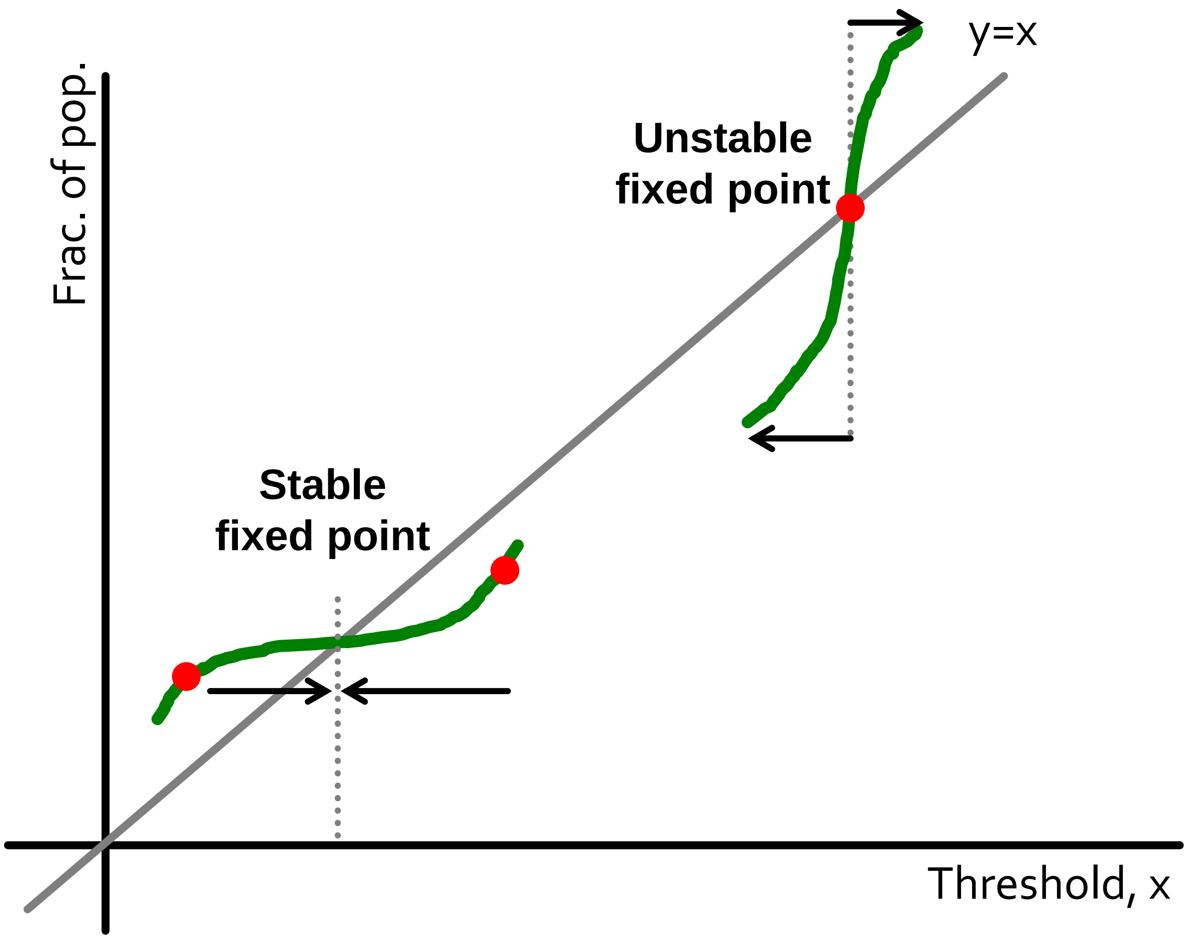
Starting Elsewhere

- **What if we start the process somewhere else?**
  - We move up/down to the next fixed point
  - **How is market going to change?**



Note: we are assuming a fully connected graph

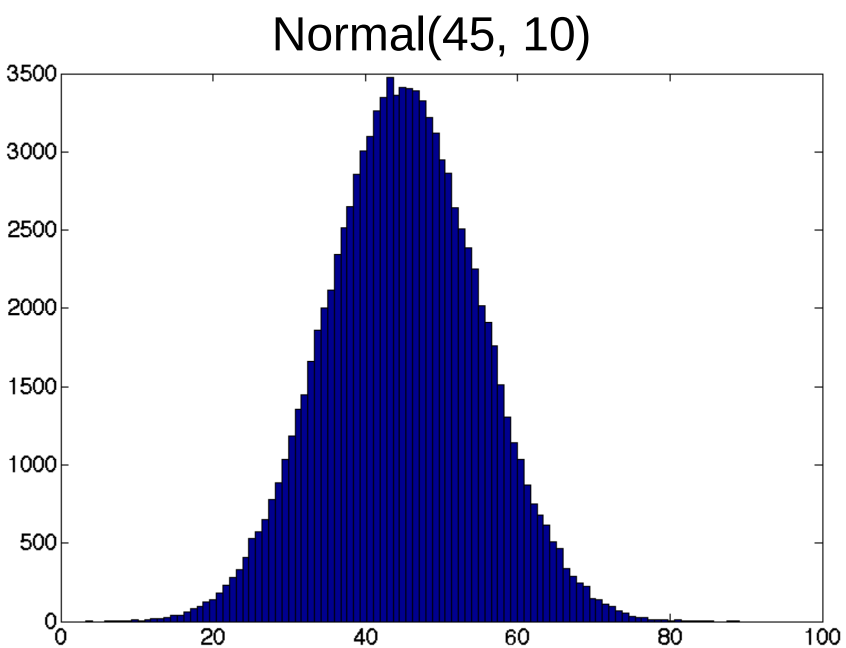
Stable vs. Unstable Fixed Point



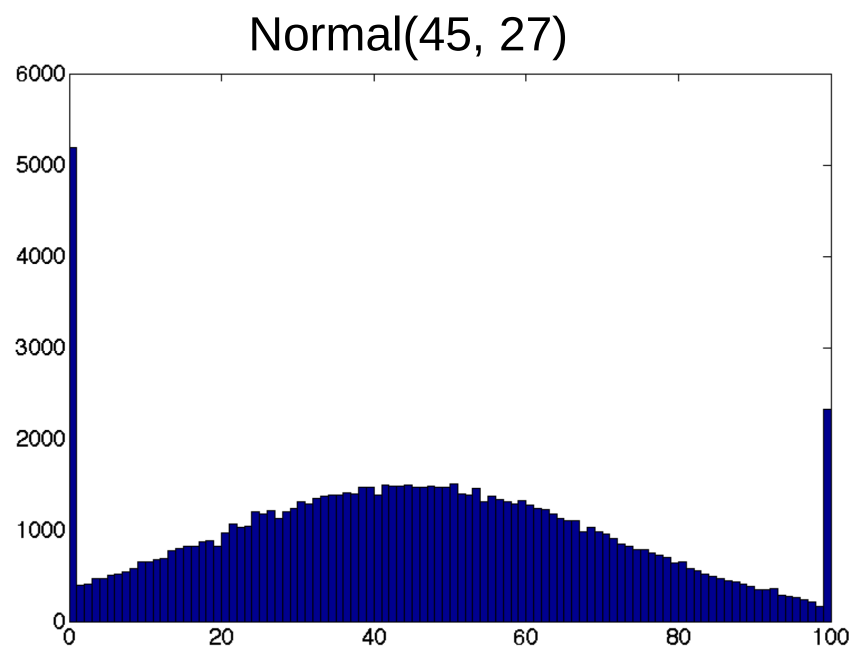
Discontinuous Transition

- Each threshold t_i is drawn independently from some distribution ***F(x) = Pr[thresh ≤ x]***
  - **Suppose:** Normal with **μ=n/2**, variance **σ**

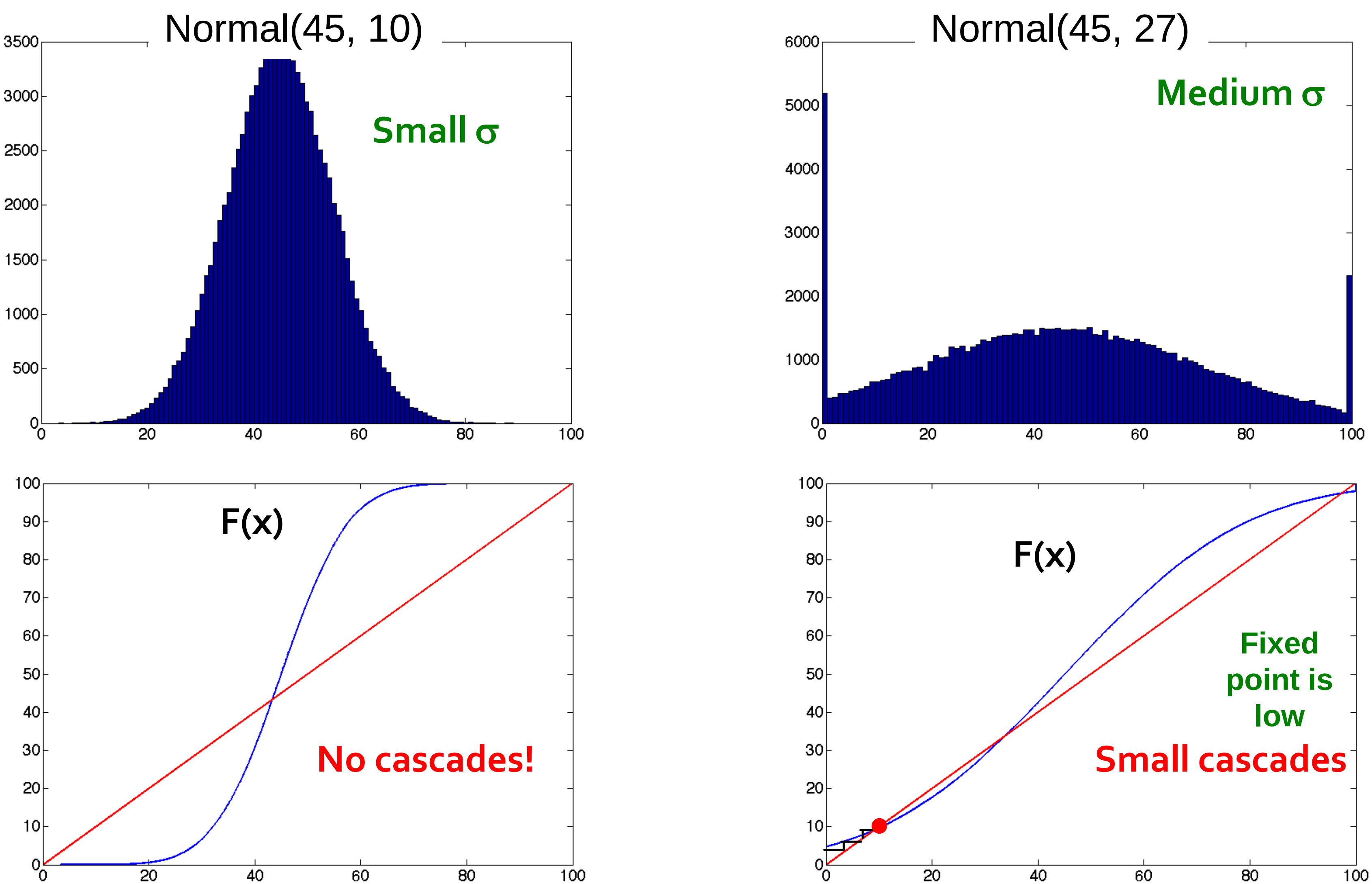
Small σ:



