EECS 4401/5326 Winter 2022
Week 2 — Additional Examples — 20/01/2022

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Example 1

Consider the knowledge base on slide 33 of the B&L lecture notes

$$S = \{On(a,b), On(b,c), Green(a), \neg Green(c)\}$$

Does $S \models Green(b)$?

Example 1

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No. Prove it.

How?

Example 1

Prove that $S \not\models Green(b)$ by giving an interpretation $\mathcal{I} = \langle D, I \rangle$ and showing that $\mathcal{I} \models S \cup \{\neg Green(b)\}$

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Let $D = \{a_i, b_i, c_i\}$,
$I(a) = a_i, I(b) = b_i, I(c) = c_i$
$I(Green) = \{a_i\}$, and
$I(On) = \{\langle a_i, b_i \rangle, \langle b_i, c_i \rangle\}$.

It is easy to show that $\mathcal{I} \models S \cup \{\neg Green(b)\}$.

There are many other such interpretations.

Example 1

Does $S \models \neg Green(b)$?

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Does $S \models \neg Green(b)$?

No.

To prove it, we can use the same interpretation as above except with $I(Green) = \{a_i, b_i\}$.

Then $\mathcal{I} \models S \cup \{Green(b)\}$.

Example 2

Does $S \models \neg \exists x On(c, x)$?

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Does $S \models \neg\exists x On(c, x)$?

No!

The same interpretation as above but with $\langle c_i, c_i \rangle \in I(On)$ satisfies S and this query.

Can also add d_i to D and add $\langle c_i, d_i \rangle$ to $I(On)$.

Example 2

Let $S' = S \cup \{\forall x \forall y. On(x, y) \supset (x = a \wedge y = b) \vee (x = b \wedge y = c)\}$.

Does $S' \models \neg\exists x On(c, x)$?

Example 2

Let $S' = S \cup \{\forall x \forall y. On(x, y) \supset (x = a \wedge y = b) \vee (x = b \wedge y = c)\}$.

Does $S' \models \neg\exists x On(c, x)$?

No, because nothing ensures that c is not equal to a or b (e.g., we can have $I(c) = a_i$).

Example 2

Let $S'' = S' \cup \{a \neq b \wedge b \neq c \wedge a \neq c\}$.

Does $S'' \models \neg\exists x On(c, x)$?

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Let $S'' = S' \cup \{a \neq b \wedge b \neq c \wedge a \neq c\}$.

Does $S'' \models \neg\exists x On(c,x)$? Yes.

Proof:

Take an arbitrary interpretation $\mathcal{I}$ and assume that $\mathcal{I} \models S''$.

$\mathcal{I} \models \forall x \forall y. On(x,y) \supset (x = a \wedge y = b) \vee (x = b \wedge y = c)$

$\mathcal{I} \models c \neq a$

$\mathcal{I} \models c \neq b$

Thus $\mathcal{I} \models \neg\exists x On(c,x)$?

Therefore $S'' \models \neg\exists x On(c,x)$?.

Exercise 1

Write a sentence in FOL that represents the following knowledge:

Every proferssor who is not member of any committee is happy.

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Write a sentence in FOL that represents the following knowledge:

Every proferssor who is not member of any committee is happy.

$$\forall x. Prof(x) \wedge \neg\exists y. Member(x,y) \supset Happy(x)$$

or

$$\forall x. Prof(x) \wedge \neg\exists y. Committee(y) \wedge Member(x,y) \supset Happy(x)$$

or

$$\forall x. Prof(x) \wedge (\forall y. Committee(y) \supset \neg Member(x,y)) \supset Happy(x)$$

Exercise 2

Write a sentence in FOL that represents the following knowledge:

There is a team member who likes every team member except Mary.

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Write a sentence in FOL that represents the following knowledge:

There is a team member who likes every team member except Mary.

$$\exists x.Member(x) \land \forall y.Member(y) \land \neg y = mary \supset Likes(x, y)$$