Large σ:

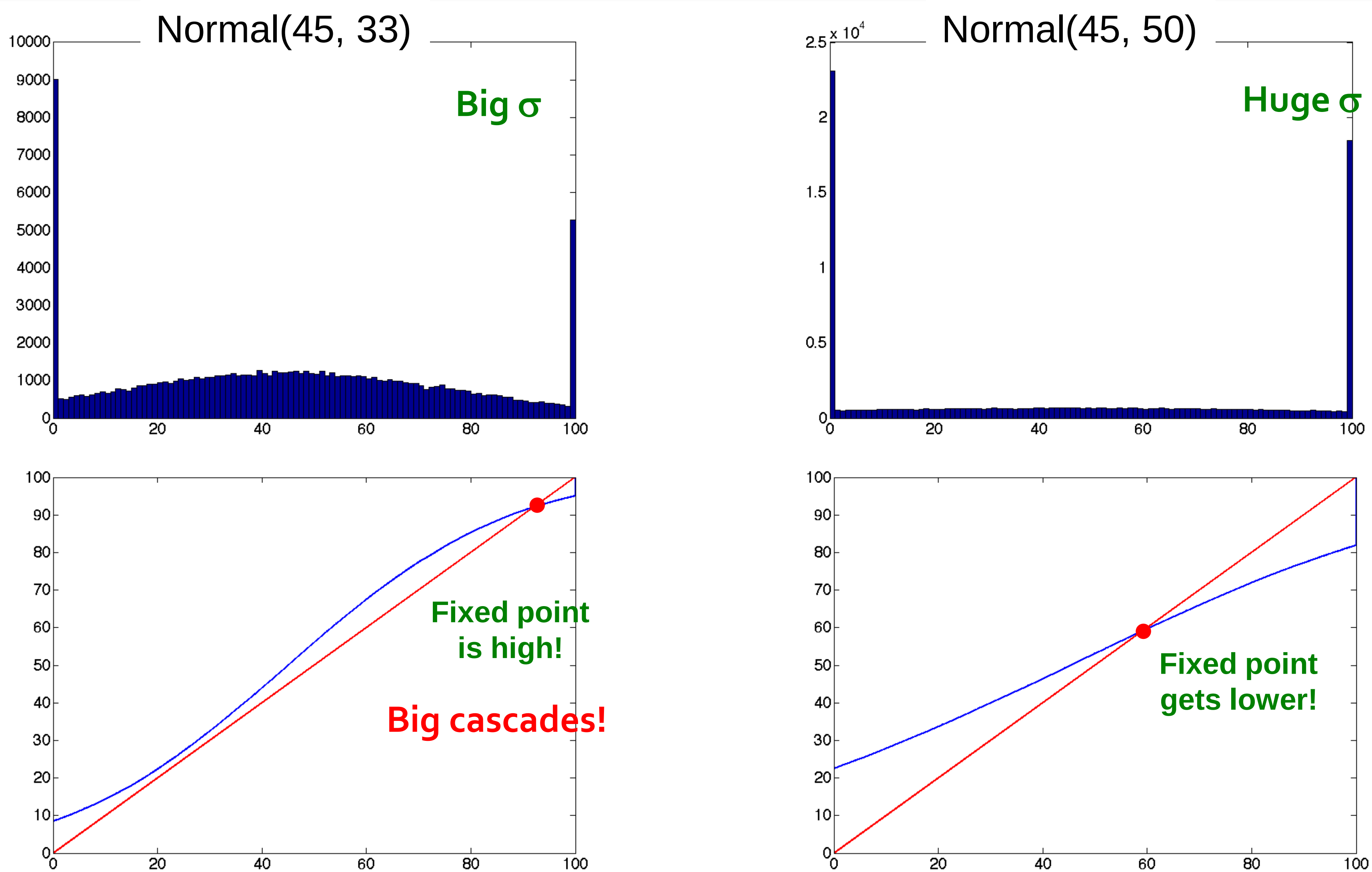


Discontinuous Transition



Bigger variance let's you build a bridge from early adopters to mainstream

Discontinuous Transition

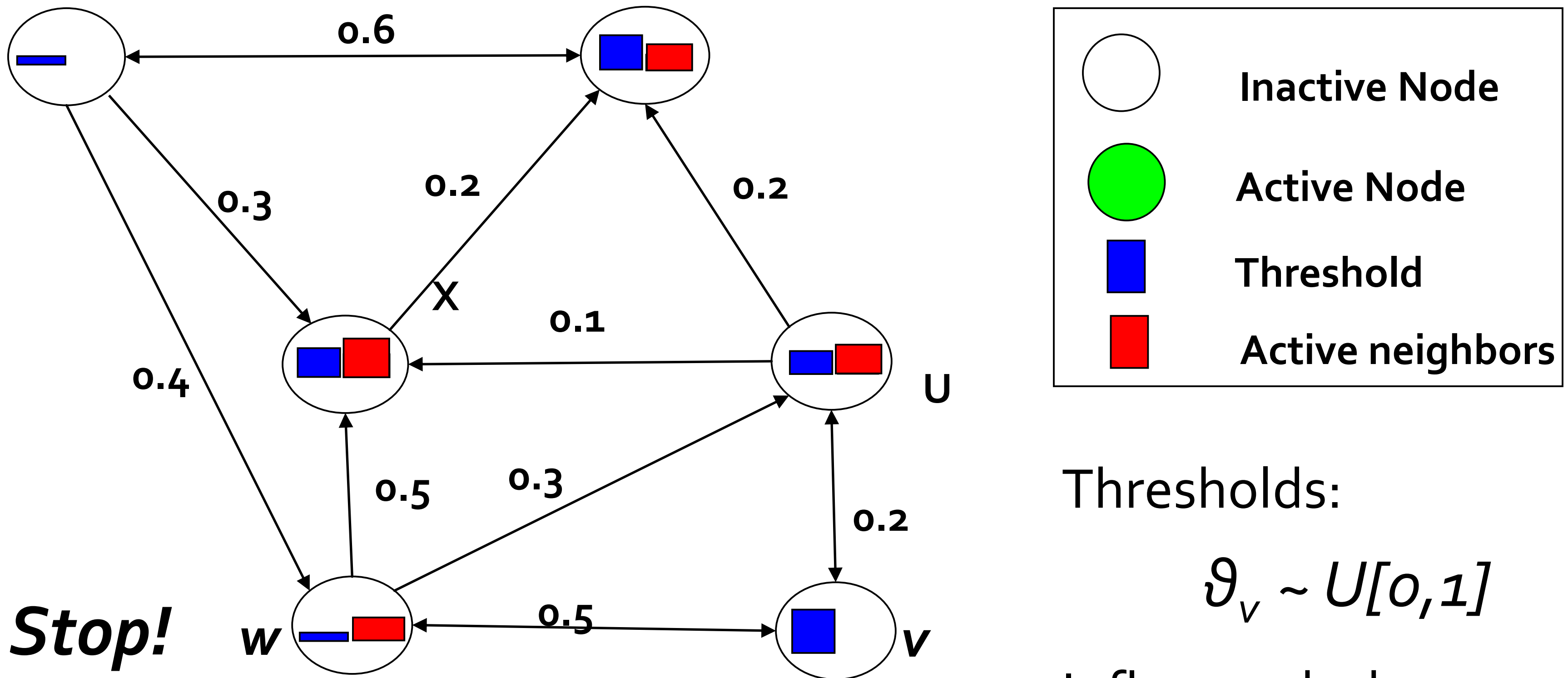


But if we increase the variance the fixed point starts going down

Weaknesses of the Model

- **No notion of social network:**
  - Some people are more influential
  - It matters who the early adopters are, not just how many
- **Models people's awareness** of size of participation **not just actual number of people participating**
  - Modeling perceptions of who is adopting the behavior vs. who you believe is adopting
  - Non-monotone behavior – dropping out if too many people adopt
  - People get "locked in" to certain choice over a period of time
- **Modeling thresholds**
  - Richer distributions
  - Deriving thresholds from more basic assumptions
    - game theoretic models

Linear Threshold Model



Example

Thresholds:

$$\vartheta_v \sim U[0,1]$$

Influenced when:

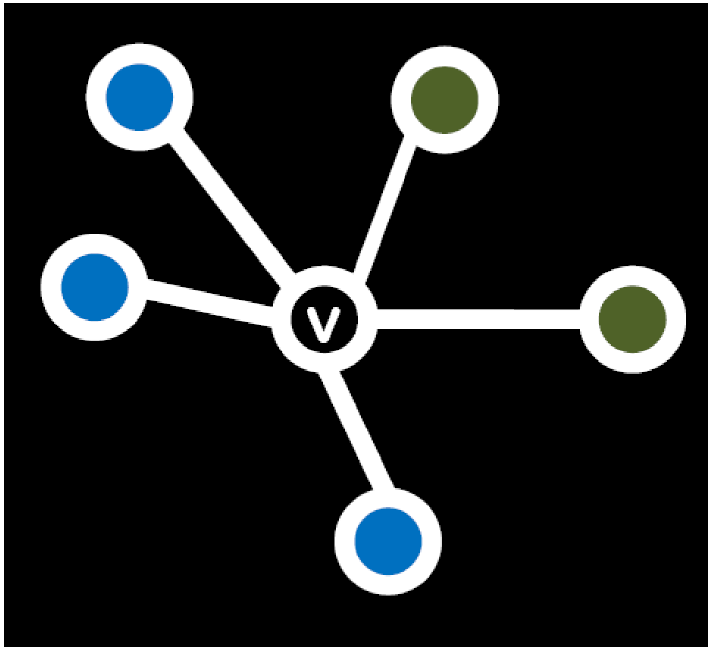
$$\sum_{w \text{ active neighbor of } v} b_{v,w} \geq \theta_v$$

Decision Based Model of Diffusion

[Morris 2000]

Game Theoretic Model of Cascades

- **Based on 2 player coordination game**
  - 2 players – each chooses technology A or B
  - Each person can only adopt **one** "behavior", **A** or **B**
  - You gain more payoff if your friend has adopted the **same** behavior as you



Local view of the network of node **v**

Example: VHS vs. BetaMax

Example: BlueRay vs. HD DVD

Blu-ray Disc

HD DVD

The Model for Two Nodes

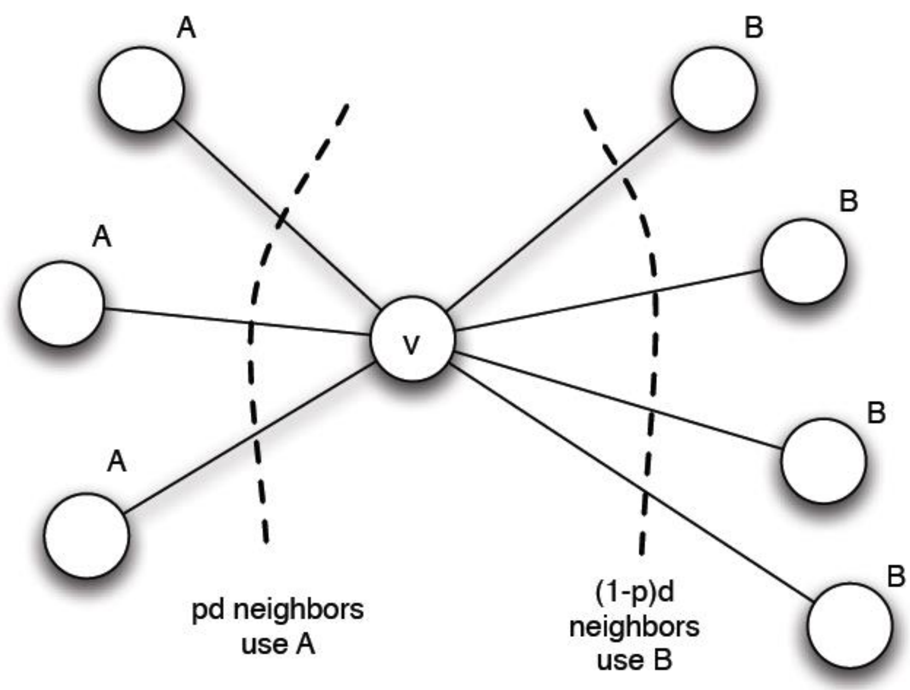
- ***Payoff matrix:***
  - If both **v** and **w** adopt behavior **A**, they each get payoff $a > 0$
  - If ***v*** and ***w*** adopt behavior ***B***, they each get payoff $b > 0$
  - If ***v*** and ***w*** adopt the opposite behaviors, they each get ***0***
- **In some large network:**
  - Each node ***v*** is playing a copy of the game with each of its neighbors
  - **Payoff**: sum of node payoffs per game

w

v

	A	B
A	a, a	o,o
B	o,o	b,b

Calculation of Node v



Threshold:
v chooses ***A*** if

$$p > \frac{b}{a+b} = q$$

p… frac. v's nbrs. with A
q… payoff threshold

- Let **v** have ***d*** neighbors
- Assume fraction ***p*** of ***v***'s neighbors adopt ***A***
  - *Payoff$_v$ = a·p·d* , if *v* chooses A
    = b·(1-p)·d , if *v* chooses B
- **Thus: *v* chooses A if: *a·p·d > b·(1-p)·d***

Example Scenario

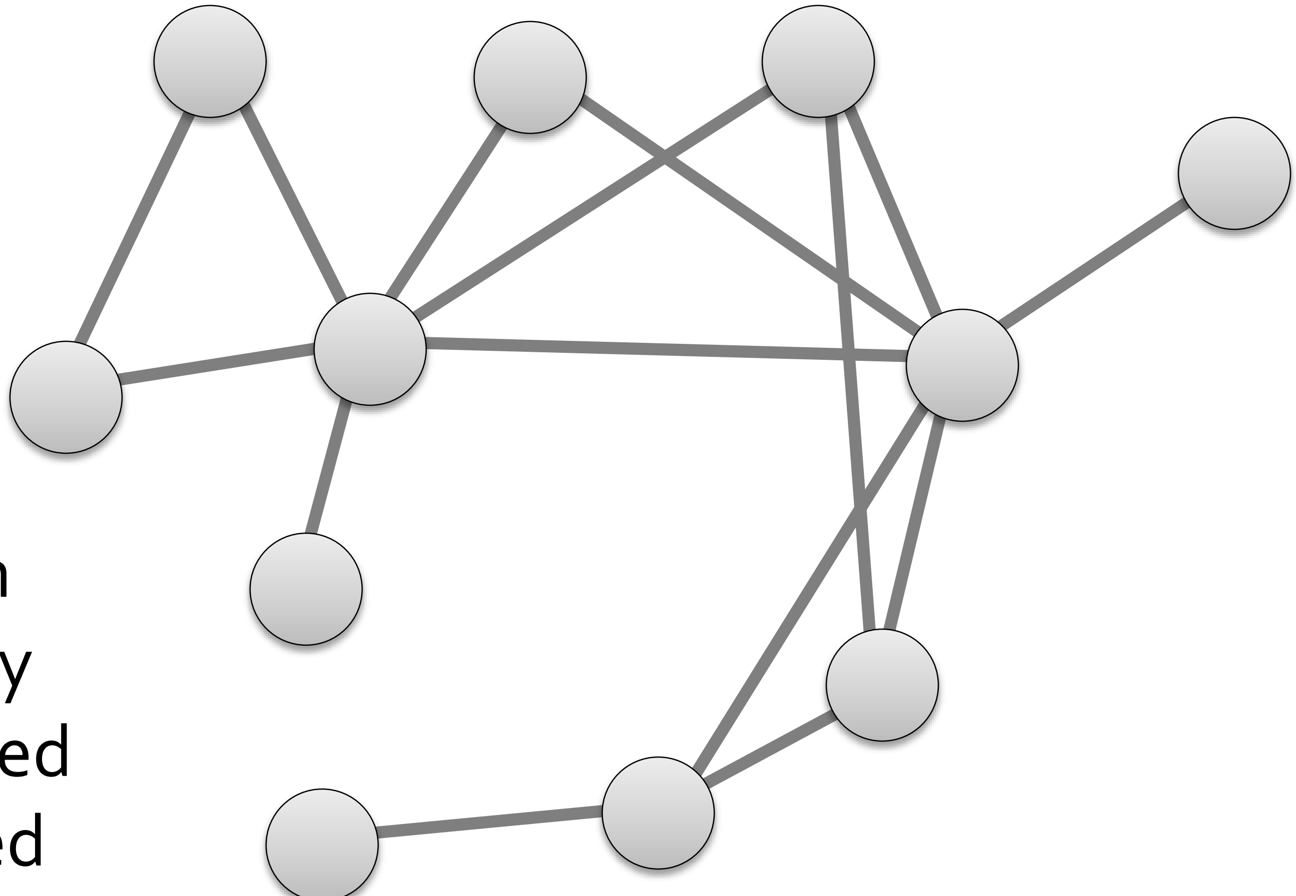
- **Scenario:**
  Graph where everyone starts with ***B***
  Small set ***S*** of early adopters of ***A***
  - Hard-wire ***S*** – they keep using ***A*** no matter what payoffs tell them to do

- **Assume payoffs are set in such a way that nodes say:**
  If more than 50% of my friends take A
  I'll also take A

  (this means: **a = b-ε** and **q>1/2**)

Example Scenario

$S = \{u, v\}$



If **more** than **q=**50% of my friends are red I'll also be red

Example Scenario

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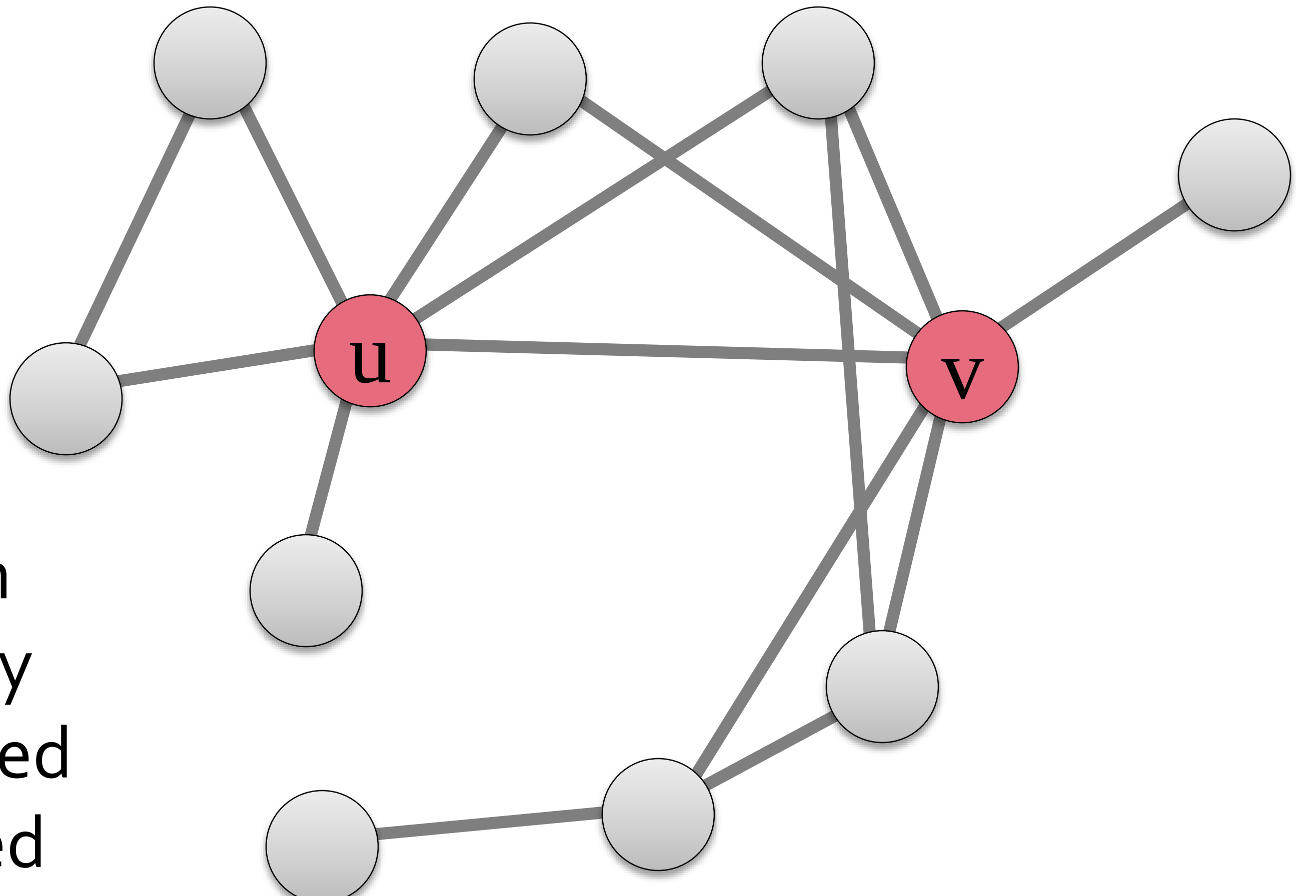

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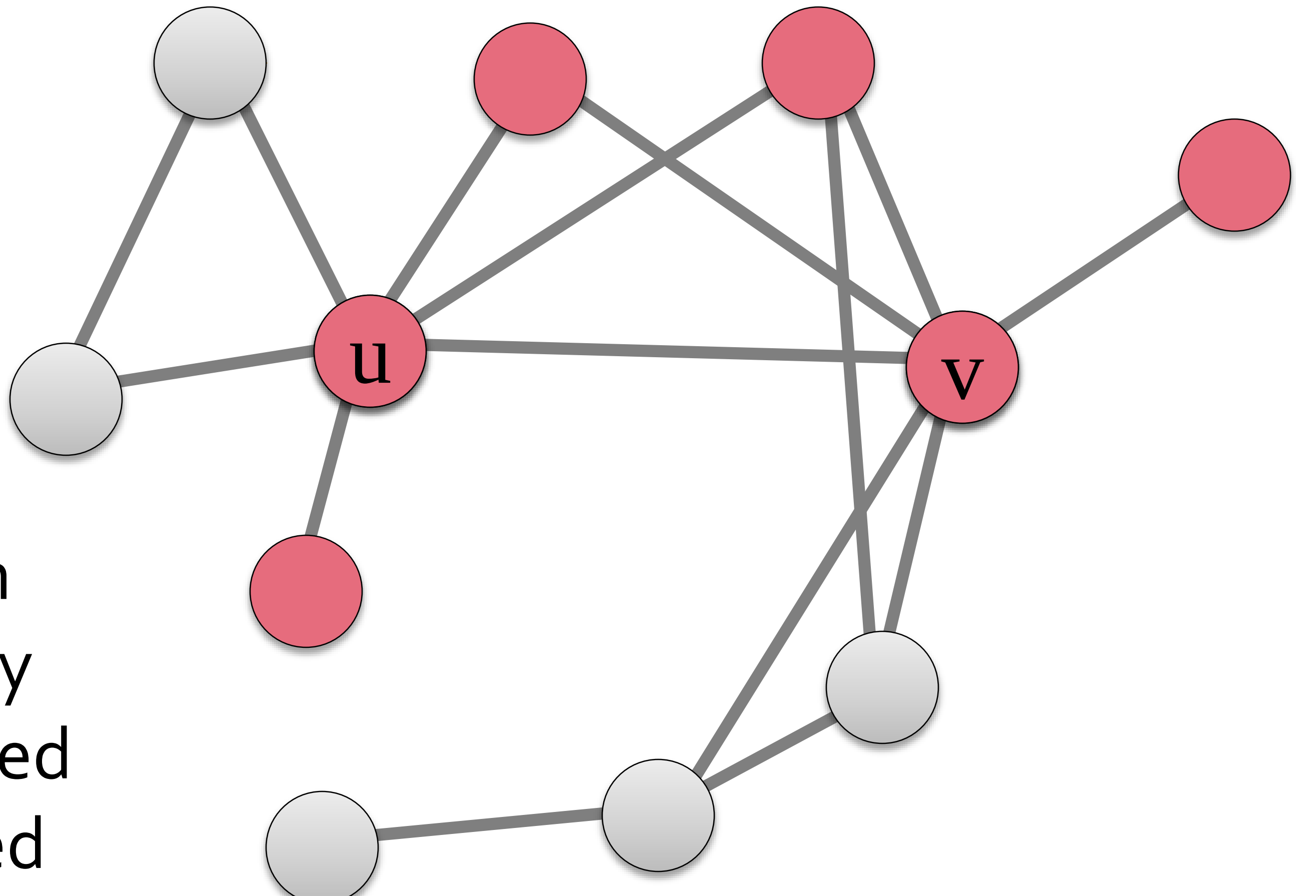

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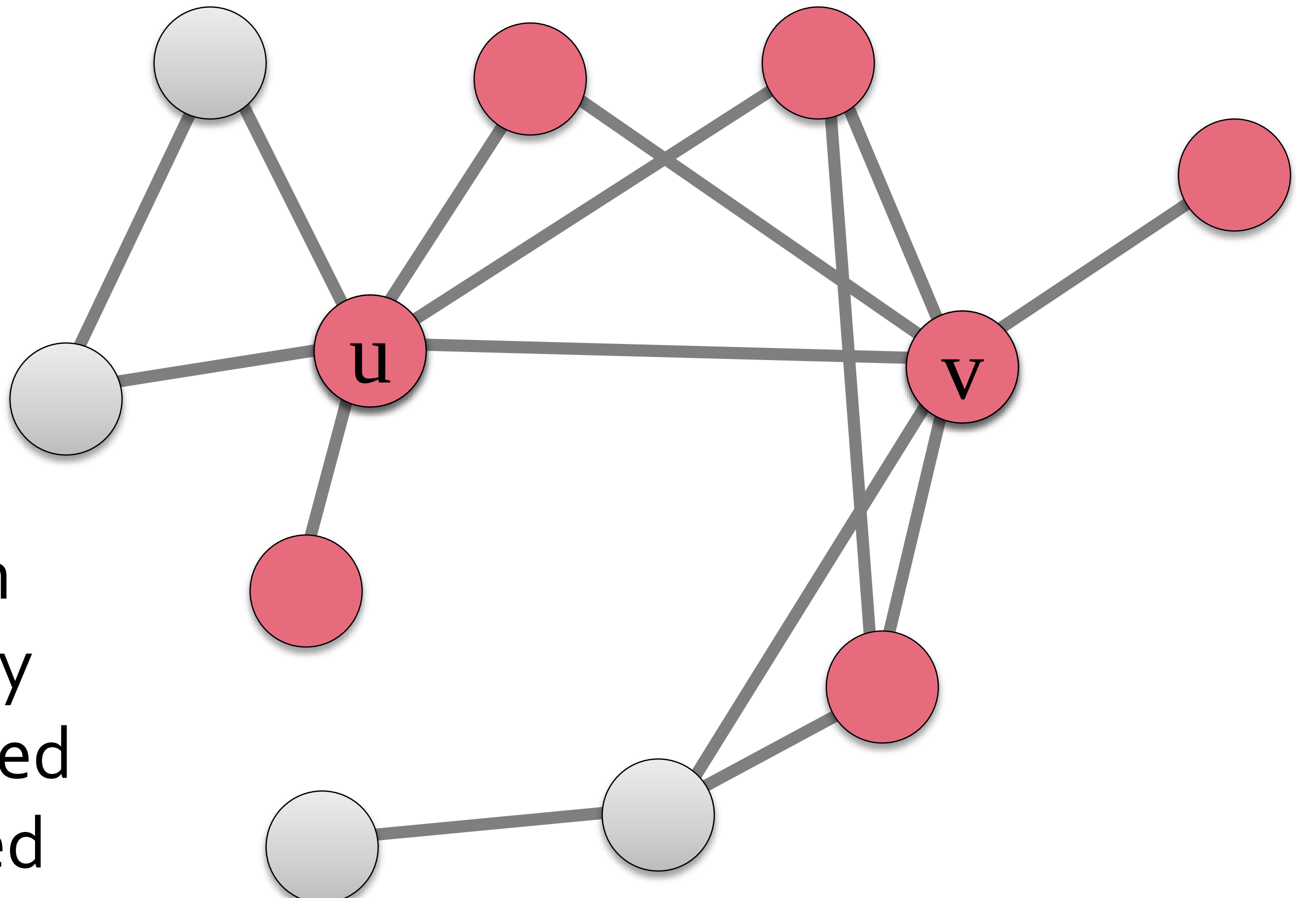

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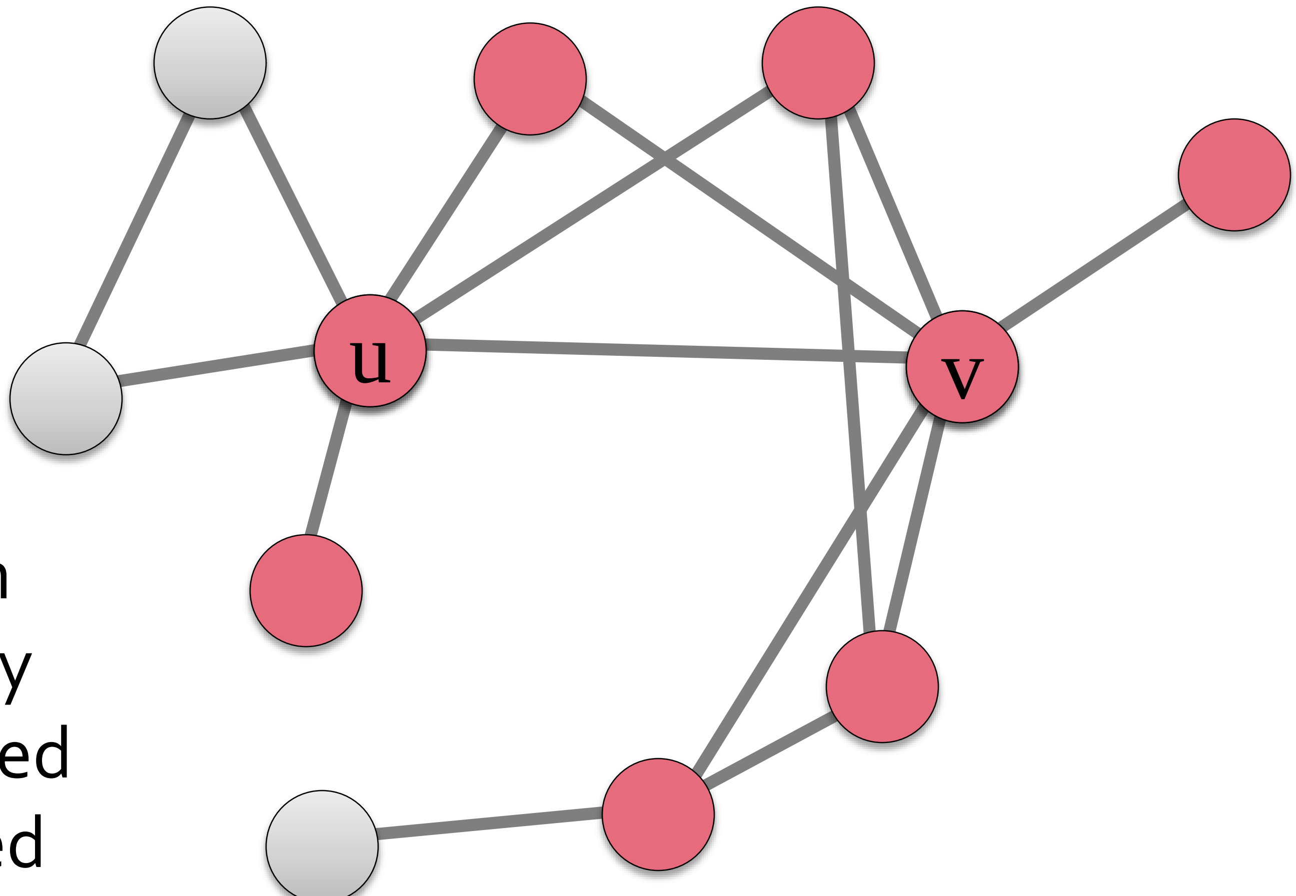

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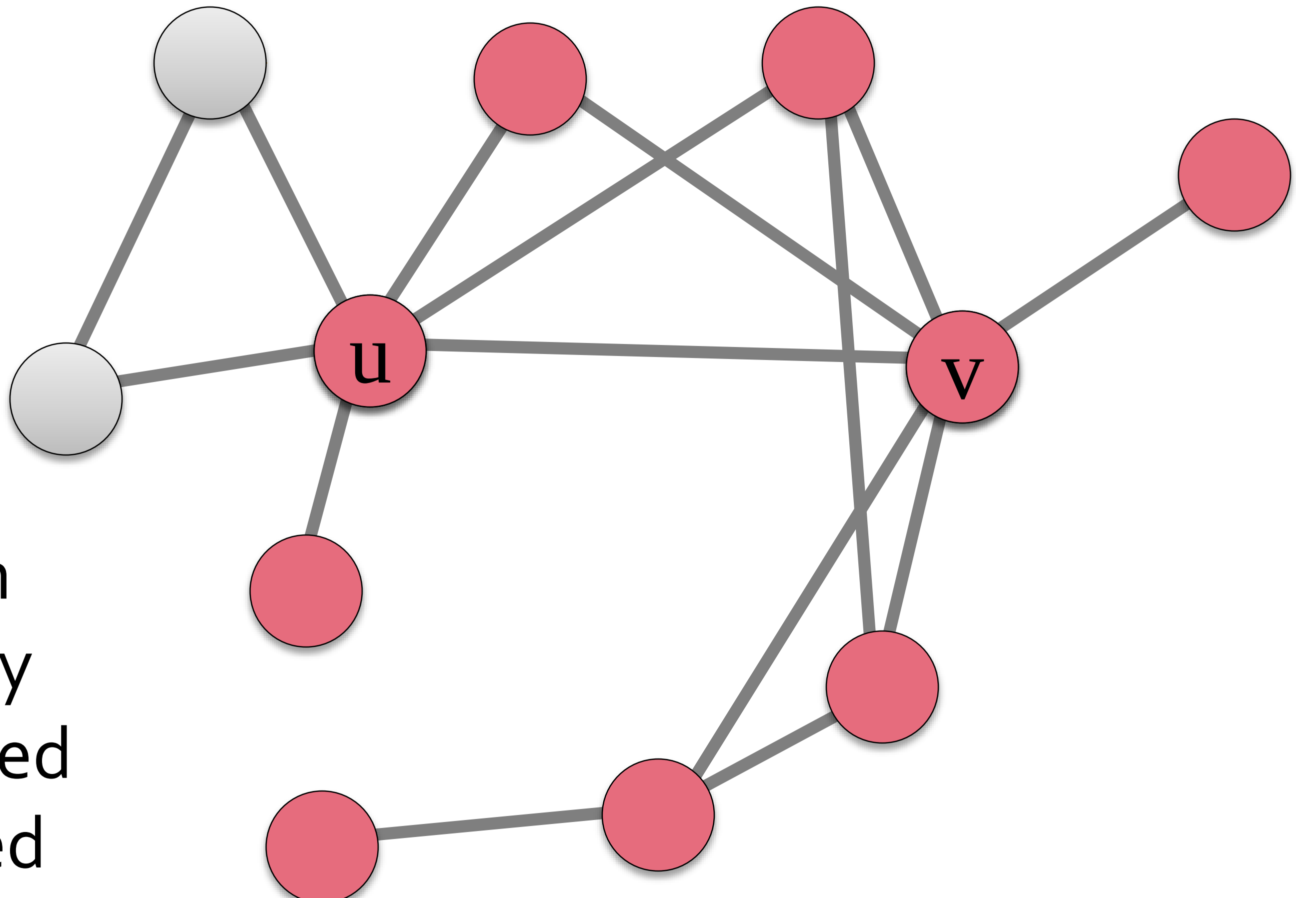


If **more** than **q=**50% of my friends are red I'll also be red

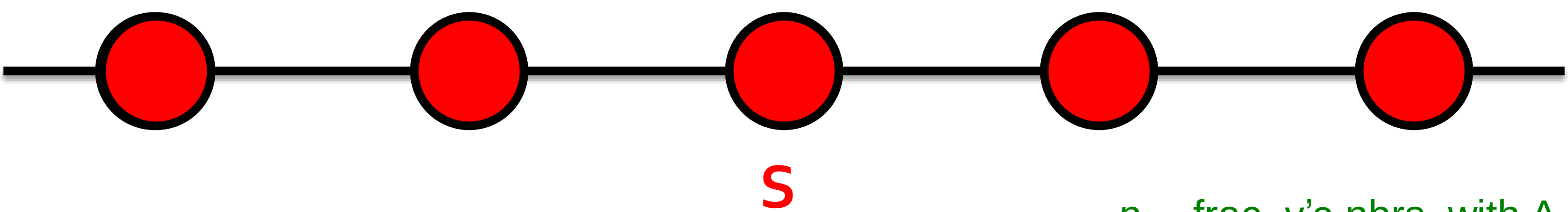
Infinite Graphs

v chooses *A* if *p>q*

$$q = \frac{b}{a+b}$$

- **Consider <u>infinite</u> graph *G***
  - (but each node has finite number of neighbors!)
- We say that a finite set ***S*** causes a cascade in *G* with **threshold *q*** if, when ***S*** adopts ***A***, eventually **every node in G adopts *A***
- Example: **Path**

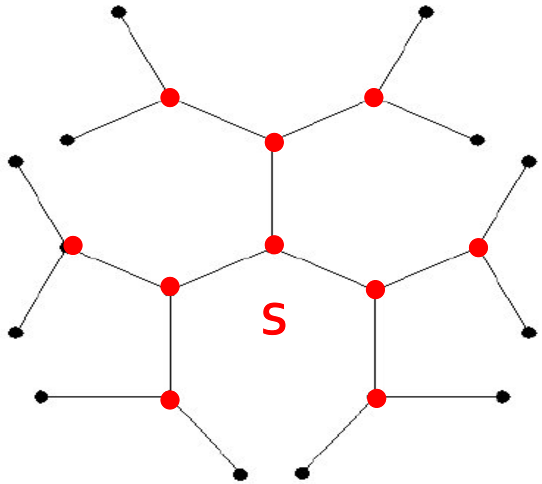
If ***q<1/2*** then cascade occurs



p… frac. v's nbrs. with A
q… payoff threshold

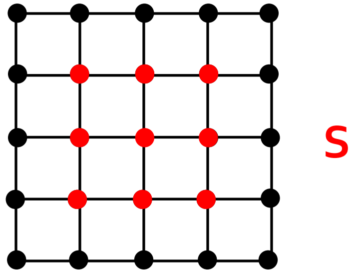
Infinite Graphs

- **Infinite Tree:**



If *q<1/3* then cascade occurs

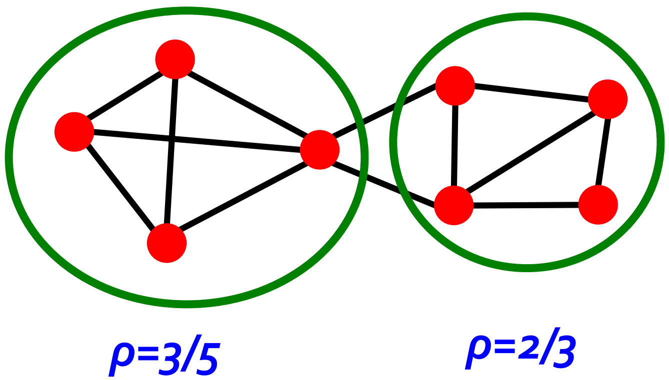
- **Infinite Grid:**



If *q<1/4* then cascade occurs

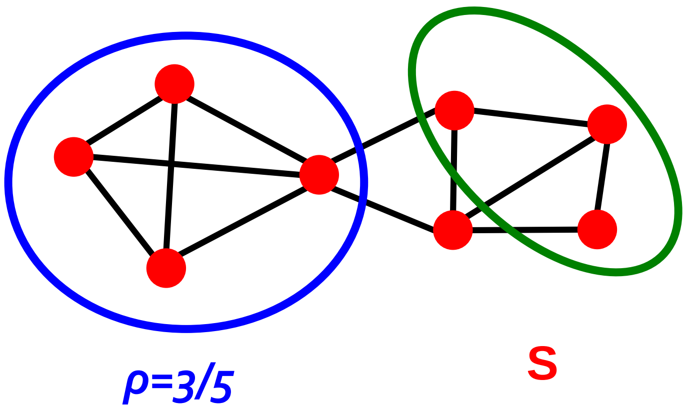
Stopping Cascades

- **What prevents cascades from spreading?**
- Def: **Cluster of density *ρ*** is a **set of nodes *C*** where each node in the set has at least ***ρ*** fraction of edges in ***C***



Stopping Cascades

- **Let S be an initial set of adopters of A**
- All nodes apply threshold ***q*** to decide whether to switch to **A**
- **Two facts:**
  - 1) If ***G\S*** contains a cluster of density ***>(1-q)*** then ***S*** can not cause a cascade
  - 2) If **S** fails to create a cascade, then there is a cluster of density ***>(1-q)*** in **G\S**



No cascade if q>2/5

Extending the Model: Allow People to Adopt A and B

Cascades & Compatibility

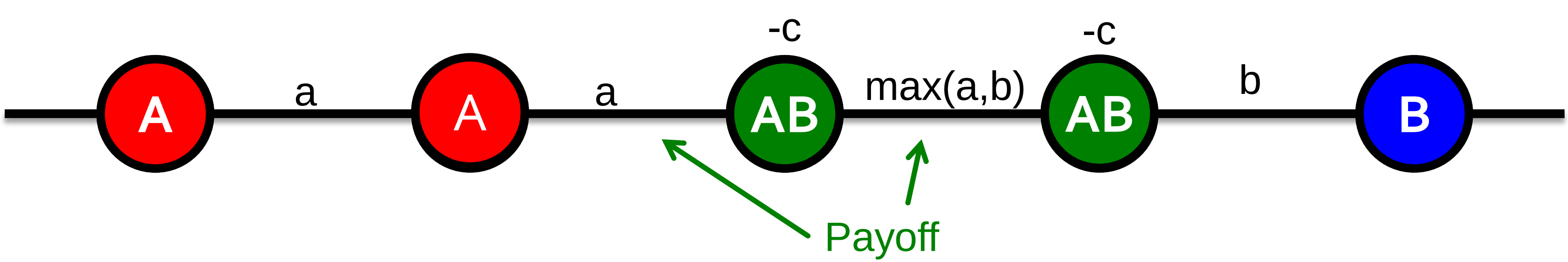
- **So far:**
  - Behaviors ***A*** and ***B*** compete
  - Can only get utility from neighbors of same behavior: ***A-A* get a, *B-B* get *b*, *A-B* get *0***
- **Let an extra strategy "*AB*"**
  - ***AB-A*** : gets ***a***
  - ***AB-B*** : gets ***b***
  - ***AB-AB*** : gets **max(*a*, *b*)**
  - **Also:** Some **cost *c*** for the effort of maintaining both strategies (summed over all interactions)
    - Note: a given node can receive *a* from one neighbor and *b* from another by playing AB, which is why it could be worth the cost *c*

w

v		A	B	AB
	A	a, a	0,0	a, a
	B	0,0	b,b	b,b
	AB	a, a	b,b	max(a,b), max(a,b)

Cascades & Compatibility: Model

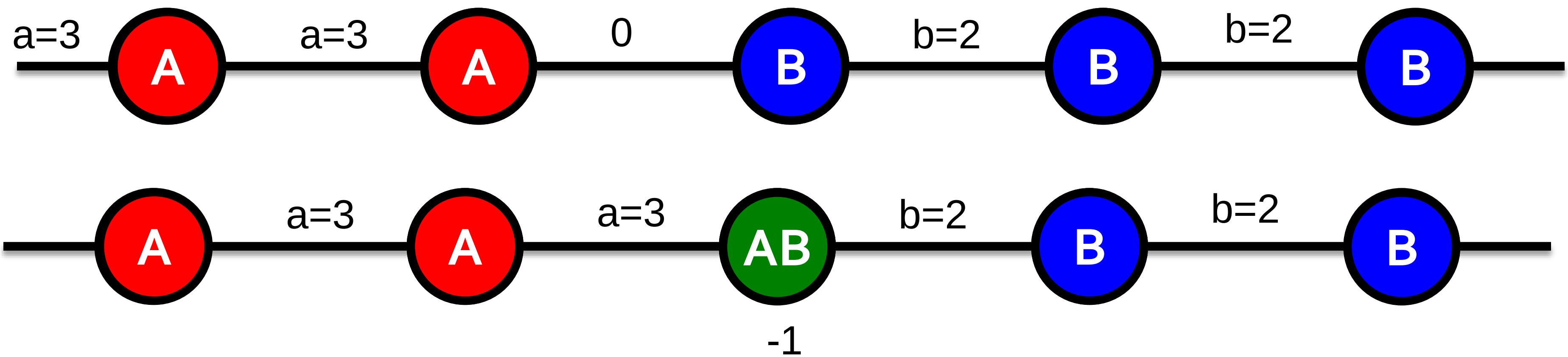
- Every node in an infinite network starts with ***B***
- Then a finite set ***S*** initially adopts ***A***
- Run the model for ***t=1,2,3,…***
  - Each node selects behavior that will optimize payoff (given what its neighbors did in at time ***t-1***)



- **How will nodes switch from *B* to *A* or *AB*?**

Example: Path Graph (1)

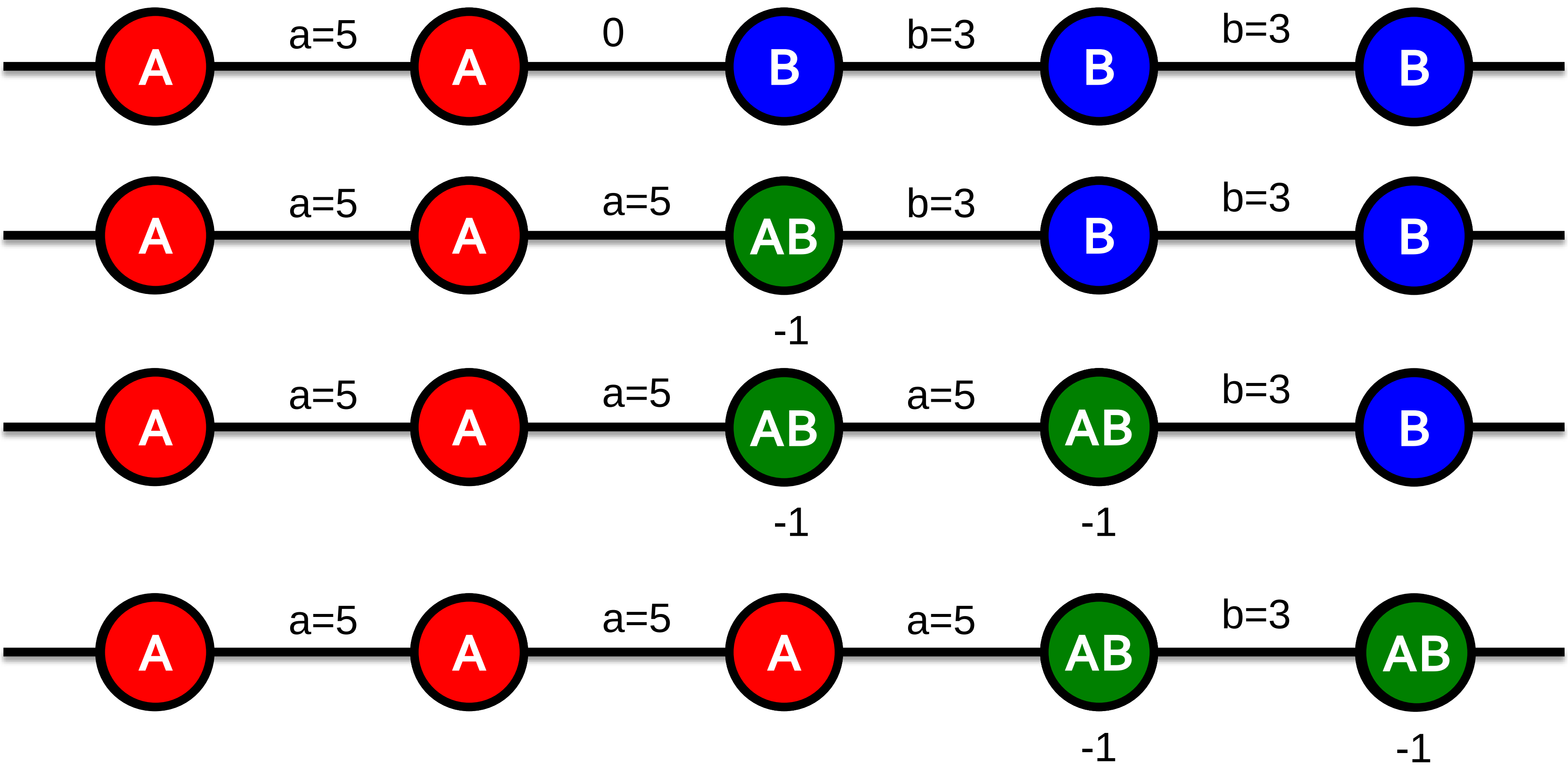
- **Path graph:** Start with all **B**s, **a** > **b** (**A** is better)
- **One node switches to A – what happens?**
  - **With just A, B:** **A** spreads if ***a*** > ***b***
  - **With A, B, AB: Does A spread?**
- **Example: a=3, b=2, c=1**



Cascade stops

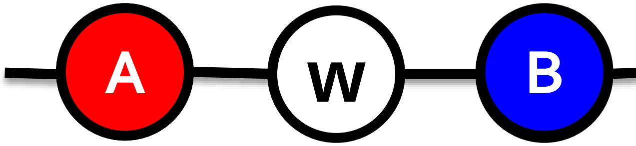
Example: Path Graph (2)

- Example: a=5, b=3, c=1

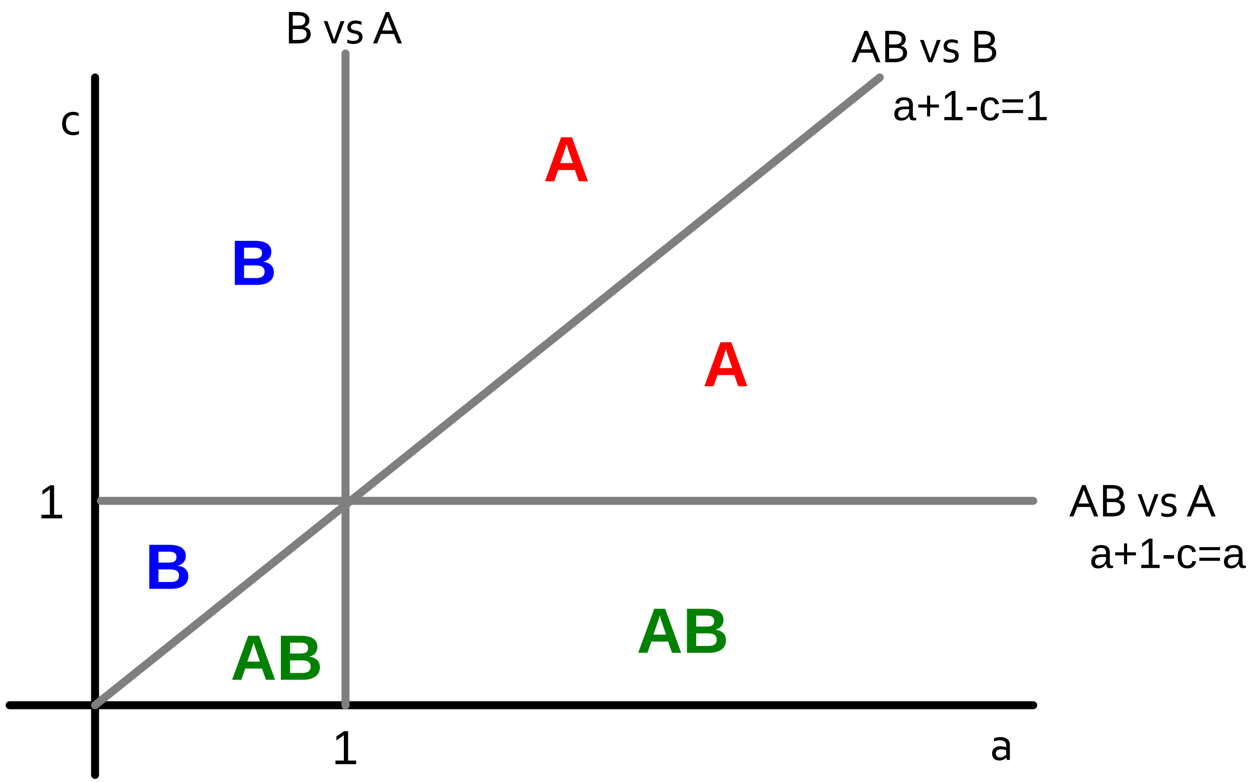


Cascade never stops!

For what pairs (c,a) does A spread?



- **Infinite path, start with all Bs**
- **Payoffs for *w*:** **A**:a, **B**:1, **AB**:a+1-c
- What does node ***w*** in **A-*w*-B** do?



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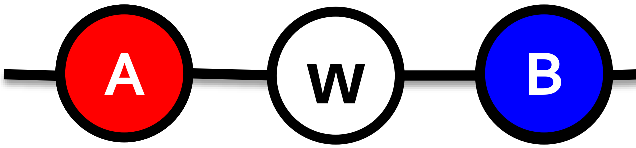

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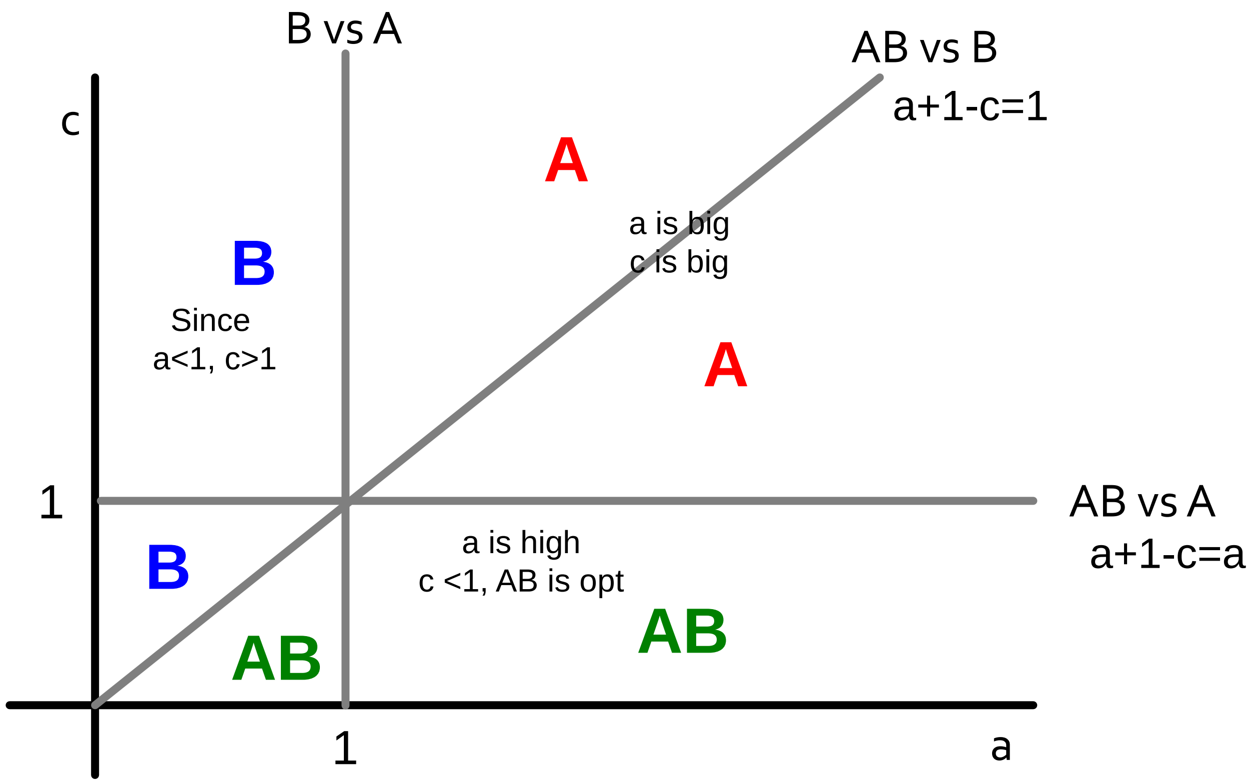
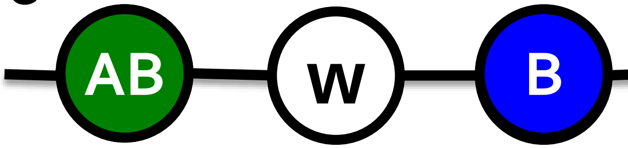
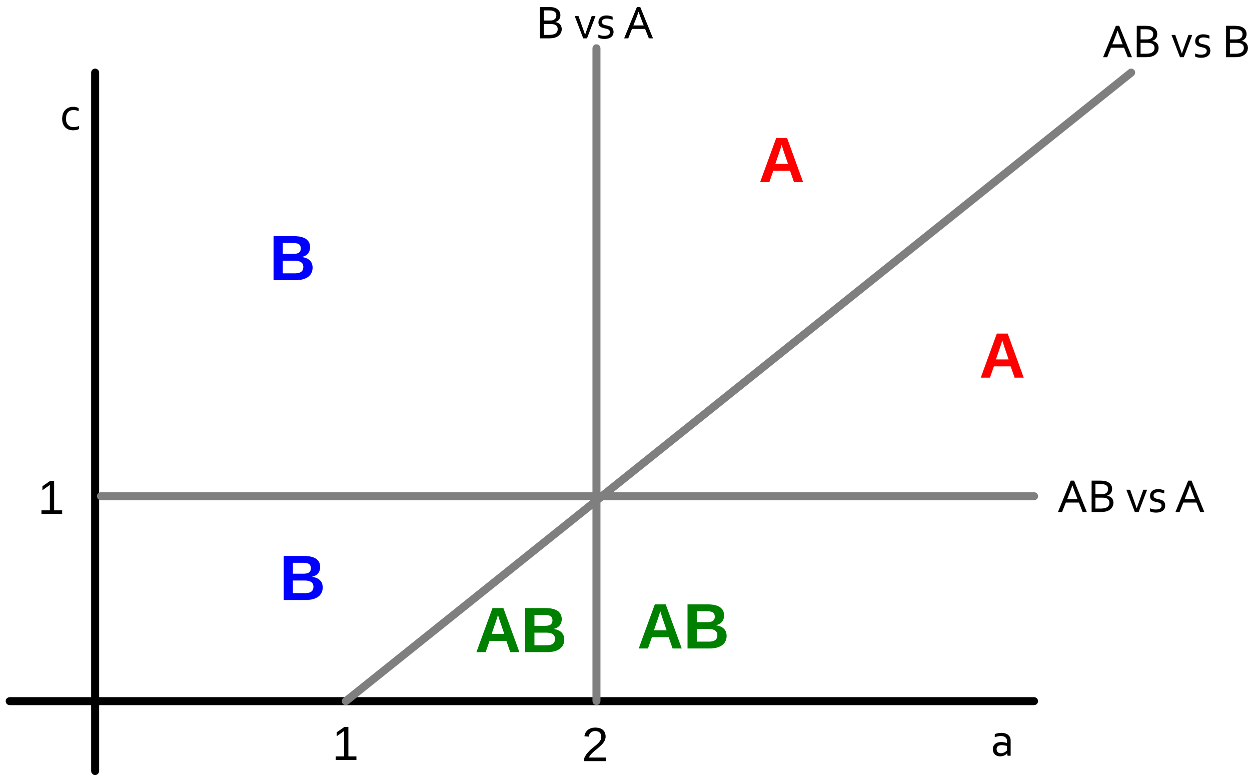


For what pairs (c,a) does A spread?

- **Same reward structure as before but now payoffs for *w* change: A**:a, **B**:1+1, **AB**:a+1-c
- **Notice: Now also AB spreads**
- What does node *w* in ***AB-w-B*** do?





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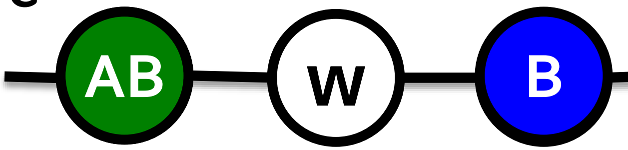
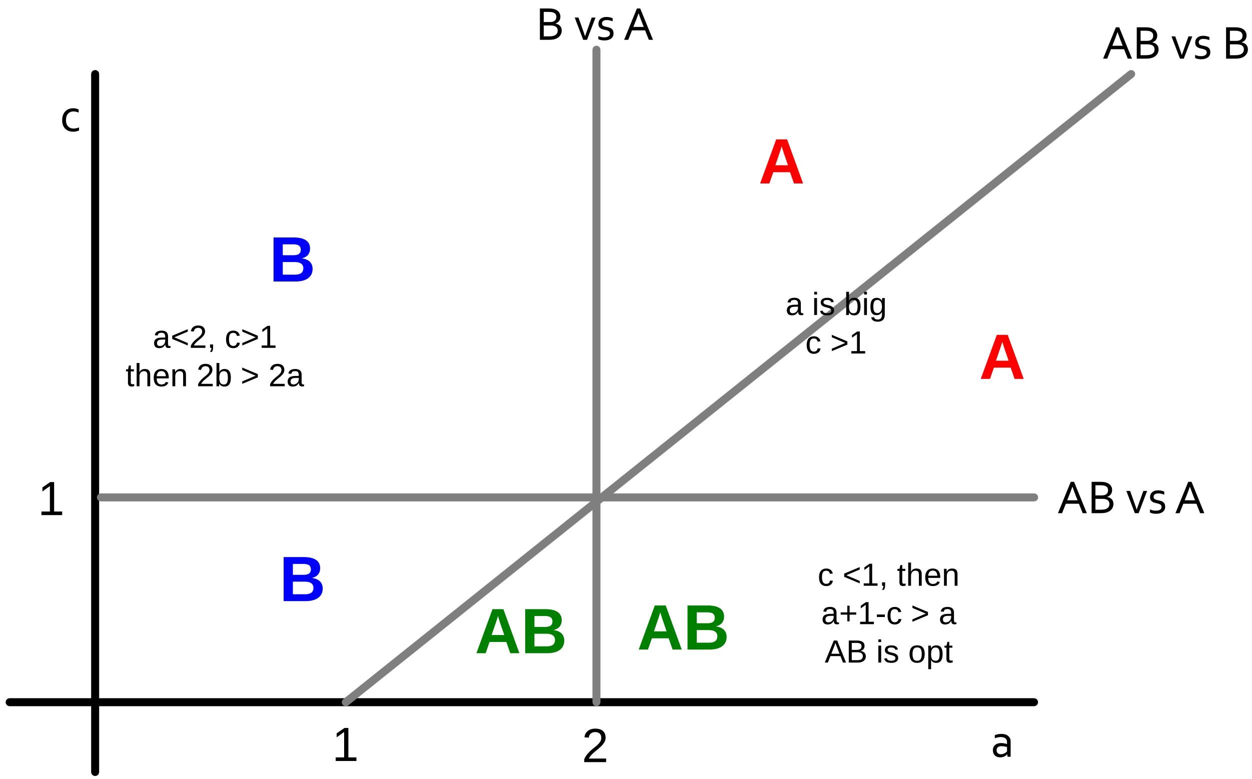
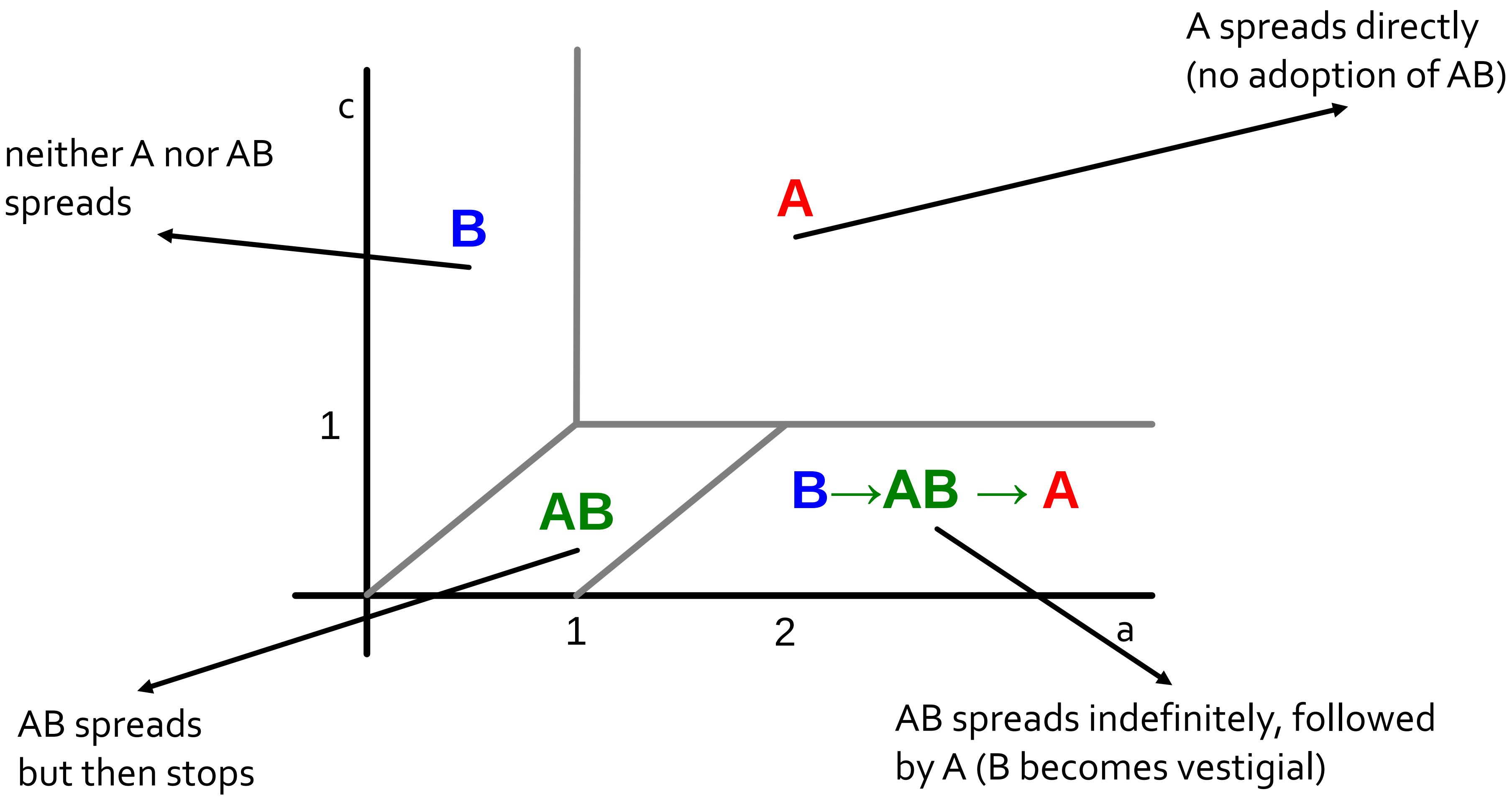
- What does node ***w*** in **AB-*w*-B** do?





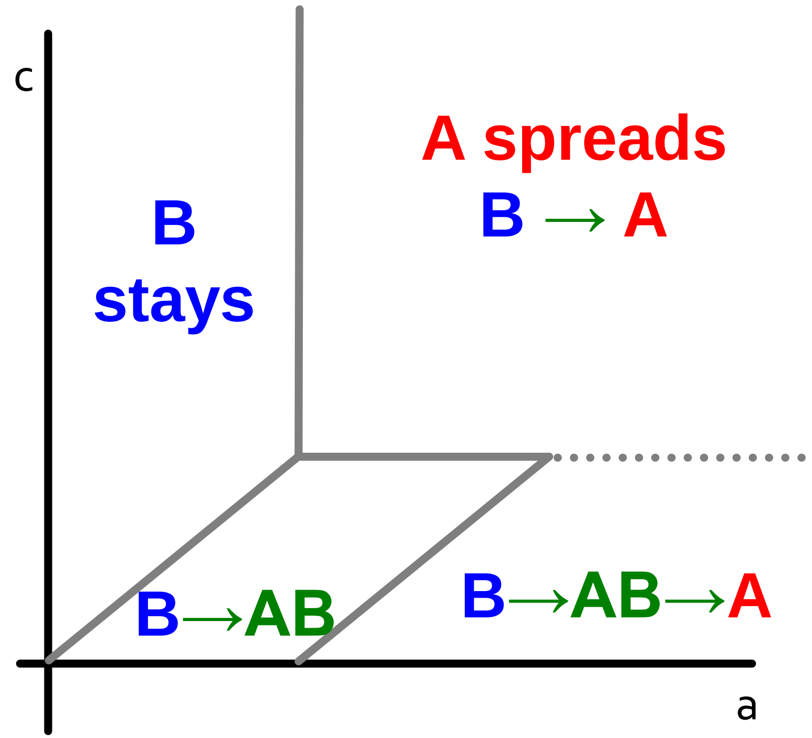
For what pairs (c,a) does A spread?

- **Joining the two pictures:**

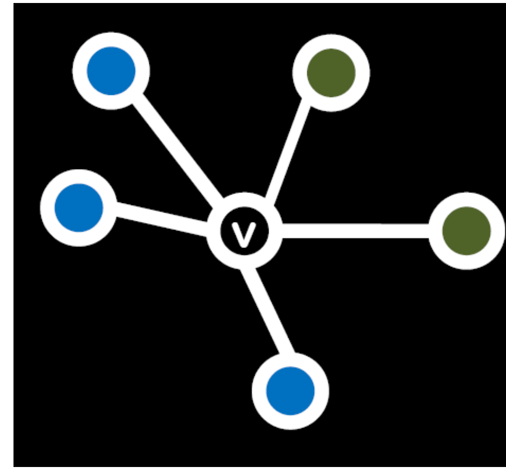


Lesson

- ***B* is the default throughout the network until new/better *A* comes along. What happens?**
  - **Infiltration:** If **B** is too compatible then people will take on both and then drop the worse one **(B)**
  - **Direct conquest:** If **A** makes itself not compatible – people on the border must choose. They pick the better one (**A**)
  - **Buffer zone:** If you choose an optimal level then you keep a static "buffer" between **A** and **B**



Models of Cascading Behavior



- **So far:**
  Decision Based Models
  - Utility based
  - Deterministic
  - "Node" centric: A node observes decisions of its neighbors and makes its own decision
  - Require us to know too much about the data
- **Next: Probabilistic Models**
  - Let's you do things by observing data
  - We lose "why people do things"